

MAE 552

Heuristic Optimization

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Lecture #36

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Multi-Objective Optimization

Multi-Objective Optimization

References: General

<http://www-fp.mcs.anl.gov/otc/Guide/OptWeb/multiobj/>

<http://www.caam.rice.edu/~indra/NBIhomepage.html>

Multi-Objective Optimization

Last time, we talked about multi-criteria optimization.

We discussed the most common method of solution being a linear convex combination of objectives using weights.

We also discussed many of the drawbacks associated with this approach.

Today, we will look at some other techniques.

Multi-Objective Optimization

Let us briefly describe what we would like to see in a Pareto frontier.

- 1 – Even or uniform sampling.
- 2 – Wide expanse.
- 3 – Distinctness of points. That is, 2 points are not distinct if they are so close to one another that the designer does not care to distinguish between them.

Multi-Objective Optimization

Let's first restate the optimization problem:

$$\min F(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_n(x) \end{bmatrix}, \quad n \geq 2$$

where $C = \{x : h(x) = 0, g(x) \leq 0, x_l \leq x \leq x_u\}$

Multi-Objective Optimization

Normal Boundary Intersection method (Dennis and Das):

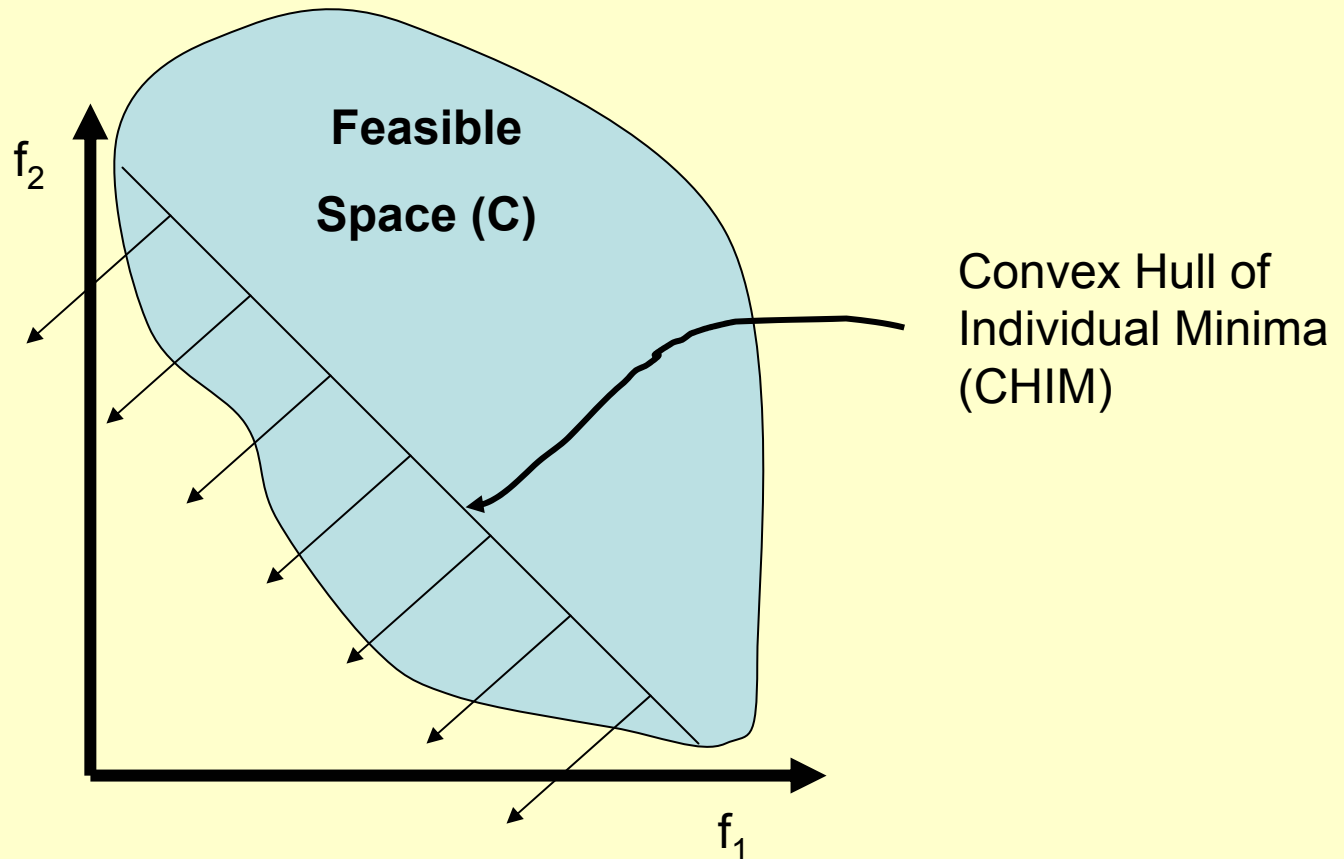
Properties:

