Dielectric behavior

Topic 9

Reading assignment

- Askeland and Phule, The Science and Engineering of Materials, 4th Ed., Sec. 18-8, 18-9 and 18-10.
- Shackelford, Materials Science for Engineering, Sec. 15.4.
- Chung, Composite Materials, Ch. 7.

Insulators and dielectric properties

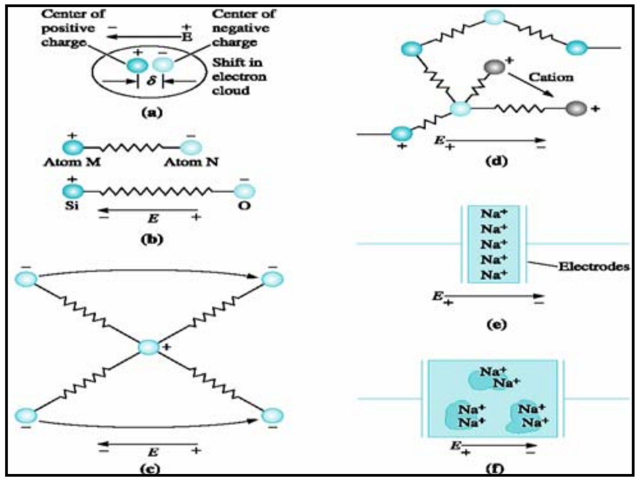
- Materials used to insulate an electric field from its surroundings are required in a large number of electrical and electronic applications.
- Electrical insulators obviously must have a very low conductivity, or high resistivity, to prevent the flow of current.
- Porcelain, alumina, cordierite, mica, and some glasses and plastics are used as insulators.

Dielectric strength

- Maximum electric field that an insulator can withstand before it loses its insulating behavior
- Lower for ceramics than polymers
- Dielectric breakdown - avalanche breakdown or carrier multiplication

Polarization in dielectrics

- Capacitor – An electronic device, constructed from alternating layers of a dielectric and a conductor, that is capable of storing a charge. These can be single layer or multi-layer devices.
- Permittivity - The ability of a material to polarize and store a charge within it.
- Linear dielectrics - Materials in which the dielectric polarization is linearly related to the electric field; the dielectric constant is not dependent on the electric field.
- Dielectric strength - The maximum electric field that can be maintained between two conductor plates without causing a breakdown.



Polarization mechanisms in materials:

(a) electronic,

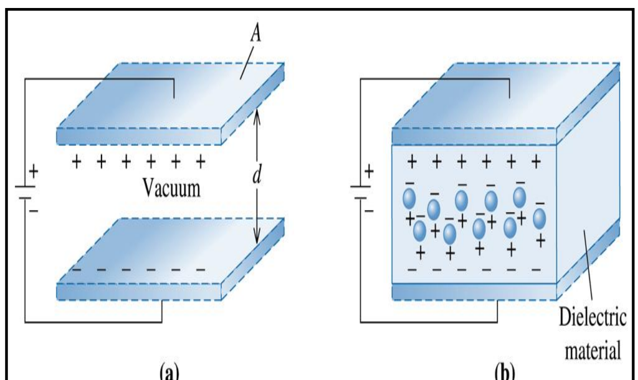
(b) atomic or ionic,

(c) high-frequency dipolar or orientation (present in ferroelectrics),

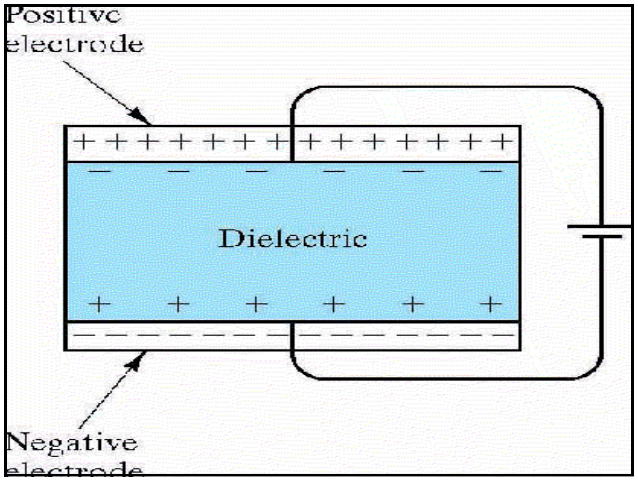
(d) low-frequency dipolar (present in linear dielectrics and glasses),

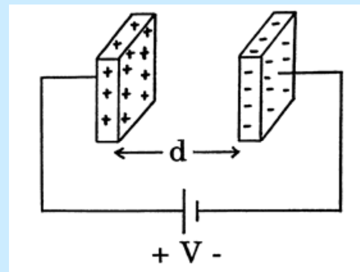
(e) interfacial-space charge at electrodes, and

(f) interfacial-space charge at heterogeneities such as grain boundaries.



A charge can be stored at the conductor plates in a vacuum (a). However, when a dielectric is placed between the plates (b), the dielectric polarizes and additional charge is stored.





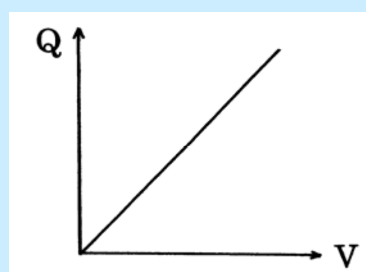
$$D_o = \frac{Q}{A} \quad ,$$

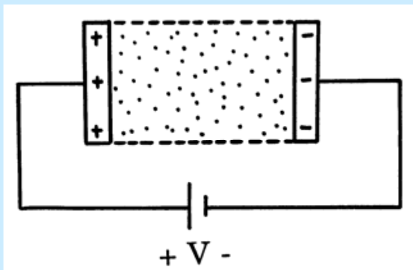
$$\Sigma = \frac{V}{d} \quad ,$$

$$D_o = \varepsilon_o \Sigma$$

$$\varepsilon_o = 8.85 \times 10^{-12} \ C/(V.m)$$

$$\text{Slope} = C_o = \frac{Q}{V} = \frac{\varepsilon_o \Sigma A}{\Sigma d} = \frac{\varepsilon_o A}{d} \quad ,$$



