MAE 473-573: Lecture #12 - Projection and Perspective

Lecture Overview:

- Miscellaneous
- Motivation
- Projection - basics
- Means for projecting images:
  - Orthographic viewing - basics
  - Perspective viewing - basics
- The mathematics of projection
- Vanishing points
- Numerical examples

Miscellaneous:

X Programming Reference:

"Xlib Programming Manual, Volume 1", Adrian Nye, O'Reilly & Associates, Inc., 1995 (last printing)

Homogeneous coordinates:
- more extensive review forthcoming
- review lectures #5 and #9 for "need to know" details

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- Motivation:

- 2D and 3D scenes must be transferred to a 2D object….
- ...Our display device - <u>the computer screen</u>!!

Must consider……….

- Specification of viewing parameters
- Level-of-Detail required on viewed model
- Is distortion of image geometry tolerable? (i.e. *perspective*)
- Clipping/windowing - what is the view volume?

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- Projection - basics
  - DEF: Transformation of points in a coordinate system of dimension n, into a coordinate system of dimension less than n. (our concern: 3D to 2D)
  - DEF: *Foreshortening* - visual effect of a perspective projection
  - Basic classes of projection: *Parallel* and *Perspective*

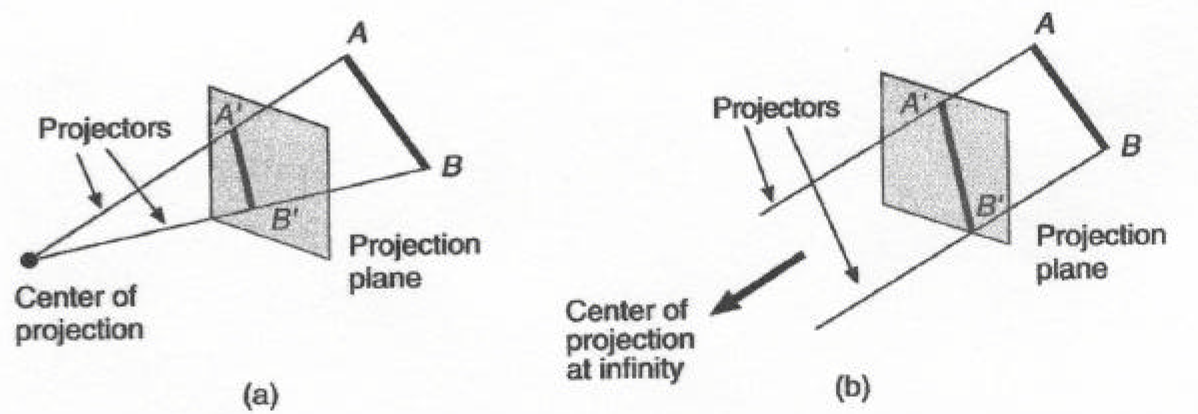

Figure 6.3 Two different projections of the same line. (a) Line AB and its perspective projection $A'B'$. (b) Line AB and its parallel projection $A'B'$. Projectors AA' and BB' are parallel.

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- Means for projecting images

Flowchart of Geometric Projection types:

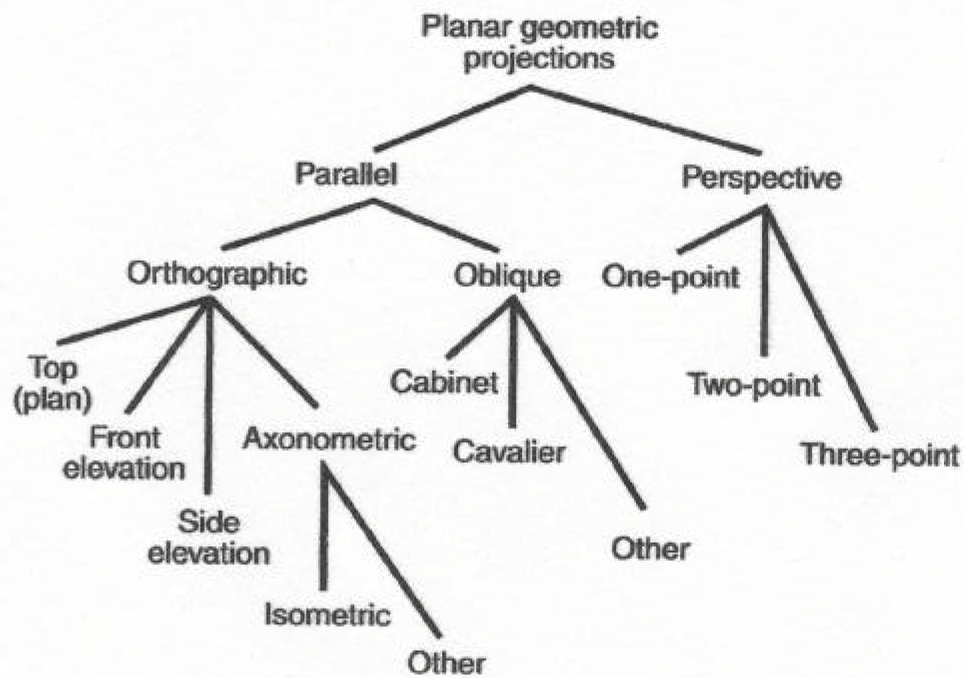

Figure 6.10 The subclasses of planar geometric projections. **Plan view** is another term for a top view. **Front** and **side** are often used without the term **elevation**.