1. Extendable to any number of objective functions.
2. Produces an even spread of Pareto Points.
3. Requires first minimizing each function individually.
4. Requires repeated solution of the NBI subproblem.

Multi-Objective Optimization

General Implementation: Consider the following Animation for a 2D example:

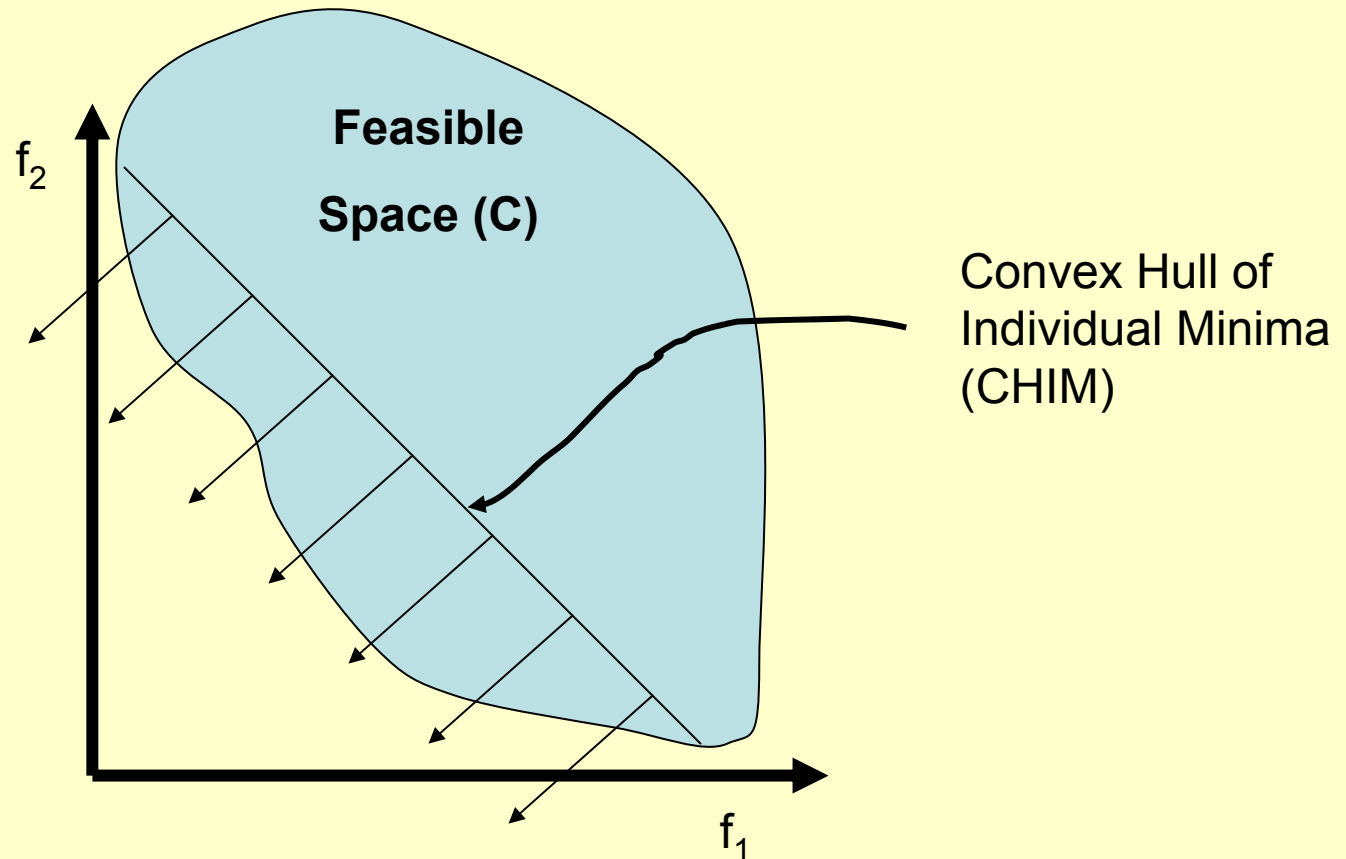
Use the NBI subproblem to find the intersections of each normal vector and the efficient frontier.