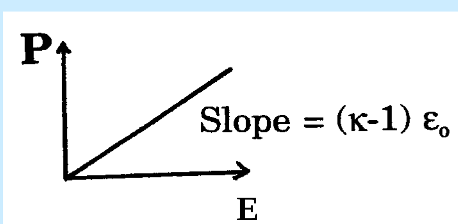


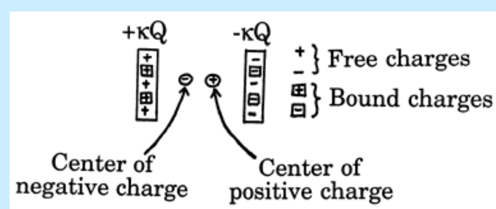
$$D_m = \kappa D_o = \frac{\kappa Q}{A} \quad ,$$

$$D_m = \kappa\, \varepsilon_o\, \Sigma = \varepsilon\, \Sigma$$

$$C_m = \frac{\kappa Q}{V} = \frac{\kappa \varepsilon_o \Sigma A}{\Sigma d} = \frac{\kappa \varepsilon_o A}{d} = \kappa C_o \quad ,$$

$$\begin{aligned} P &= D_m - D_o \\ &= \kappa\, \varepsilon_o \Sigma - \varepsilon_o \Sigma \\ &= (\kappa - 1)\, \varepsilon_o \Sigma \end{aligned}$$





$$\frac{\kappa Q - Q}{Q} = \kappa - 1 \quad ,$$

$$\chi = \kappa - 1 = \frac{P}{\varepsilon_o \Sigma} \quad ,$$

(bound charge)d = (κ – 1) Qd

$$\frac{\text{Dipole moment}}{\text{Volume}} = \frac{(\kappa - 1)Qd}{Ad} = \frac{(\kappa - 1)Q}{A} = P \quad ,$$

$$V = \frac{\kappa Q}{C_m}$$

$$\kappa Q = D_m A = \varepsilon \Sigma A$$

$$C_m = \frac{\kappa \varepsilon_o A}{x} = \frac{\varepsilon A}{x}$$

$$V = \frac{\varepsilon \Sigma A}{\frac{\varepsilon A}{x}} = \Sigma x$$

Material	Dielectric constant,[a] κ	Dielectric strength (kV/mm)
Al_2O_3 (99.9%)	10.1	9.1[b]
Al_2O_3 (99.5%)	9.8	9.5[b]
BeO (99.5%)	6.7	10.2[b]
Cordierite	4.1–5.3	2.4–7.9[b]
Nylon 66-reinforced with 33% glass fibers (dry-as-molded)	3.7	20.5
Nylon 66-reinforced with 33% glass fibers (50% relative humidity)	7.8	17.3
Acetal (50% relative humidity)	3.7	19.7
Polyester	3.6	21.7

Source: Data from *Ceramic Source '86*, American Ceramic Society, Columbus, OH, 1985, and *Design Handbook for Du Pont Engineering Plastics*.

[a] At 10^3 Hz.

[b] Average root-mean-square (RMS) values at 60 Hz.

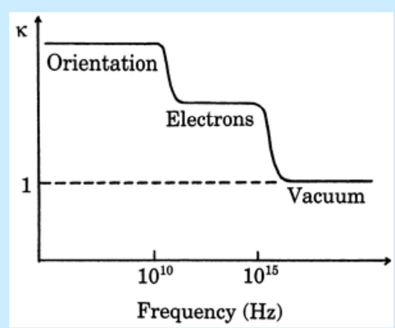
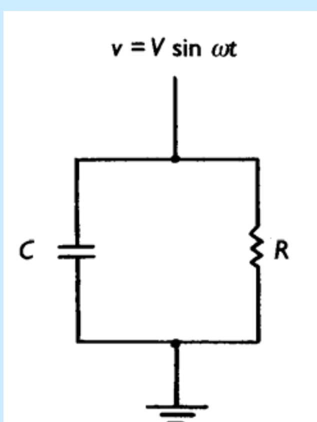


Table 7.6 Values of the relative dielectric constant κ of various dielectric materials at 1 kHz (Data from Ceramic Source '86, American Ceramic Society, Columbus, Ohio, 1985, and Design Handbook for DuPont Engineering Plastics).

Material	κ
Al_2O_3 (99.5%)	9.8
BeO (99.5%)	6.7
Cordierite	4.1-5.3
Nylon-66 reinforced with glass fibers	3.7
Polyester	3.6

TABLE 18-8 *Properties of selected dielectric materials*

Material	Dielectric Constant (at 60 Hz)	Dielectric Constant (at 10^6 Hz)	Dielectric Strength (10^6 V/m)	tan δ (at 10^6 Hz)	Resistivity (ohm · cm)
Polyethylene	2.3	2.3	20	0.00010	$>10^{16}$
Teflon	2.1	2.1	20	0.00007	10^{18}
Polystyrene	2.5	2.5	20	0.00020	10^{18}
PVC	3.5	3.2	40	0.05000	10^{12}
Nylon	4.0	3.6	20	0.04000	10^{15}
Rubber	4.0	3.2	24		
Phenolic	7.0	4.9	12	0.05000	10^{12}
Epoxy	4.0	3.6	18		10^{15}
Paraffin wax		2.3	10		10^{13}-10^{19}
Fused silica	3.8	3.8	10	0.00004	10^{11}-10^{12}
Soda-lime glass	7.0	7.0	10	0.00900	10^{15}
Al_2O_3	9.0	6.5	6	0.00100	10^{11}-10^{13}
TiO_2		14-110	8	0.00020	10^{13}-10^{18}
Mica		7.0	40		10^{13}
$BaTiO_3$		2000-5000	12	~0.0001	10^{8}-10^{15}
Water		78.3			10^{14}



$$\Sigma = \hat{\Sigma} e^{i\omega t} = \hat{\Sigma}(\cos \omega t + i \sin \omega t),$$

$$D_m = \hat{D}_m e^{i(\omega t - \delta)} = \hat{D}_m [\cos(\omega t - \delta) + i \sin(\omega t - \delta)],$$

$$\hat{D}_m e^{i(\omega t - \delta)} = \varepsilon \hat{\Sigma} e^{i\omega t},$$

$$\varepsilon = \frac{\hat{D}_m}{\hat{\Sigma}} e^{-i\delta} = \frac{\hat{D}_m}{\hat{\Sigma}} (\cos \delta - i \sin \delta),$$

$$\tan \delta = -\frac{\text{Imaginary part of } \kappa}{\text{Real part of } \kappa},$$

$$i_c = \frac{dQ}{dt} = C\frac{dv}{dt} \quad ,$$

$$v = V \sin \omega t$$

$$\omega = 2\pi f = \frac{2\pi}{T} \quad ,$$

$$i_c = C\frac{dv}{dt} = \omega CV \cos\omega t \quad ,$$

$$= \frac{V}{1/\omega C}\cos\omega t$$

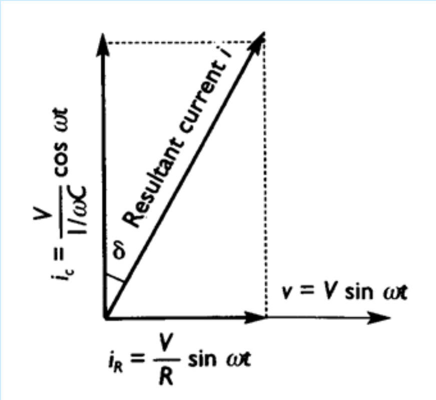
$$\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= \sin\omega t \cos\frac{\pi}{2} + \cos\omega t \sin\frac{\pi}{2}$$

