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Electrostrictive behavior

Electrostrictive behavior (or electrostriction) is a phenomenon in which the dimension changes in response to an applied electric field, due to the energy increase associated with the polarization induced by the electric field in the material. The polarization may be due to atoms becoming egg-shaped rather than spheres, bonds between ions changing in length, or orientation of the permanent electric dipoles in the material. An electrostrictive material is centrosymmetrical in crystal structure, in contrast to piezoelectric or ferroelectric materials, which are noncentrosymmetrical. The dimensional changes can be in all directions, in contrast to the directional dimensional change in piezoelectricity or ferroelectricity. Also in contrast to piezoelectric or ferroelectric behavior, no voltage of field is created by electrostriction and an applied stress does not cause an electric field (i.e., no converse effect). Thus, electrostriction can be used for actuation, but not sensing. Electrostriction is a second-order effect, i.e., the strain is proportional to the square of the electric field, or

$$\mathbf{S} = \mathbf{M}\Sigma^2 \quad (1)$$

where \mathbf{S} is the strain, Σ is the electric field and \mathbf{M} is the electrostrictive coefficient. The second-order effect is due to the anharmonicity of the “springs” that connect the adjacent ions. The anharmonicity is associated with an asymmetric potential well (Fig. 1), so that a spring tends to extend more easily than contracting. Thus, upon increase in energy, the average bond distance increases. In contrast, piezoelectricity or ferroelectricity is a first order effect. The strain due to electrostriction is small compared to that due to piezoelectricity or ferroelectricity. For example, a field of 10^4 V/m produces 23 nm per meter in quartz, but electrostrictive glass produces only 1 nm per meter. However, electrostrictive materials exhibit essentially no hysteresis upon cycling the electric field, whereas piezoelectric/ferroelectric materials exhibit hysteresis due to non-linearity between strain and electric field at high fields (> 100 V/mm), as shown in Fig. 2.

The most important electrostrictive materials are ceramics based on $\text{Pb}(\text{Mg}_{1/3}\text{Nb}_{2/3})\text{O}_3$, i.e., PMN or lead magnesium niobate, which exhibits strains as high as 0.1% (i.e., 10^{-3} m per centimeter) at moderate fields ($\leq 10^6$ V/m). The phenomenon is called giant electrostriction. These materials are also relaxor ferroelectrics. They are ferroelectrics below the Curie temperature, but electrostrictive above the Curie temperature. No poling treatment is needed for electrostrictive materials and these materials do not age (depole).

Example problem

1. An electrostrictive material produces a strain of 2×10^{-4} at an electric field of 5×10^4 V/m. What is the electrostrictive coefficient \mathbf{M} of this material?

Solution:

From Eq. (1),

$$\begin{aligned} \mathbf{M} &= \frac{\mathbf{S}}{\Sigma^2} = \frac{2 \times 10^{-4}}{(5 \times 10^4 \text{ V/m})^2} \\ &= \underline{\underline{8 \times 10^{-14} \text{ m}^2/\text{V}^2}} \end{aligned}$$

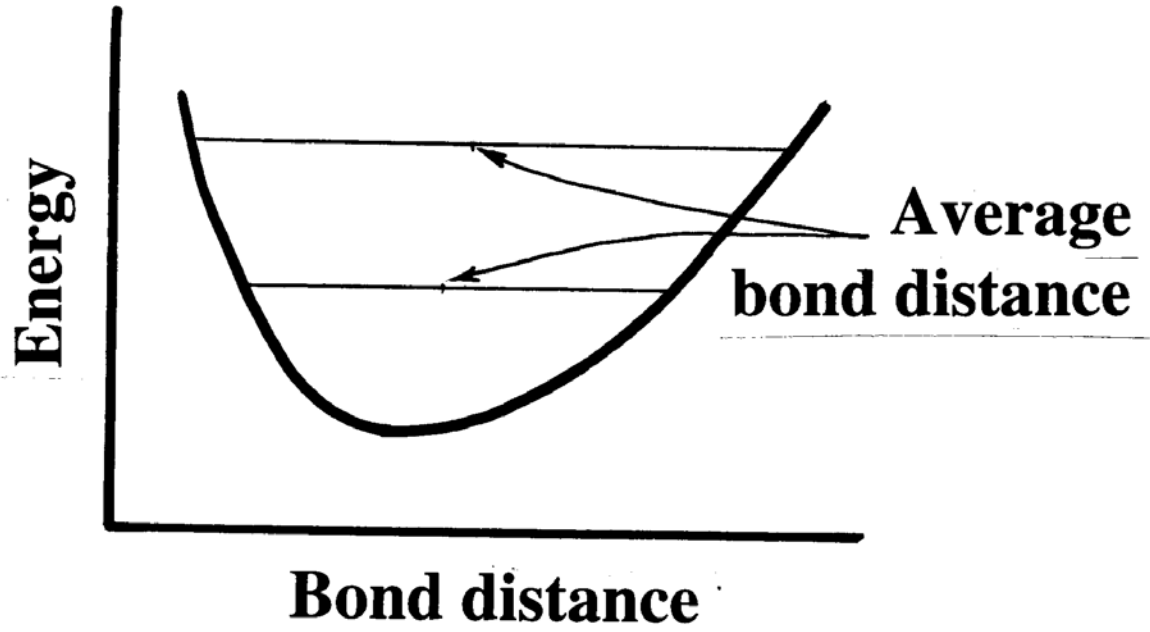


Fig. 1 Potential energy vs. bond distance, showing anharmonicity.