Multi-Objective Optimization

General Implementation: Consider the following Animation for a 2D example:

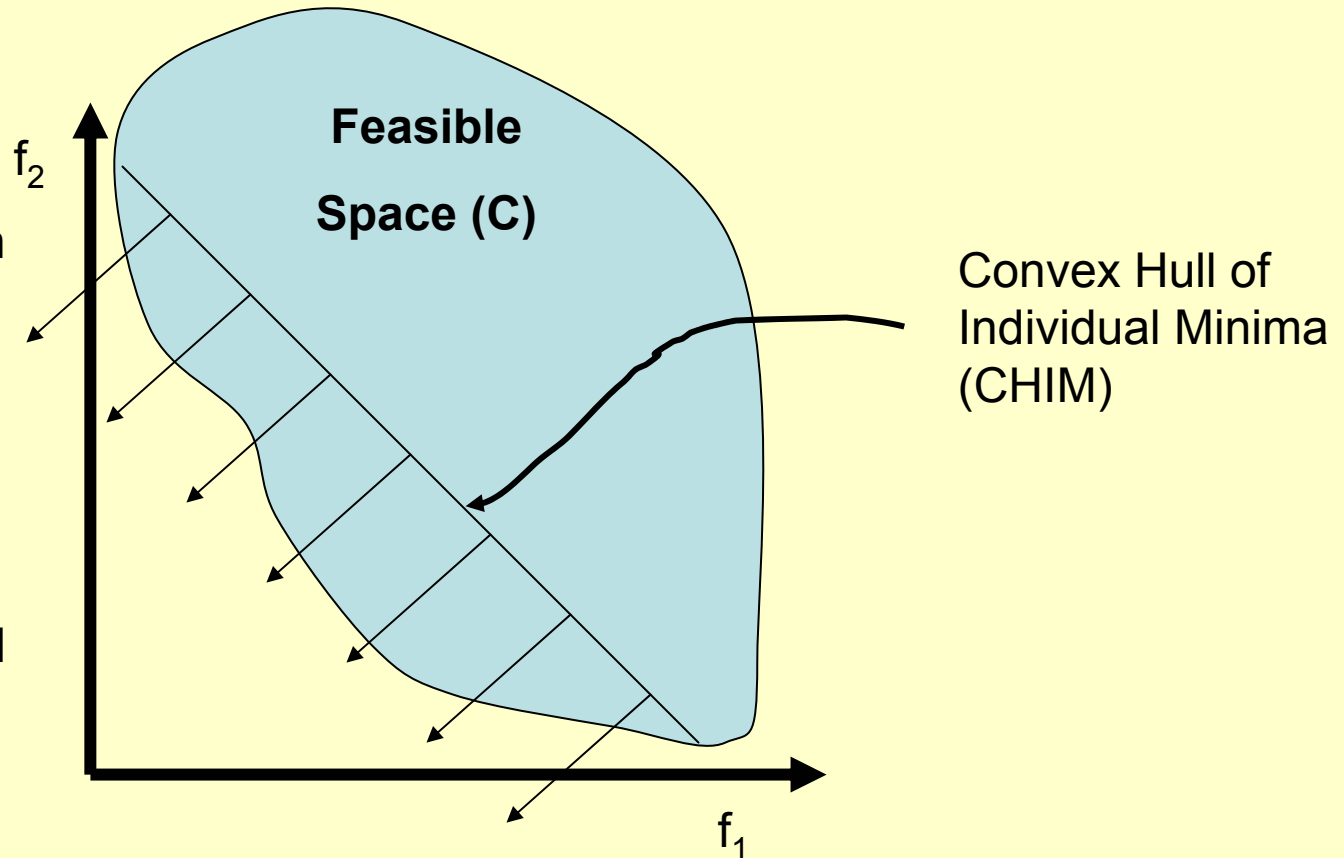
The vector parameter β in the subproblem defines the take-off points.



Multi-Objective Optimization

General Implementation: Consider the following Animation for a 2D example:

This approach has the advantage that an even spread of β 's produces an even spread of points on the CHIM which in turn corresponds to an even spread of Pareto points.