$$= \cos\omega t \quad ,$$

$$i_c = \frac{V}{1/\omega C}\sin\left(\omega t + \frac{\pi}{2}\right) \quad ,$$

$$i_R = \frac{v}{R} = \frac{V}{R}\sin\omega t \quad ,$$



$$i_c = \frac{V}{1/\omega C}\sin\left(\omega t + \frac{\pi}{2}\right) \quad ,$$

$$i_R = \frac{v}{R} = \frac{V}{R}\sin\omega t \quad ,$$

$$\tan\delta = \frac{V/R}{V\omega C} = \frac{1}{\omega CR} \quad ,$$

$$\text{Energy stored} = \int_0^{\tau} v i_C \, dt \; ,$$

$$= \int_0^{\tau} V^2 \omega C \sin \omega t \cos \omega t dt \; ,$$

$$= \int_0^{\tau} \frac{V^2 \omega C}{2} \sin 2\omega t \, dt \; ,$$

$$= -\frac{V^2 \omega C}{4\omega} \left[\cos 2\omega t \right]_0^{\tau} \; ,$$

$$= -\frac{1}{4} CV^2 (\cos 2\omega\tau - 1) \; ,$$

Maximum energy stored = ½ CV^2

This occurs when

cos 2ωt = -1

Energy loss per cycle due to conduction through the resistor R

$$\text{Energy loss} = \frac{V^2}{R} \int_0^{2\pi/\omega} \sin \omega t \sin \omega t \; dt$$

$$= \frac{V^2}{\omega R} \int_0^{2\pi} \frac{1}{2} (1 - \cos 2\omega t) \, d(\omega t)$$

$$= \frac{V^2}{\omega R} \left[\frac{1}{2} \left(\omega t - \frac{1}{2} \sin 2\omega t \right) \right]_0^{2\pi}$$

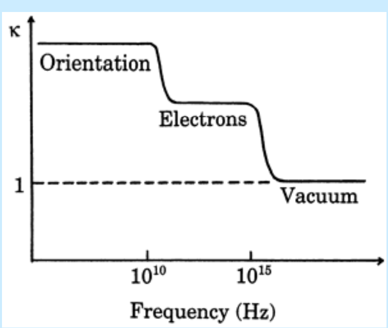
$$= \frac{V^2}{\omega R} \left[\frac{1}{2} (2\pi - 0 - 0 + 0) \right]$$

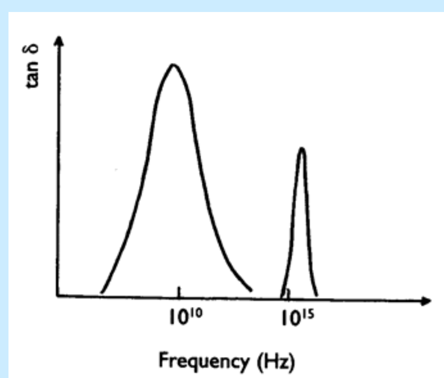
$$= \frac{V^2 \pi}{\omega R} \; .$$

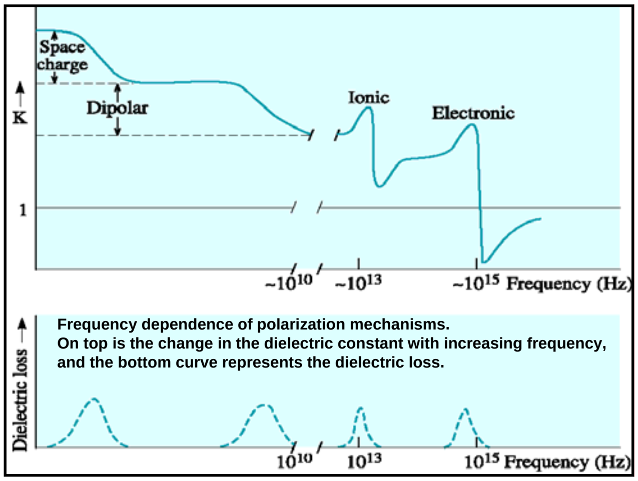
The smaller is R, the greater is the energy loss.

$$\frac{\text{Energy lost per cycle}}{2\pi \times \text{maximum energy stored}} = \frac{V^2\pi / \omega R}{2\pi C V^2 / 2}$$

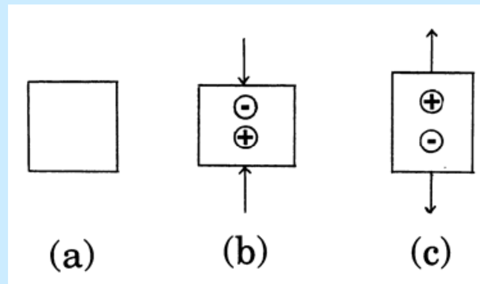
$$= \frac{1}{\omega C R} = \tan\delta$$



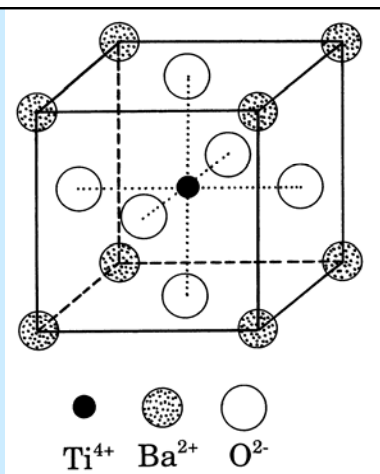


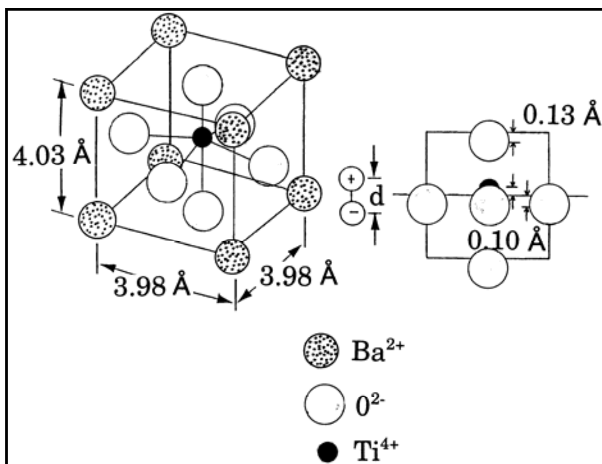


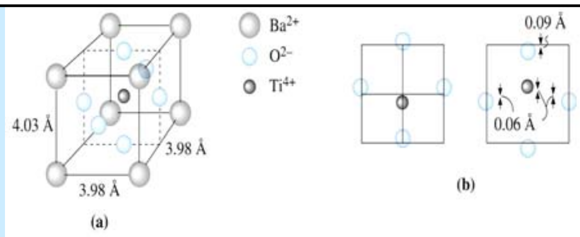
Frequency dependence of polarization mechanisms.
On top is the change in the dielectric constant with increasing frequency, and the bottom curve represents the dielectric loss.