- Parallel Projections:

Orthographic - Direction of projection, and the normal to the projection plane are in same direction (...unlike *Oblique* projection)

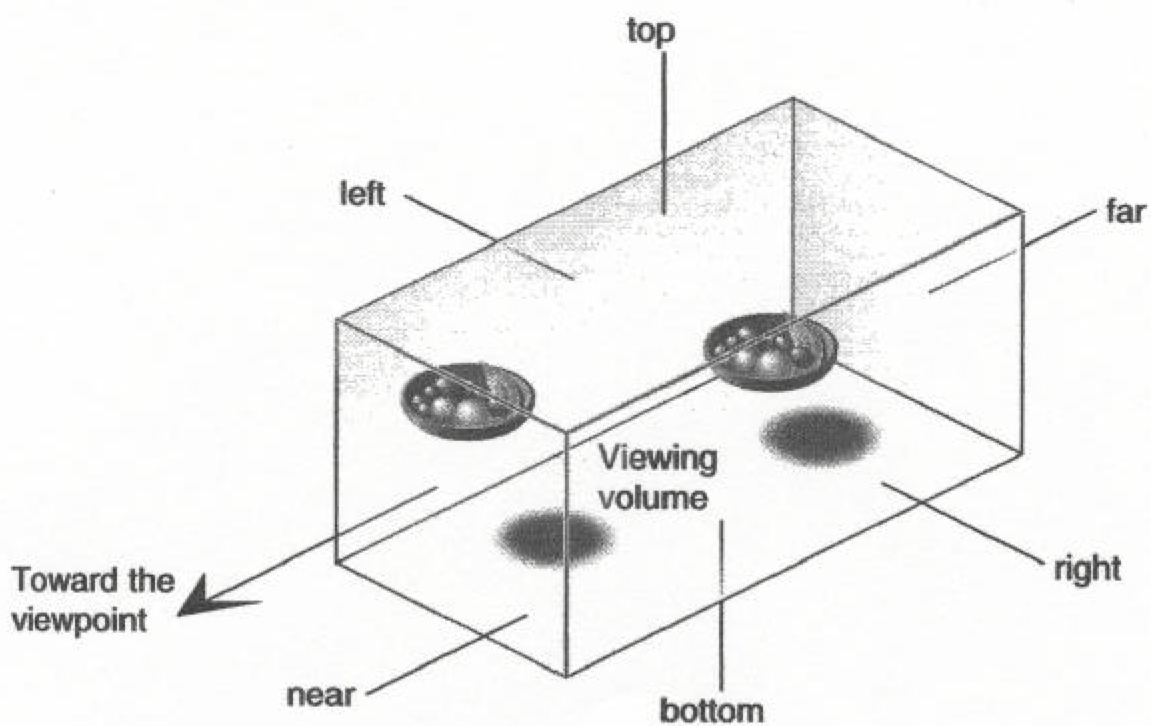

Figure 3-15 Orthographic Viewing Volume

Orthographic parallel projection (cont.)

- Common types: Top, front, side, isometric elevation
- Projection plane is perpendicular to direction of projection
- Typically used in engineering drawings
- Distances and angles can be accurately measured
- Drawback: each projection results in *one face* of an object

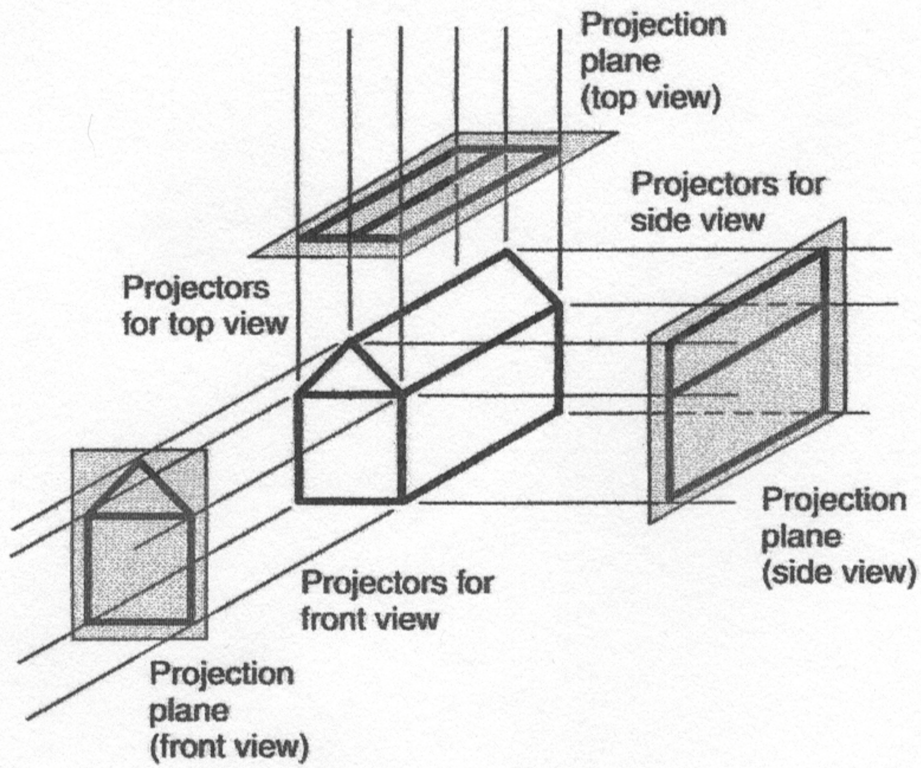

Figure 6.7 Construction of three orthographic projections.

Axonometric orthographic projection:

- Projection plane NOT normal to the direction of projection

- Result: several faces shown at once

- Resemble perspective projection, only foreshortening is uniform

- Isometric - most common - projection plane normal makes <u>equal angles</u> with each principal (x,y,z) axis

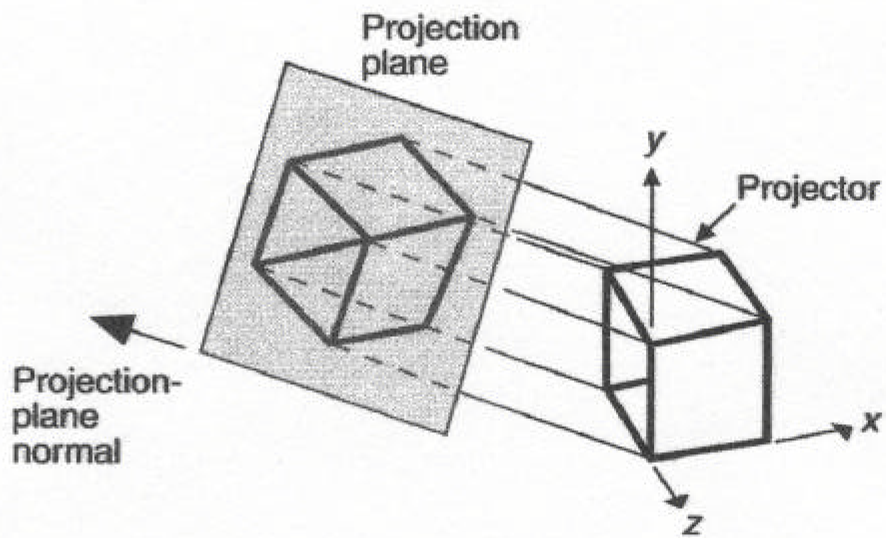

Figure 6.8 Construction of an isometric projection of a unit cube. (Adapted from [CARL78], Association for Computing Machinery, Inc.; used by permission.)