Multi-Objective Optimization

The subproblem is defined as follows:

$$\begin{aligned} & \max_{x,t} t \\ & s.t. \Phi\beta + t\hat{n} = F(x) - F^* \\ & \quad x \in C \end{aligned}$$

Terms defined on next page.

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C we showed as the feasible space.

x_i^* is the optimal design configuration for objective i .

f_i^* is the optimum value of objective i ($= f_i(x_i^*)$).

F* is the vector of individual objective optimums
(denotes the utopia point).

Φ is $n \times n$ payoff matrix such that each column is given by:

$$F_i^* - F^*$$

So Φ shifts the axes in the performance space such that the utopia point is at the origin.

Multi-Objective Optimization

So anyway, NBI works very well for the case of 2 objectives but may potentially be unable to find certain points in 3 or more dimensions termed “peripheral points”.

For more information, see:

- I. Das and J. E. Dennis. Normal-Boundary Intersection: An Alternate Approach for Generating Pareto-optimal Points in Multicriteria Optimization Problems. ICASE-NASA Tech. Report 96-62. Submitted to SIAM J. on Optimization, July 1996.

Multi-Objective Optimization

Goal Programming:

Properties:

1. Extendable to any number of objective functions.
2. Not useful for populating the Pareto frontier.
3. Requires relatively intimate knowledge of desired values for various objectives.

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Essentially, for each objective, the user must set one of the following:

A maximum allowable value,

A minimum allowable value,

A desired range for the value.

The job is then to minimize the deviation of each objective from its goal.

Multi-Objective Optimization

So if all objectives are minimization, the goal approach limits the value to some ceiling and allows it to be less than that ceiling.

So this method is not guaranteed to find a Pareto solution.

Nor is it useful to populate the Pareto frontier.

It is useful to find the design parameters that will satisfy a given set of performance values.

Multi-Objective Optimization

There are some other less interesting but common approaches to multi-criteria optimization. You can read about some of them on the reference web pages.

Does anyone know which of the methods we discussed this semester might be well suited to solution of this type of problem?

(Recall that we are seeking a group of solutions).

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Recently, much attention has been paid to the usefulness of Genetic programming for solution to these problems.

How might we adapt our genetic algorithm to deal with multiple objectives?

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Recall the progression of our algorithm as follows:

```
Initialize Population.  
Evaluate Population.  
t = 0  
while(not converged)  
    t = t + 1;  
    Variation of P(t)  
    Evaluation of P(t)  
    Select P(t) from P(t-1)  
end
```

Multi-Objective Optimization

So what phase of this algorithm must be adjusted to accommodate multiple objectives?

Clearly, the evaluation phase must take into account Pareto optimality.

There are a number of ways to account for Pareto optimality in a GA. Some of which involve using a fitness parameter and some of which do not.

Multi-Objective Optimization

One method using a fitness parameter is to assign the fitness of a design as the number of designs in the current population that it dominates.

A method that does not use a fitness parameter would be conditional entry. It goes as follows:

Generate children as you normally would from a population. Then, on an individual basis, do the following:

Multi-Objective Optimization

Find a member in the population that the child dominates. If such a design exists, replace it with the child.

If such a design does not exist, do the following:

Find a member of the population that dominates the child. If such a design exists, discard the child. Otherwise, grow the population and add the child in.

Multi-Objective Optimization

So clearly, this is not a steady state type GA but it helps to allow as many efficient points as possible participate in the evolution.

So how then do we account for constraints in such a case? Clearly we are not using some penalty function in our assessment of fitness.

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There are a couple of options that we can choose from.

The first of which is to treat each constraint as an implicit objective. That is, consider that the presence of each constraint implies the existence of an objective.

That objective is satisfaction of this constraint.

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We have now converted our n-objective problem into one of $n + \text{num_of_cons}$ which is a drawback.

Another drawback is that we lose all information that tells us just how much better 1 design is over another.

So we have alleviated one of the drawbacks of the previous approach.