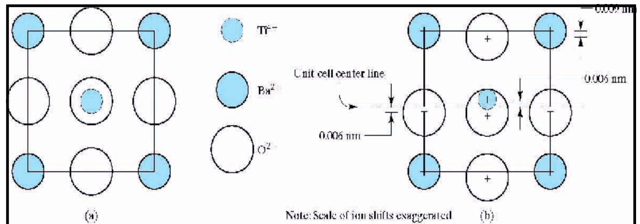
Quartz – polarization only under stress

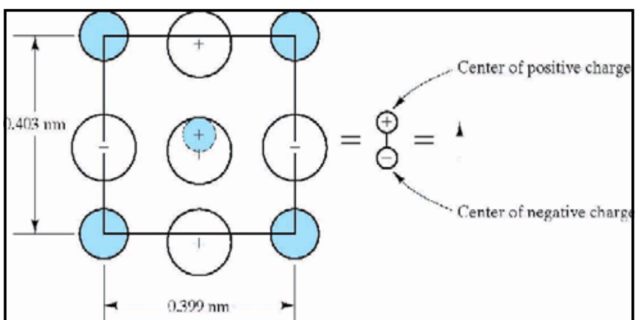


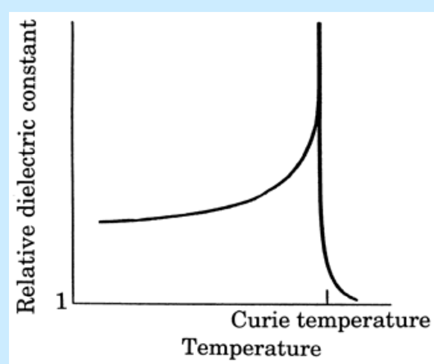




(a) The oxygen ions are at face centers,
Ba+2 ions are at cube corners and
Ti+4 is at cube center in cubic BaTi03.
(b) In tetragonal BaTi03 ,the Ti+4 is off-center and
the unit cell has a net polarization.







Different polymorphs of $BaTiO_3$ and accompanying changes in lattice constants and dielectric constants.

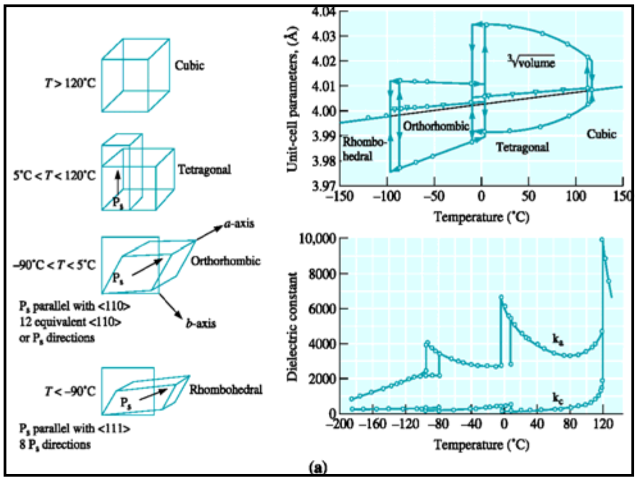
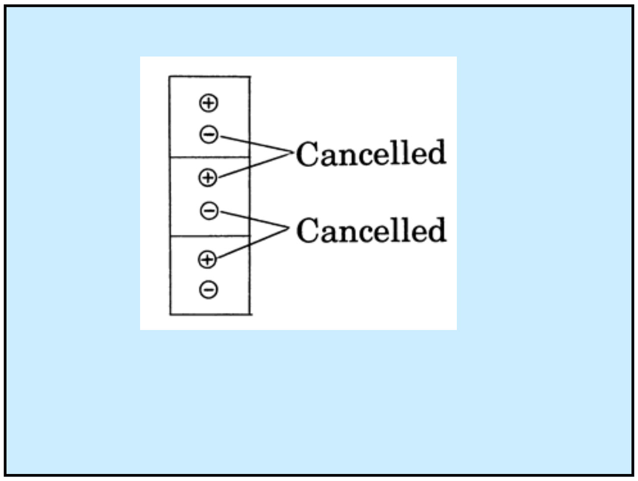
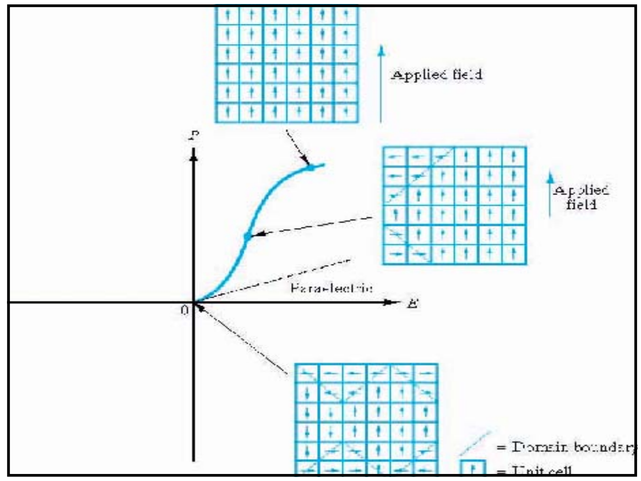