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• Perspective projections: Have finite center of projection, and vanishing point(s)

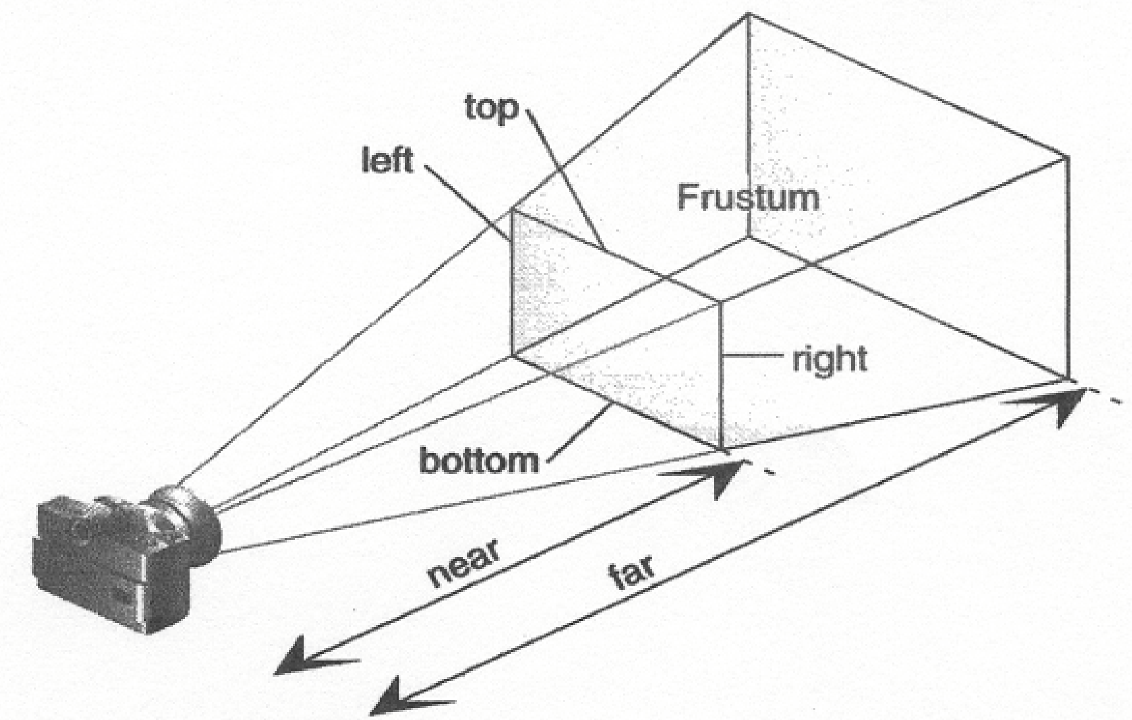

Figure 3-13 Perspective Viewing Volume Specified by glFrustum()

DEF: *Vanishing point* - The perspective projections of any set of parallel lines NOT parallel to the projection plane converge to a finite *vanishing point (VP)*.

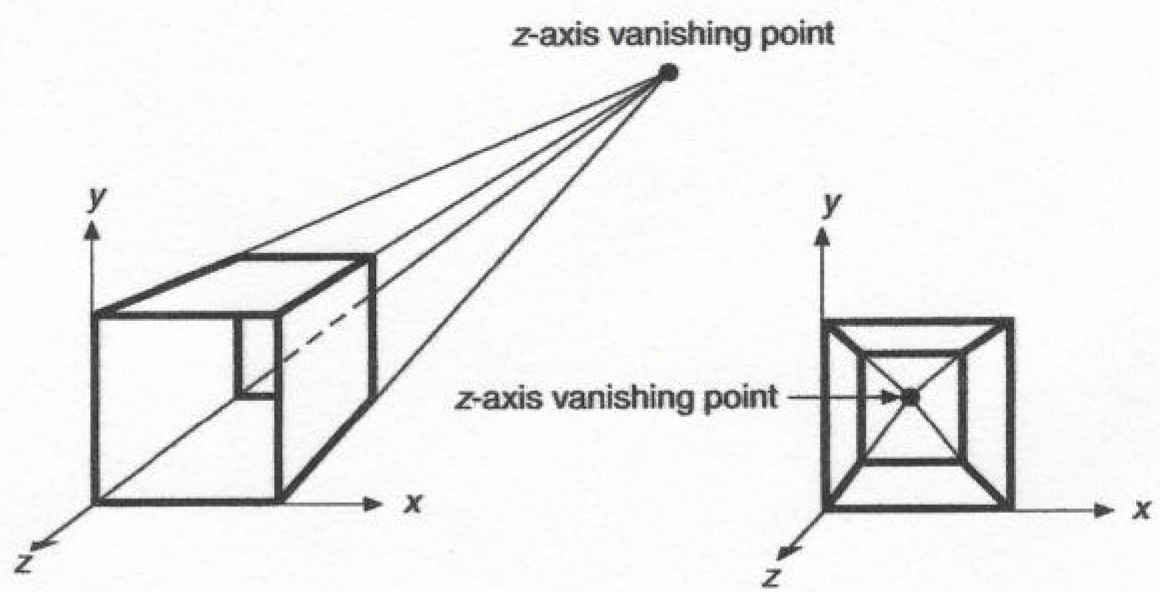

Figure 6.4 One-point perspective projections of a cube onto a plane cutting the z axis, showing vanishing point of lines perpendicular to projection plane.