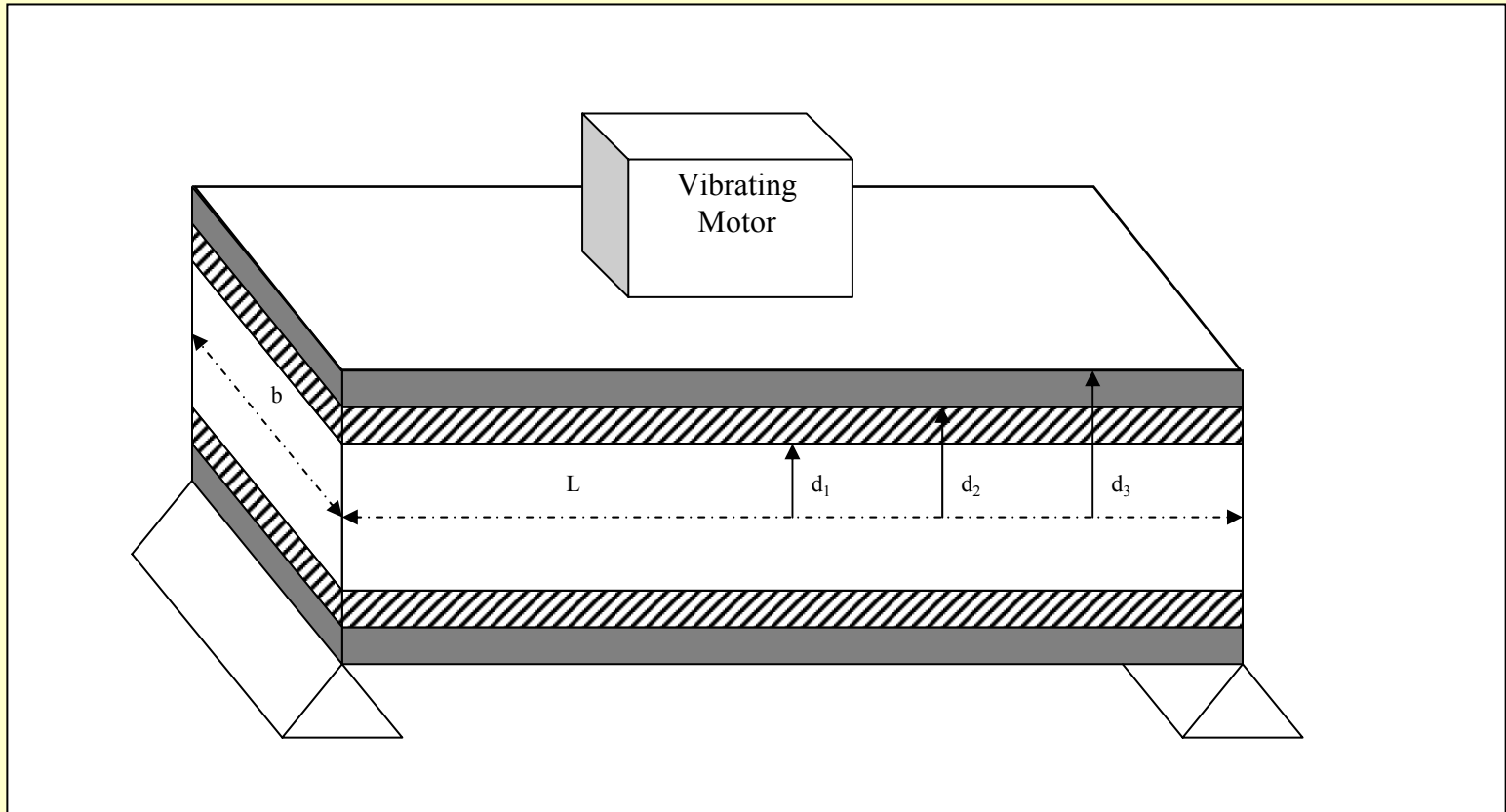
Multi-Objective Optimization

Another way to do it is to add a single additional objective to our problem whereby that objective is satisfaction of all constraints.

So we could pool all of our constraint violations into a term like a penalty term and use a seek 0 (which is minimize by our standard penalty formulations) approach to optimizing it as though it were an objective.

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Consider the following example of a problem that I solved using a MOGA.