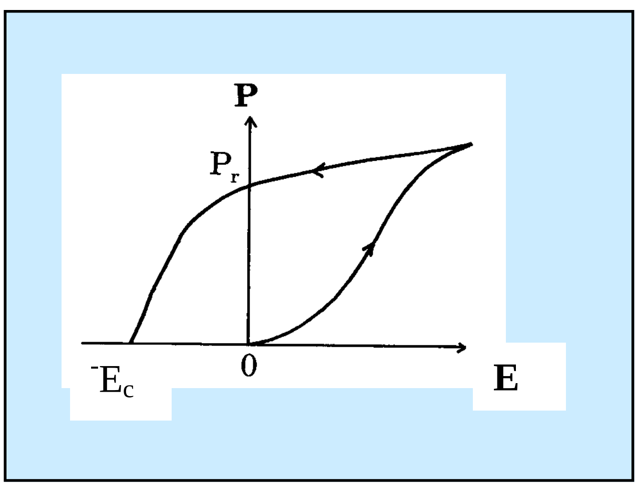
Table 7.3 Contribution to dipole moment of a $BaTiO_3$ unit cell by each type of ion.			
Ion	Charge (C)	Displacement (m)	Dipole moment (C.m)
Ba^{2+}	$(+2)(1.6 \times 10^{-19})$	0	0
Ti^{4+}	$(+4)(1.6 \times 10^{-19})$	$+0.10(10^{-10})$	6.4×10^{-30}
$2O^{2-}$ (side of cell)	$2(-2)(1.6 \times 10^{-19})$	$-0.10(10^{-10})$	6.4×10^{-30}
O^{2-} (top and bottom of cell)	$(-2)(1.6 \times 10^{-19})$	$-0.13(10^{-10})$	4.2×10^{-30}
			Total = 17×10^{-30}

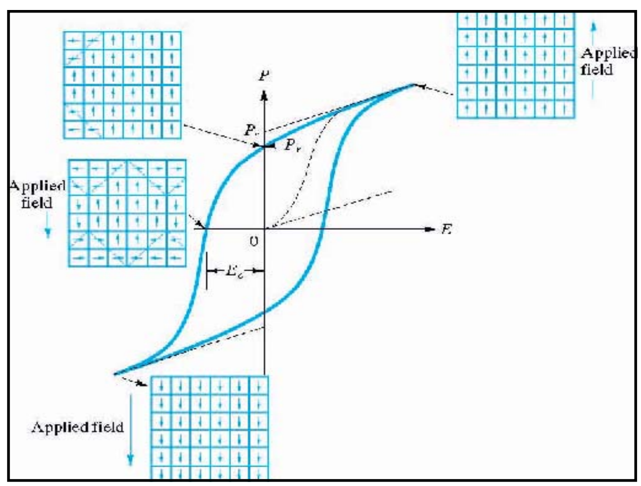
$$\mathbf{P} = \frac{17 \times 10^{-30}\,\text{C.m}}{4.03 \times 3.98^2 \times 10^{-30}\,\text{m}^3}$$

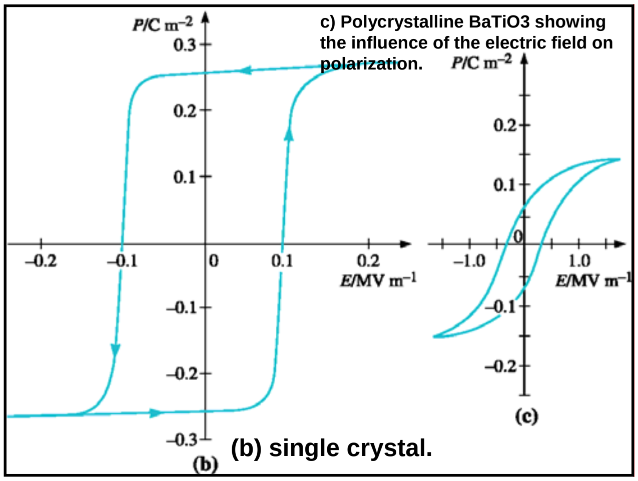
$$= 0.27\ \text{C.m}^{-2}$$


Cancelled
Cancelled

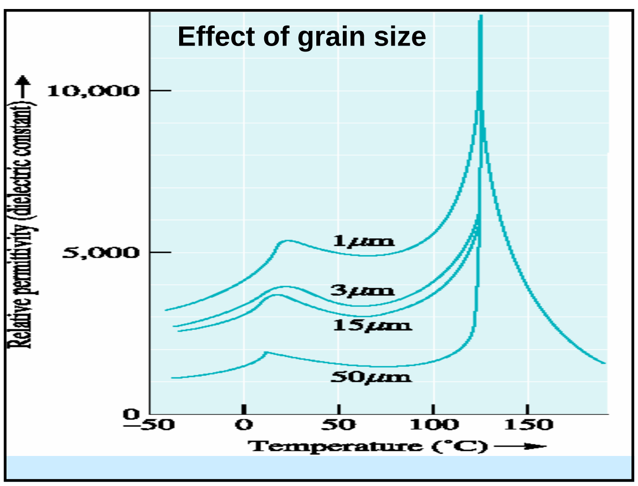

Applied field
P
Applied field
Paraelectric
0
E
= Domain boundary

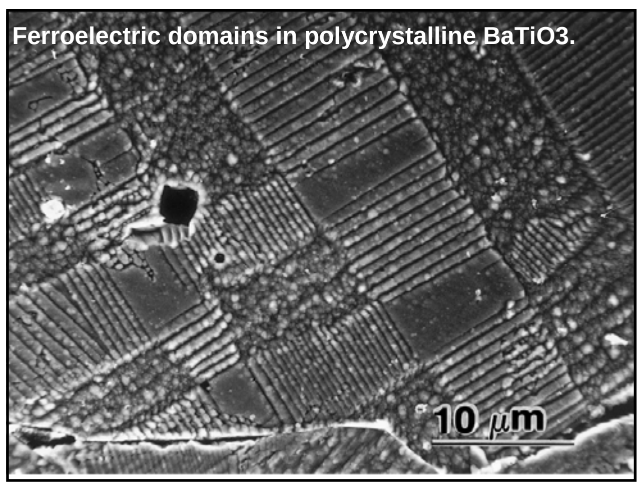

P
P_r
$-E_c$
0
E

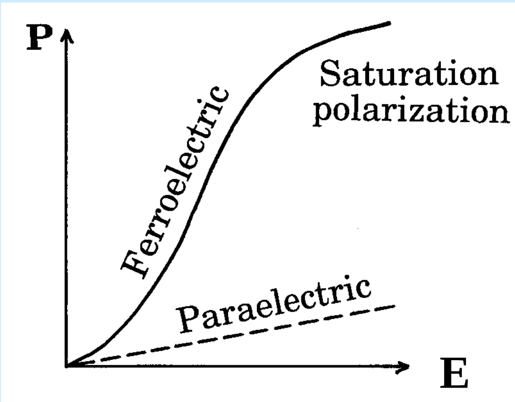

P
P_r
Applied field
Applied field
0
E
E_c
Applied field



The effect of temperature and grain size on the dielectric constant of barium titanate. Above the Curie temperature, the spontaneous polarization is lost due to a change in crystal structure and barium titanate is in the paraelectric state. The grain size dependence shows that similar to yield-strength dielectric constant is a microstructure sensitive property.