- One, Two, Three point perspective
  - Governed by the number of VP's
  - There can be at most 3 VP's (x,y,z)

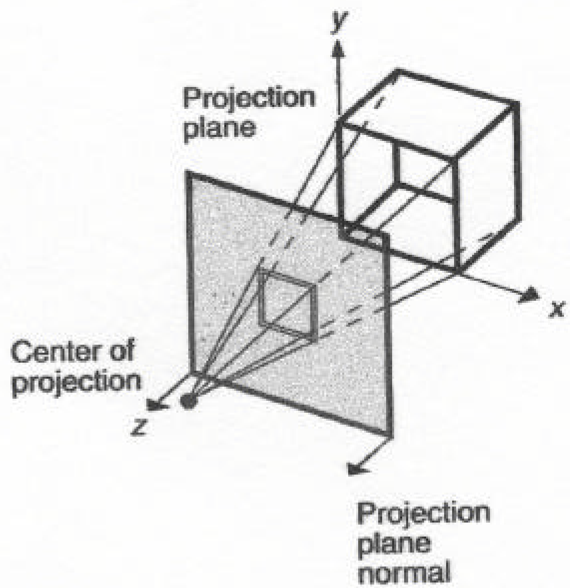

Figure 6.5 Construction of one-point perspective projection of cube onto plane cutting the z axis. The projection-plane normal is parallel to z axis. (Adapted from [CARL78], Association for Computing Machinery, Inc.; used by permission.)

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- 2-point perspective commonly used in architecture/industrial design
- 3-point perspective rarely used; not much more realistic than 2 VP's

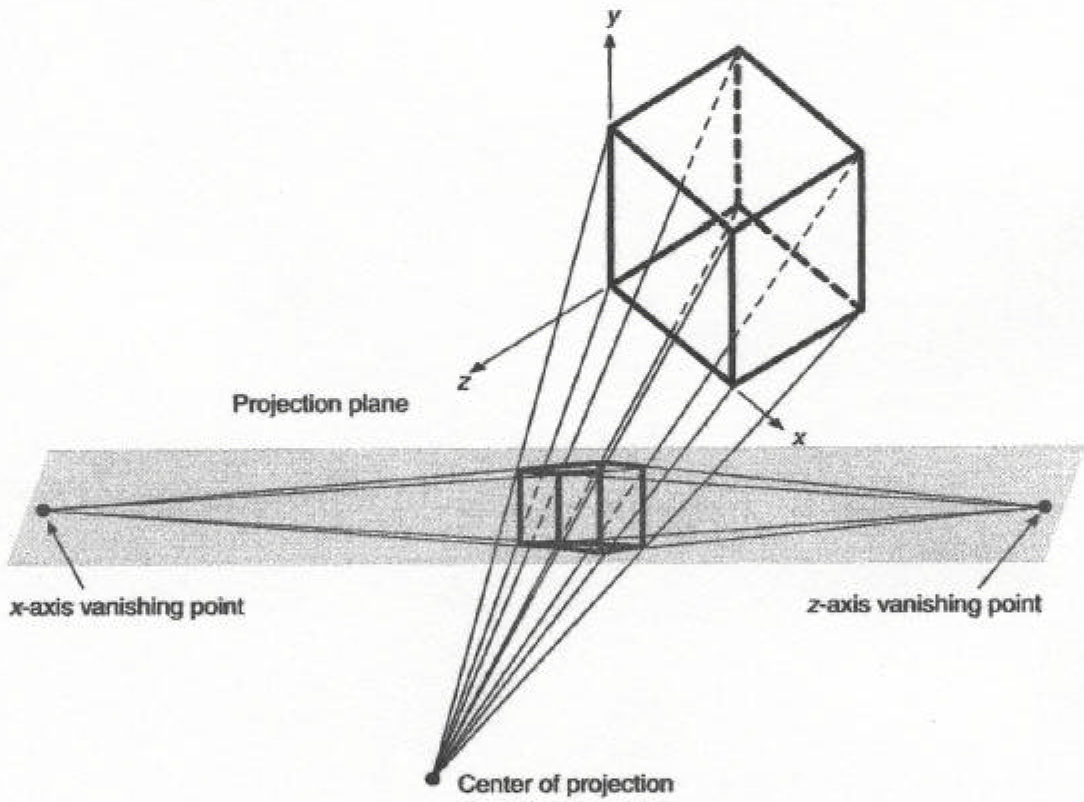
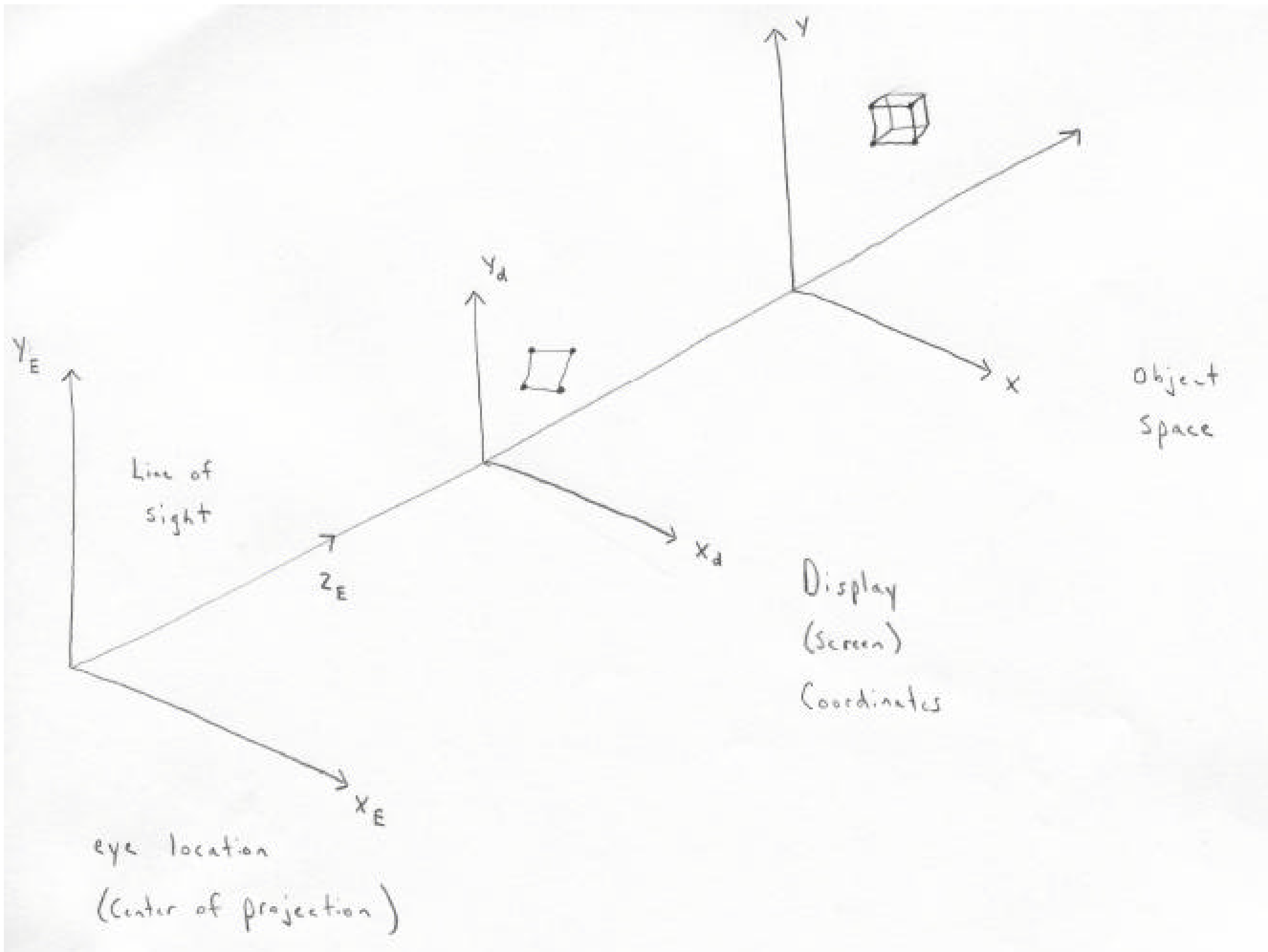