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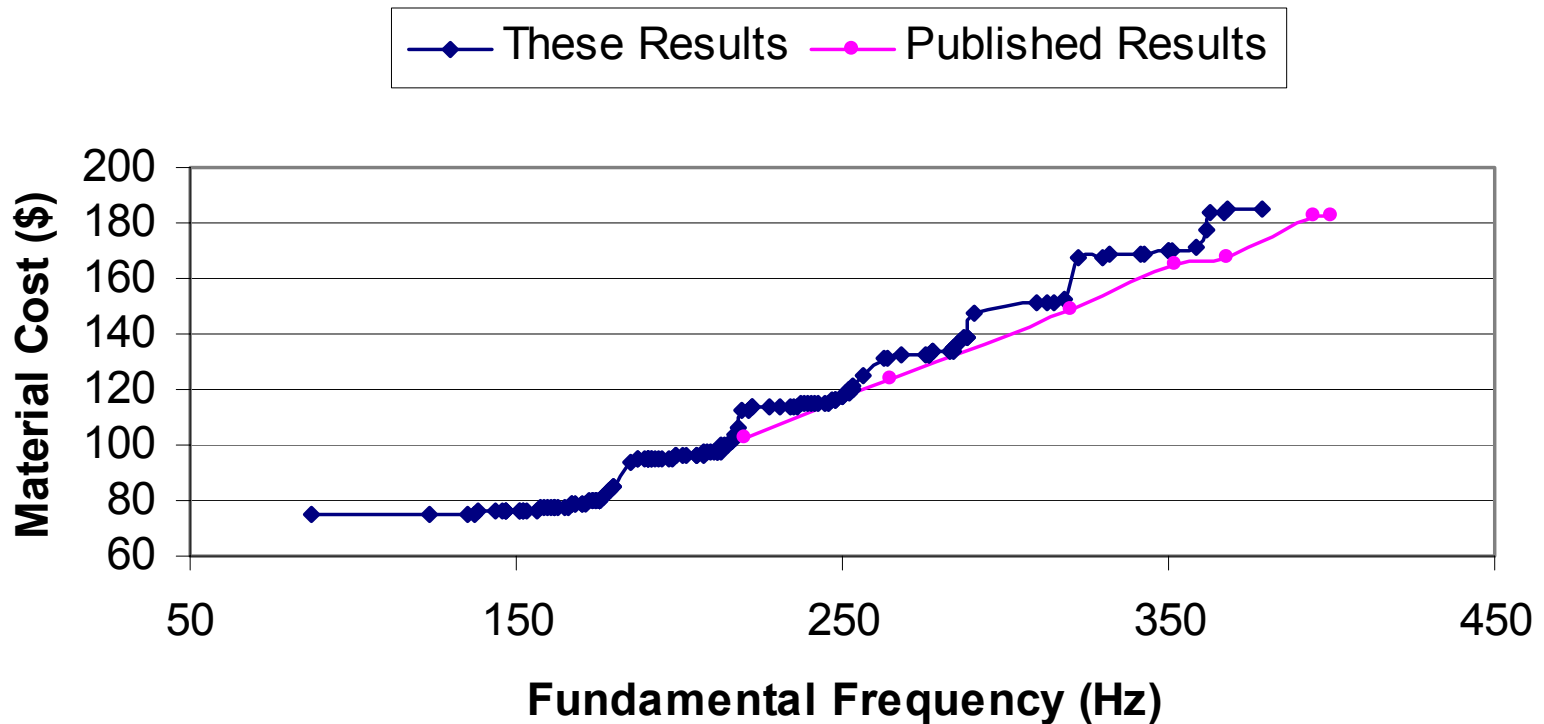
The objectives are to minimize the material cost and to maximize the fundamental frequency of the beam.

There are a number of constraints for geometry, material selections, and mass.

The following figure shows two curves. The blue curve is the data generated here and the pink curve is data generated by the I-SHOT method.

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Resulting Pareto Set



Multi-Objective Optimization

The I-SHOT method of Azarm, Reynolds, and Narayanan is in brief a repeated application of a single-objective GA using special combinatorial rules for the objectives. It can be read about in:

Azarm, S., Reynolds, B. J., and Narayanan, S., 1999, “Comparison of Two Multiobjective Optimization Techniques with and within Genetic Algorithms,” *ASME Design Engineering Technical Conferences*, DETC99/DAC-8584.

Multi-Objective Optimization

A comparison of the results is as follows:

Azarm et. Al.:

- 1 Pareto point for every 7,909 points visited.
- 53,963 total points visited.
- Frequency ranged from ~ 220 – 400 Hz.
- Cost ranged from ~ \$100 - \$180.

These Results:

- 1 Pareto point for every 195 points visited.
- 33,076 total points visited.
- Frequency ranged from ~ 88 – 380 Hz.
- Cost ranged from ~ \$75 - \$185.