Depoling

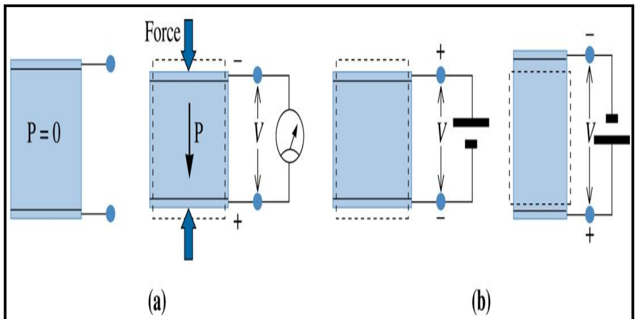
Piezoelectric aging rate r

$$\frac{u_2 - u_1}{u_1} = r \log \frac{t_2}{t_1},$$

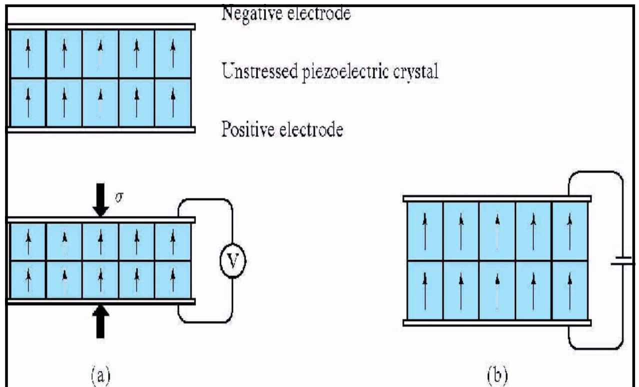
u : parameter such as capacitance
t: number of days after polarization

Ferroelectric - A material that shows spontaneous and reversible dielectric polarization.

Piezoelectric – A material that develops voltage upon the application of a stress and develops strain when an electric field is applied.

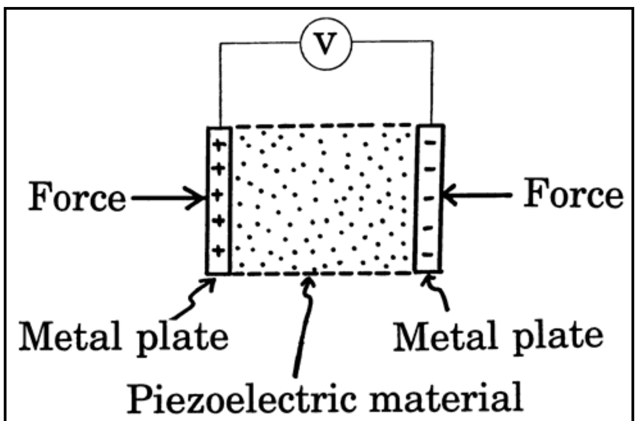


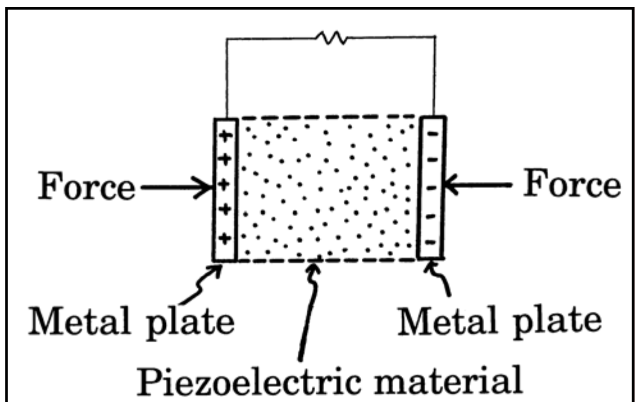
The (a) direct and (b) converse piezoelectric effect.
In the direct piezoelectric effect, applied stress causes a voltage to appear.
In the converse effect (b), an applied voltage leads to development of strain.