Figure 6.6 Two-point perspective projection of a cube. The projection plane cuts the *x* and *z* axes.

- The mathematics of projections - Coordinate system definition:



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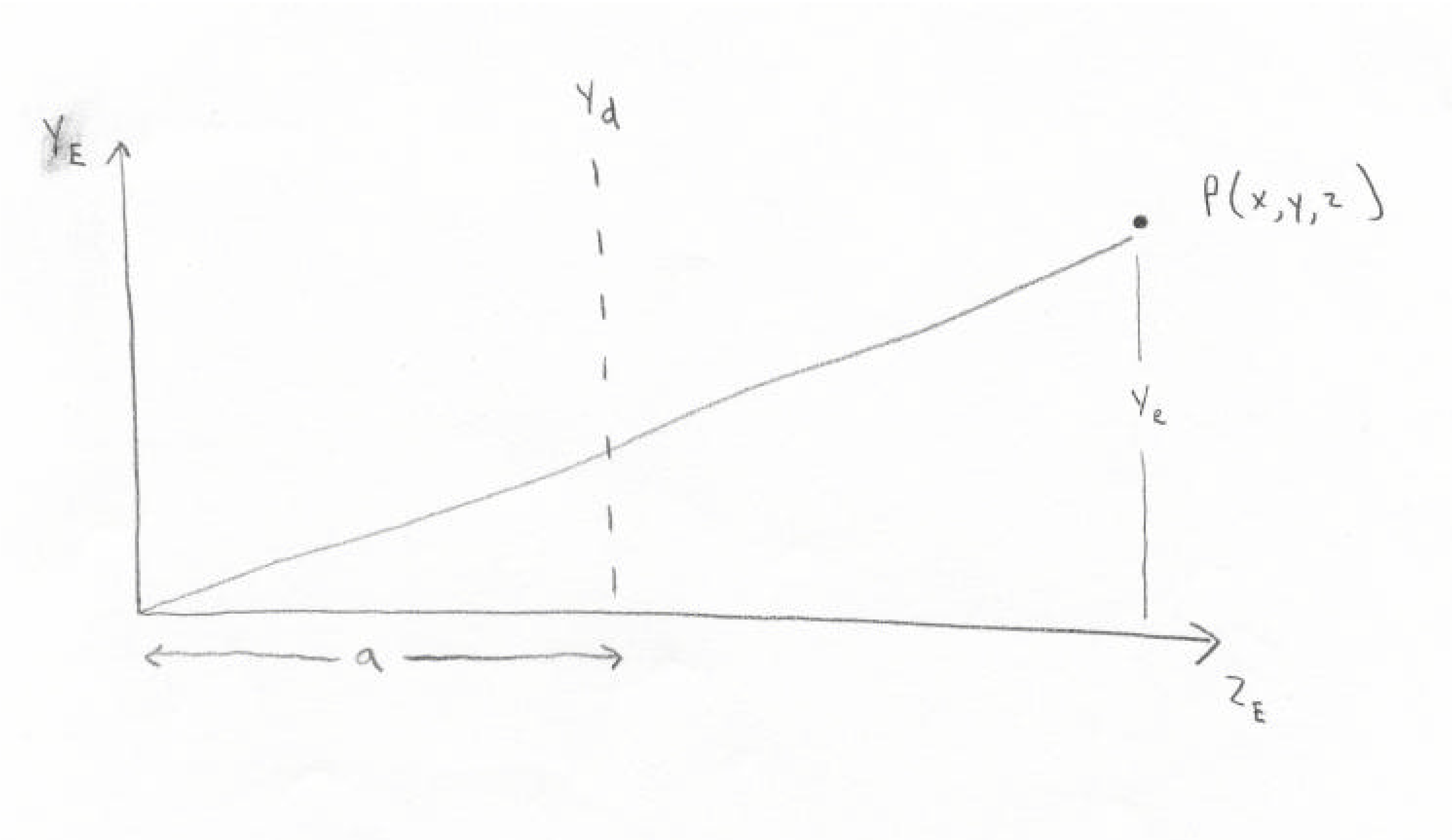
Projection to 2D is as follows:

- For simplicity, assume Projection plane is normal to z-axis (at z = a)
- Observing the y-coordinate projection:



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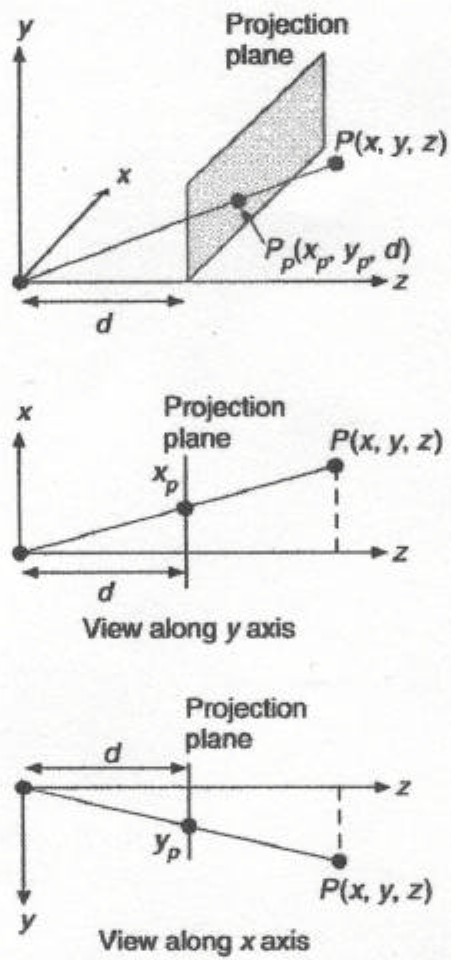
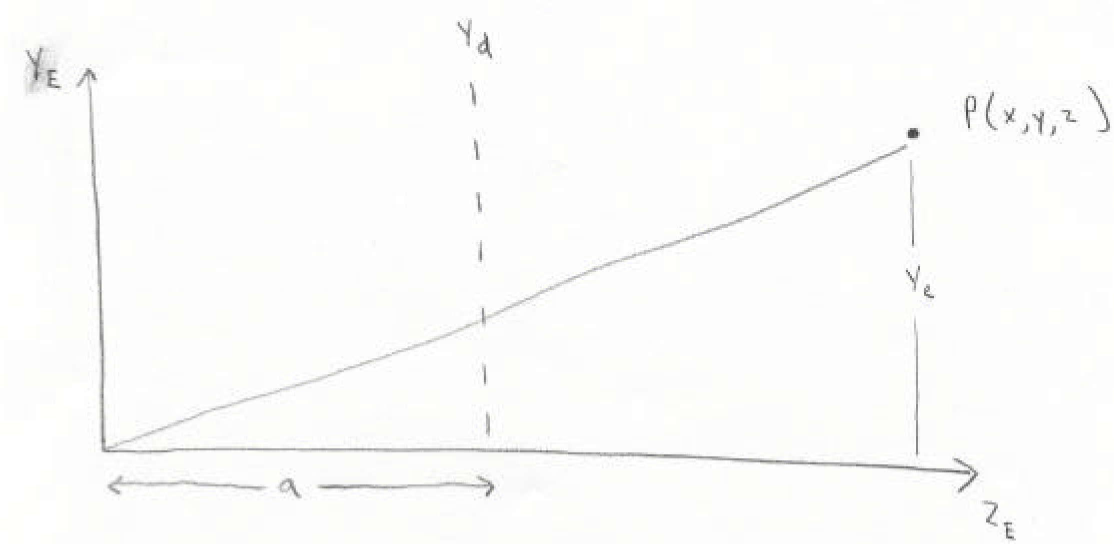


Figure 6.31
Perspective projection.



Using proportions: $y_d/a = y_e/z_e$; $y_d = (a/z_e)^* y_e$

Similarly, $x_d/a = x_e/z_e$; $x_d = (a/z_e)^* x_e$

Notes:

- a: distance from eye (camera) to display screen

- z_e: "line of sight" distance from eye to object coordinates
  Causes the perspective projection of more distant objects to be smaller than that of closer objects (i.e. adds perspective)

- (x_d, y_d) may have to be "windowed" to fit on screen

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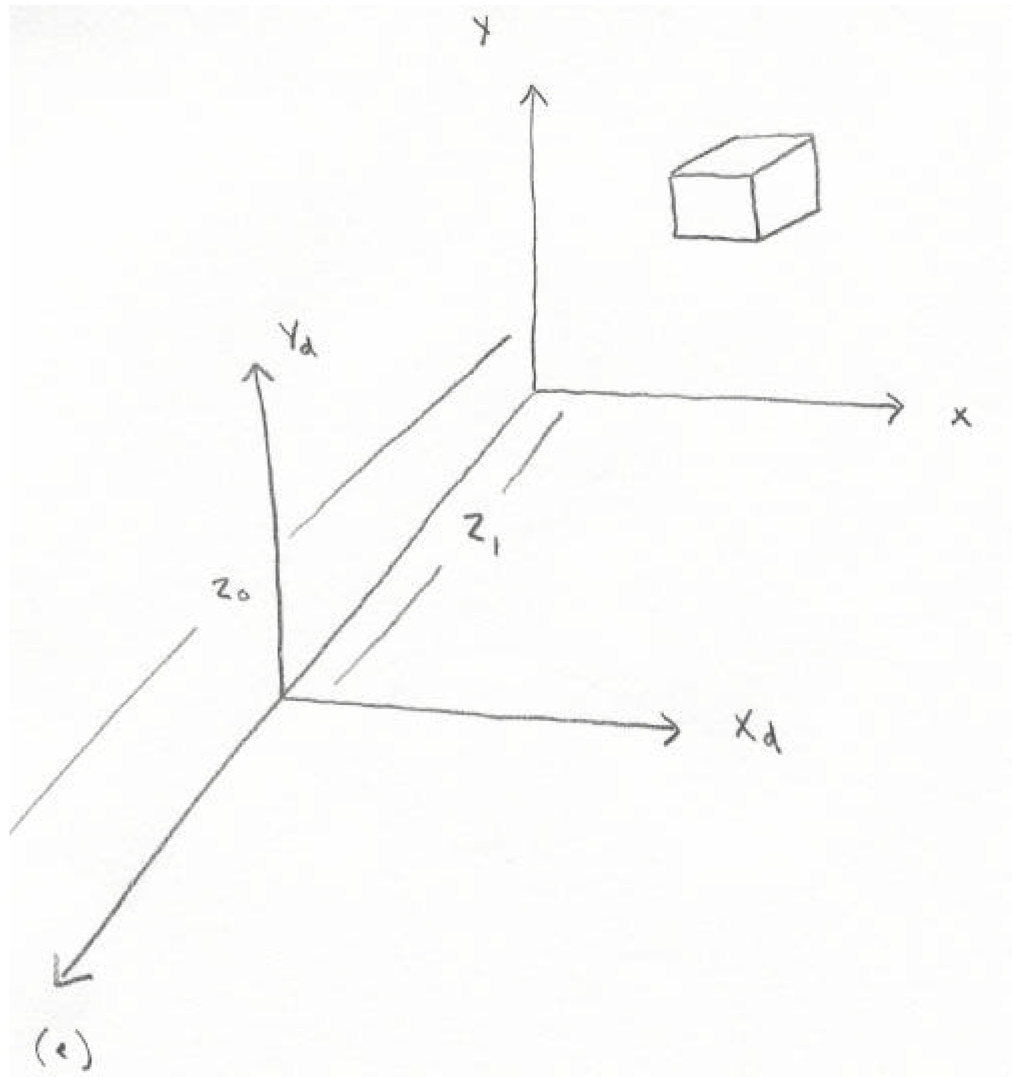
Central projection:

Viewing assumed to take place along the z-axis of the object coords.



Using simple proportions:

$$y_d/(z_0-z_1) = y/(z_0-z)$$

$$x_d/(z_0-z_1) = x/(z_0-z)$$

Notes:

- Eye is located at $(0, 0, z_0)$
- Display coordinates (x_d, y_d) are located at z_1 along the z-axis
- Object must be displayed/oriented as desired (in object space) when projection transforms take place….
- Distortion can occur if center of projection is too close to the object

A Matrix Viewing Transform:

$$\begin{bmatrix} x' \\ y' \\ z' \\ H \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -r & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \\ 1-rz \end{bmatrix}$$

Normalize to get:

$$= \begin{bmatrix} \frac{x}{1-rz} \\ \frac{y}{1-rz} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_d \\ y_d \\ 0 \\ 1 \end{bmatrix}$$

In Terms of Central Projection:

Recall: $y_d/(z_0-z_1) = y/(z_0-z)$
$x_d/(z_0-z_1) = x/(z_0-z)$

Assume $z_1 = 0$ (Display plane and object plane are <u>one and the same</u>)

Comparing this to:

$$= \begin{bmatrix} \frac{x}{1-rz} \\ \frac{y}{1-rz} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_d \\ y_d \\ 0 \\ 1 \end{bmatrix}$$

Leads to: $r = 1/z_0$

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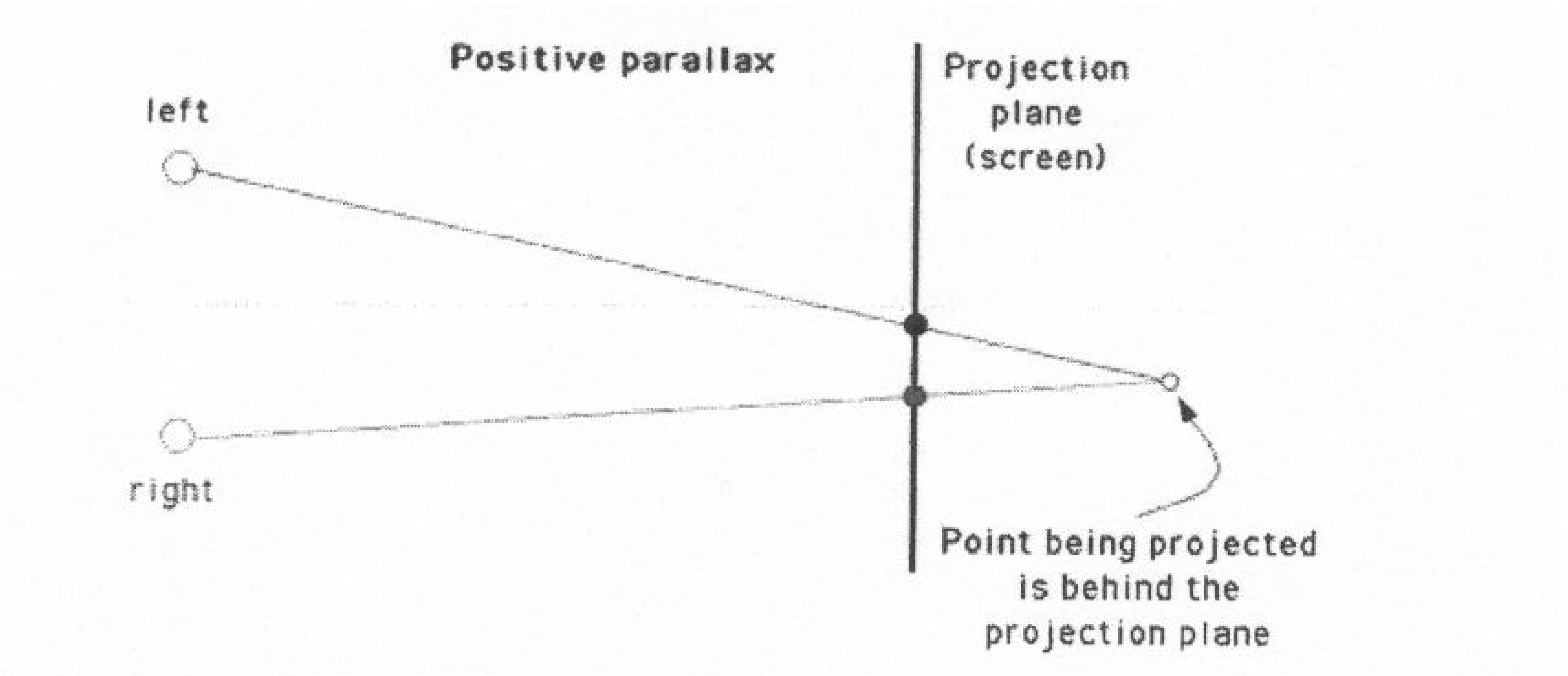
Question from last class - “What if the object is IN FRONT of the projection plane?”