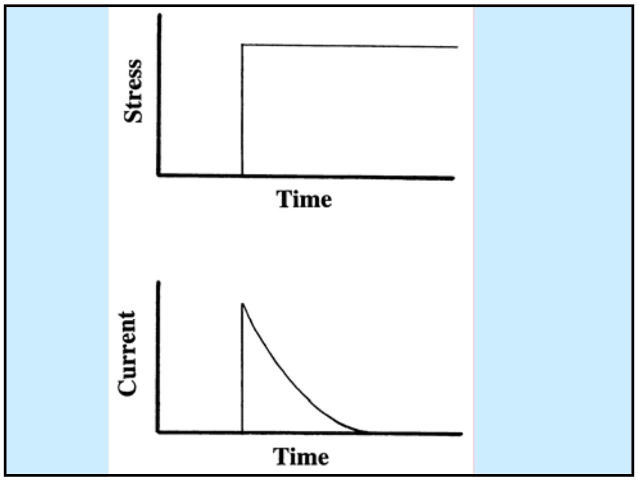


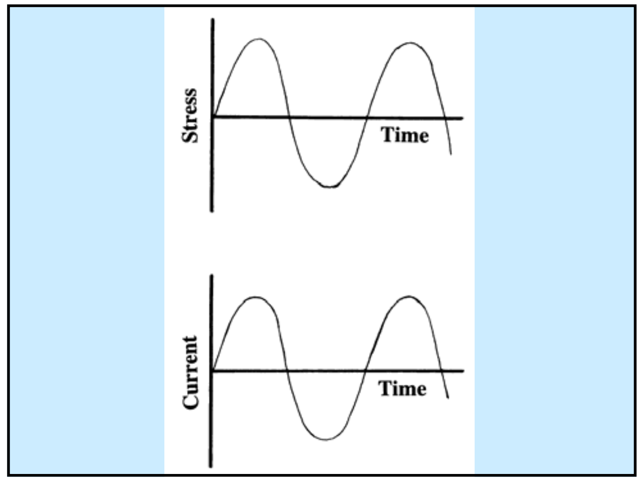
Direct piezoelectric effect

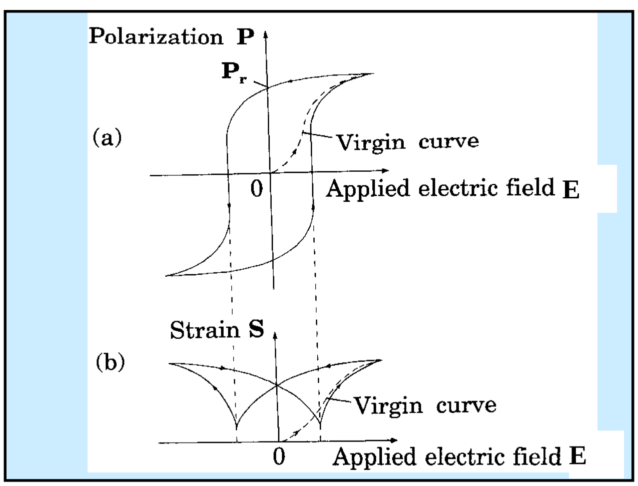
Reverse (converse) piezoelectric effect

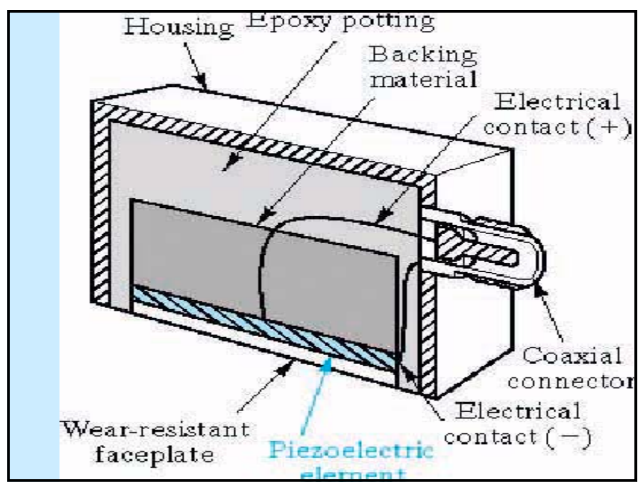





Stress
Time
Current
Time


Stress
Time
Current
Time


Polarization P
P_r
(a)
Virgin curve
0
Applied electric field E
Strain S
(b)
Virgin curve
0
Applied electric field E


Housing
Epoxy potting
Backing material
Electrical contact (+)
Coaxial connector
Electrical contact (−)
Wear-resistant faceplate
Piezoelectric element

Direct piezoelectric effect

$$\mathbf{P} = \mathbf{d}\sigma$$

$$\partial\mathbf{P} = \mathbf{d} \quad \partial\sigma$$

$$\mathbf{d} = \varepsilon_o \Sigma \left| \frac{\partial \kappa}{\partial \sigma} \right|$$

d: Piezoelectric coupling coefficient (piezoelectric charge coefficient)

Table 7.1 The piezoelectric constant **d** (longitudinal) for selected materials

Material	Piezoelectric constant **d** (C/N = m/V)
Quartz	2.3×10^{-12}
$BaTiO_3$	100×10^{-12}
$PbZrTiO_6$	250×10^{-12}
$PbNb_2O_6$	80×10^{-12}

$$P = D_m - D_o$$

$$= \kappa\, \varepsilon_o\Sigma - \varepsilon_o\Sigma$$

$$= (\kappa - 1)\, \varepsilon_o\Sigma$$

$$\partial\Sigma = \frac{\partial \mathbf{P}}{\varepsilon_o(\kappa - 1)}$$

$$\partial\Sigma = \frac{\mathbf{d}\,\partial\sigma}{\varepsilon_o(\kappa - 1)}$$

$$\partial\mathbf{V} = \ell\partial\Sigma,$$

$$\partial\mathbf{V} = \frac{\ell\mathbf{d}\partial\sigma}{\varepsilon_o(\kappa - 1)}$$

$$\partial\mathbf{V} = \frac{\ell\mathbf{d}\partial\sigma}{\varepsilon_o(\kappa - 1)}$$

$$\mathbf{g} = \frac{\mathbf{d}}{(\kappa - 1)\varepsilon_o}$$

$$\partial\mathbf{V} = \ell\mathbf{g}\partial\sigma$$

g: piezoelectric voltage coefficient

TABLE 18-9 The piezoelectric coefficients d and g for selected materials

Material	d Coefficient (pC/N)	g Coefficient (mV/N)
Quartz (SiO_2)	2.3	50×10^{-3}
$BaTiO_3$*	190	12×10^{-3}
PZT*	268 to 480	12×10^{-3} to 35×10^{-3}
$PbNb_2O_6$*	80	
$PbTiO_3$	47	
$LiNbO_3$	6	
$LiTaO_3$	5.7	

*Assumes strain and poling axis along same direction.

Reverse piezoelectric effect

$$\mathbf{S} = \mathbf{d}\Sigma$$

$$\partial\mathbf{S} = \mathbf{d}\partial\Sigma$$

$$\frac{\Sigma}{\sigma} = \frac{\mathbf{S}}{\mathbf{P}}$$

$$\frac{\partial\Sigma}{\partial\sigma} = \frac{\partial\mathbf{S}}{\partial\mathbf{P}}$$

Reverse piezoelectric effect

$$\frac{\Sigma}{\sigma} = \frac{\mathbf{S}}{(\kappa - 1)\varepsilon_o\Sigma}$$

$$\frac{\partial\Sigma}{\partial\sigma} = \frac{\partial\mathbf{S}}{(\kappa - 1)\varepsilon_o\partial\Sigma}$$

S = dΣ

$$\frac{\Sigma}{\sigma} = \frac{\mathbf{S}}{(\kappa - 1)\varepsilon_o\Sigma}$$

$$\frac{\Sigma}{\sigma} = \frac{\mathbf{d}}{(\kappa - 1)\varepsilon_o}$$

$$\frac{\partial\Sigma}{\partial\sigma} = \frac{\mathbf{d}}{(\kappa - 1)\varepsilon_o}$$

$$\frac{\Sigma}{\sigma} = \frac{\mathbf{d}}{(\kappa - 1)\varepsilon_o}$$

$$\mathbf{g} = \frac{\mathbf{d}}{(\kappa - 1)\varepsilon_o}$$

$$\Sigma = \mathbf{g}\sigma$$
$$\partial\Sigma = \mathbf{g}\partial\sigma$$

Hooke's law

$$\sigma = \mathbf{ES}$$

$$\Sigma = \mathbf{g}\sigma$$

$$\Sigma = \mathbf{gES}$$

S = dΣ

$$\mathbf{S} = \frac{\Sigma}{\mathbf{gE}}$$

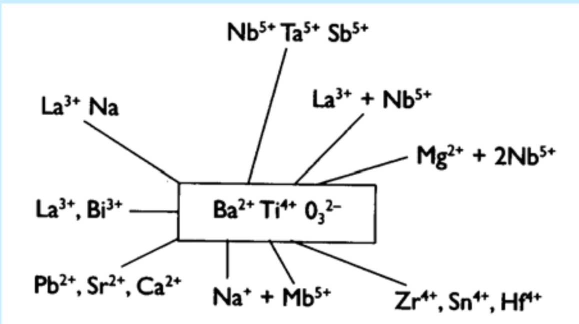
$$\mathbf{d} = \frac{1}{\mathbf{gE}}$$

$$\mathbf{E} = \frac{1}{\mathbf{gd}}$$

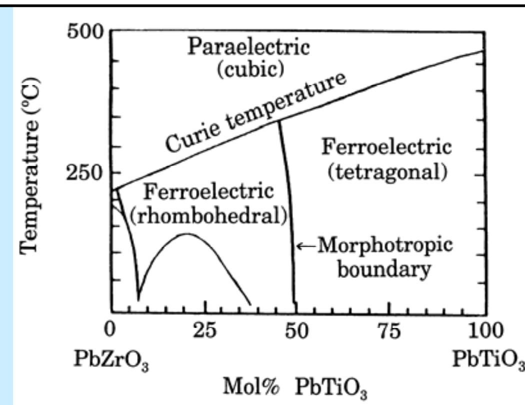
Electromechanical coupling factor (electromechanical coupling coefficient) k