Brings up the concept of parallax:



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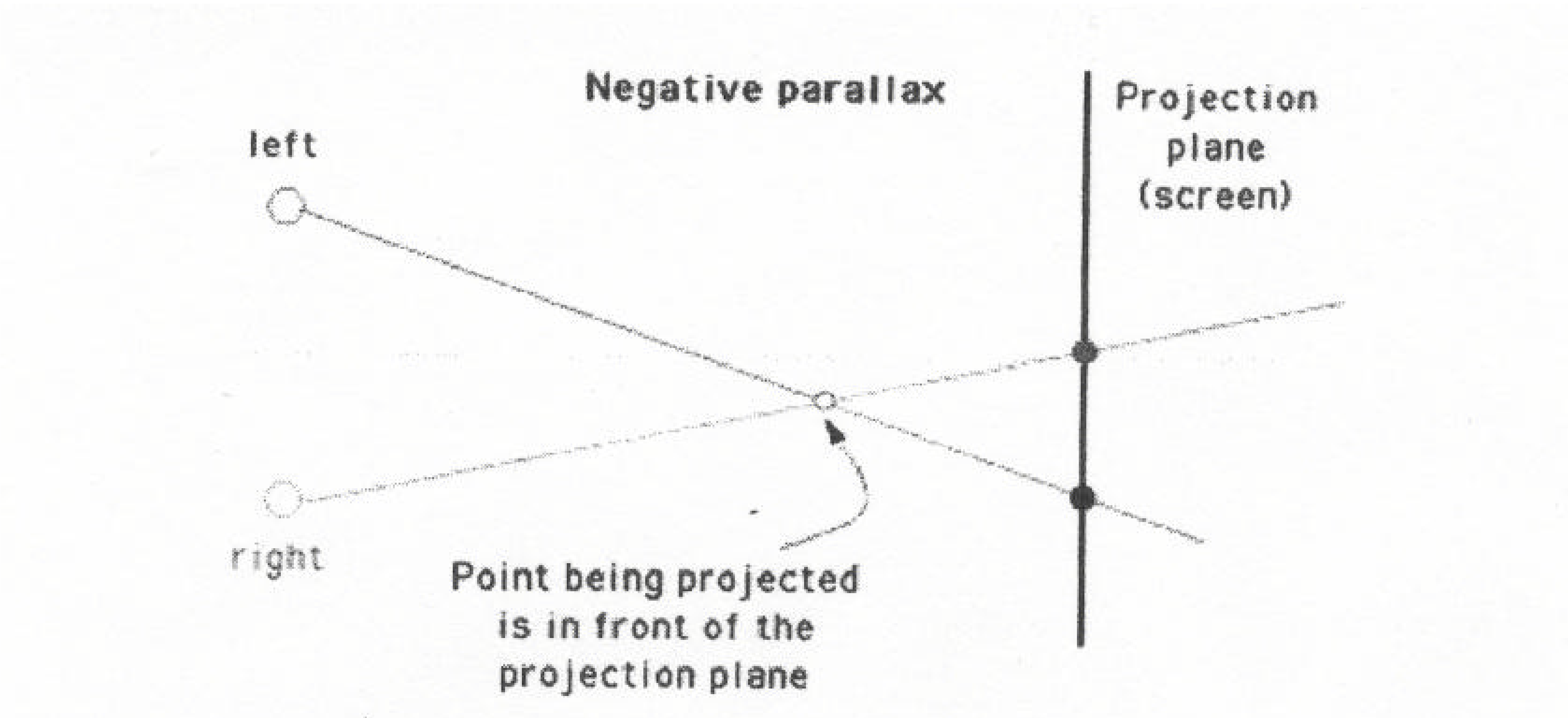

We'll see more of this when we talk about <u>stereo</u>!!

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Vanishing point determination:

One-point perspective

As z approaches ∞, x_d and y_d approach 0.

X_{vp}, $Y_{vp} = 0$; $Z_{vp} = -1/r$

What about Multi-point perspective?

Important! <u>Multiple faces</u> of an object must be viewed for projection to be useful! To accomplish this, employ:

a. single-point projection, AND (preceded by)

b. a translation(s) or rotation(s) of the object

Multi-point perspective (cont.)

Two-point perspective

For example, lets perform a rotation about the y-axis in addition to our z-plane projection transformation:

$$\begin{bmatrix} x' \\ y' \\ z' \\ H \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -r & 1 \end{bmatrix} * \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Multi-point perspective (cont.)

$$= \begin{bmatrix} x\cos\theta + z\sin\theta \\ y \\ 0 \\ 1 + rx\sin\theta - rz\cos\theta \end{bmatrix} = \begin{bmatrix} x_d \\ y_d \\ 0 \\ 1 \end{bmatrix}$$

Normalized…….

$$= \begin{bmatrix} \dfrac{x\cos\theta + z\sin\theta}{1 + rx\sin\theta - rz\cos\theta} \\ \dfrac{y}{1 + rx\sin\theta - rz\cos\theta} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_d \\ y_d \\ 0 \\ 1 \end{bmatrix}$$

So - what are the Vanishing points?

As x approaches infinity:

$x_d = \cos\Theta / r \sin\Theta$
$y_d = 0$

As z approaches infinity:

$x_d = -\sin\Theta / r \cos\Theta$
$y_d = 0$

$$Z_{vp} = -1/r$$

Three-point perspective

Employ a z-plane perspective transformation, augmented by TWO successive rotations………...

- Examples (handouts)
  - Single-point Perspective
  - Two-point Perspective