$$\mathbf{k}^2 = \frac{\text{output mechanical energy}}{\text{input electrical energy}}$$

$$\mathbf{k}^2 = \frac{\text{output electrical energy}}{\text{input mechanical energy}}$$



Substitution of A and B sites in $BaTiO_3$



PZT: $PbZrO_3$-$PbTiO_3$ solid solution or lead zirconotitanate

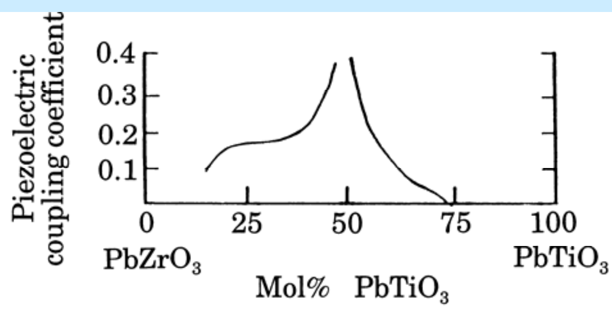


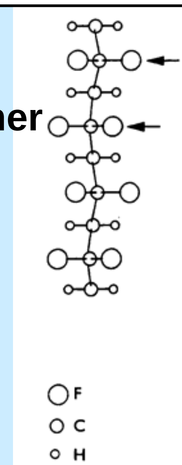
Table 7.4 Properties of commercial PZT ceramics

Property	PZT-5H (soft)	PZT4 (hard)
Permittivity (κ at 1 kHz)	3400	1300
Dielectric loss (tan δ at 1 kHz)	0.02	0.004
Curie temperature (T_c, °C)	193	328
Piezoelectric coefficients (10^{-12} m/V)		
d_{33}	593	289
d_{31}	-274	-123
d_{15}	741	496
Piezoelectric coupling factors		
k_{33}	0.752	0.70
k_{31}	-0.388	-0.334
k_{15}	0.675	0.71

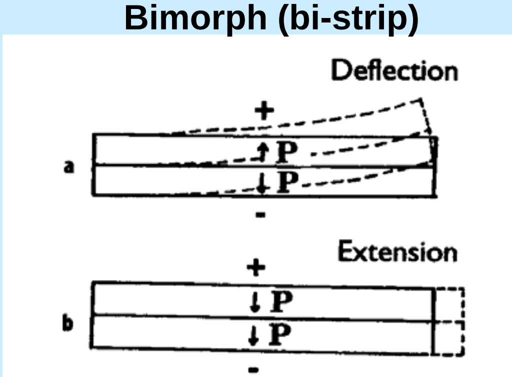
Table 7.2 Measured longitudinal piezoelectric coupling coefficient d, measured relative dielectric constant κ, calculated piezoelectric voltage coefficient g and calculated voltage change resulting from a stress change of 1 kPa for a specimen thickness of 1 cm in the direction of polarization.

Material	d (10^{-13} m/V)*	κ†	g (10^{-4} m²/C)†	Voltage change (mV)†
Cement paste (plain)	0.659 ± 0.031	35	2.2	2.2
Cement paste with steel fibers and PVA	208 ± 16	2700	8.7	8.7
Cement paste with carbon fibers	3.62 ± 0.40	49	8.5	8.5
PZT	136	1024	15	15

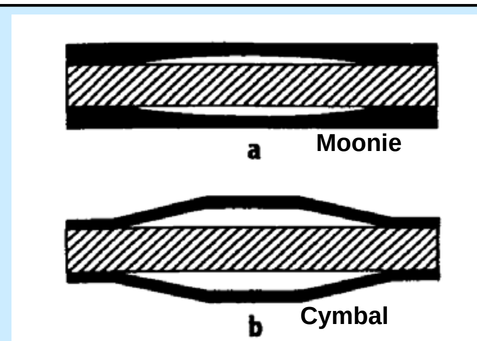
Piezopolymer



Bimorph (bi-strip)



Cantilever beam configuration for actuation



Composites with piezoelectric/ferroelectric material sandwiched by metal faceplates fo enhancing the piezoelectric coupling coefficient

Pyroelectric - **The ability of a material to spontaneously polarize and produce a voltage due to changes in temperature.**

$$p = \frac{dP}{dT} = \varepsilon_o \Sigma \frac{d\kappa}{dT},$$

p = pyroelectirc coefficient
P = polarization

Table 7.5 Pyroelectric coefficient (10^{-6} C/m^2.K)

$BaTiO_3$	20
PZT	380
PVDF	27
Cement paste	0.002

$$V = \frac{Px}{(\kappa - 1)\ \varepsilon_o}$$

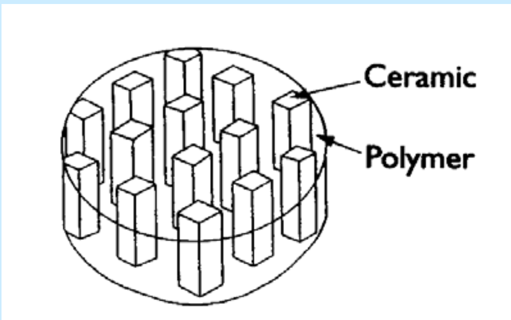
Voltage sensitivity

$$\frac{dV}{d\sigma} = \frac{P}{(\kappa - 1)\ \varepsilon_o}\frac{dx}{d\sigma} + \frac{x}{(\kappa - 1)\ \varepsilon_o}\frac{dP}{d\sigma}$$

Compliance

Piezoelectric coupling coefficient d

Piezoelectric composite



- When any material undergoes polarization (due to an applied electric field), its ions and electronic clouds are displaced, causing the development of a mechanical strain in the material. polarization.
- This phenomenon is known as the electrostriction.

Examples of ceramic capacitors.
(a)Single-layer ceramic capacitor (disk capacitors).
(b) Multilayer ceramic capacitor (stacked ceramic layers).