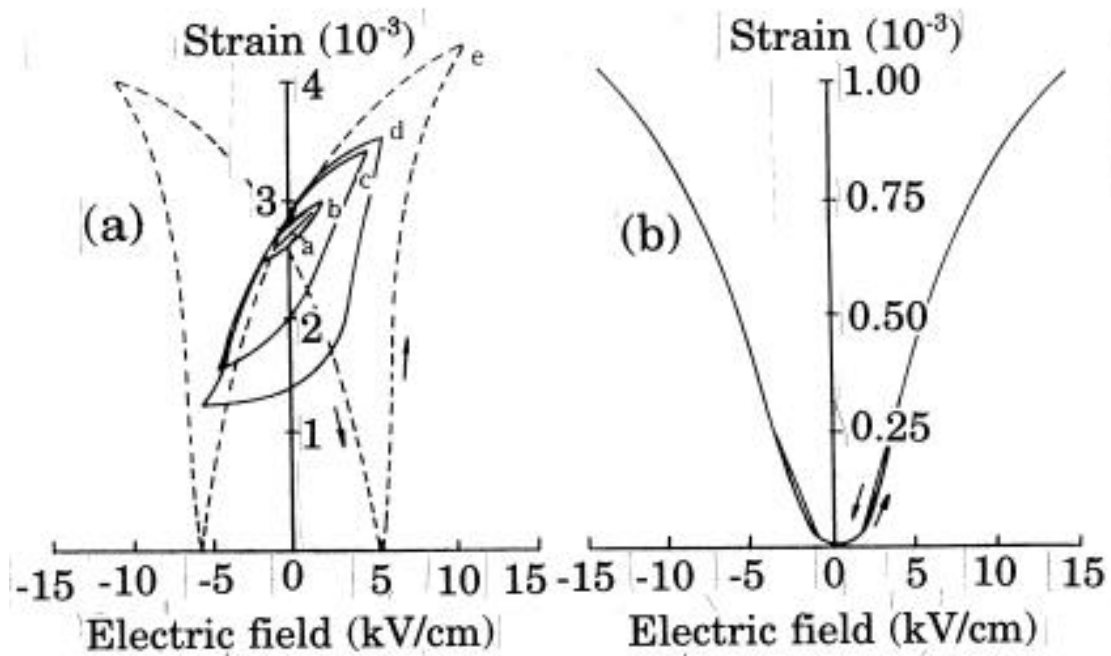


Fig. 2 Strain induced by electric field in (a) piezoelectric PLZT, (b) electrostrictive 0.9 $\text{Pb}(\text{Mg}_{1/3}\text{Nb}_{2/3})\text{O}_3$ -0.1 PbTiO_3 .

Electrorheology

Rheology refers to the science behind the deformation and flow of materials, as expressed in terms of the elasticity and viscosity of the materials. The materials can be liquids or solids.

Elasticity refers to the capacity of a material to return to its original dimensions after

deformation. Elasticity occurs at stresses below the elastic limit. Beyond the elastic limit (also called the proportional limit), plasticity can occur. The relationship between stress and strain in the elastic (Hookean) regime is described by Hooke's law, i.e.,

$$\text{Stress} = \text{modulus} \times \text{strain},$$

where the modulus describes the stiffness. Under tension, Hooke's law is expressed as

$$\sigma = E\varepsilon, \quad (2)$$

Where σ is the tensile stress,

ε is the tensile strain, and

E is the Young's modulus.

Under shear, Hooke's law is expressed as

$$\tau = G\gamma, \quad (3)$$

Where τ is the shear stress,

γ is the shear strain, and

G is the shear modulus.

Viscosity refers to the resistance of a material to flow. For an ideal viscous fluid, the stress is linearly related to the shear rate (or shear strain rate=rate of change of the shear strain= $d\gamma/dt=\dot{\gamma}$). This linear relationship is known as Newton's law. It is expressed as,

$$\tau = \eta \frac{d\gamma}{dt} = \eta \dot{\gamma} \quad (4)$$

where the proportionality constant is η , the viscosity (or coefficient of viscosity). The unit of viscosity is Pa.s, where $1 \text{ Pa.s} = 10 \text{ dyne.s/cm}^2 = 10 \text{ poise} = 10 \text{ P}$. The viscosity is 0.00100 Pa.s for water, and 1.49 Pa.s for glycerin. A fluid is Newtonian if the viscosity η is independent of the shear rate. Water is Newtonian. If η changes with $\dot{\gamma}$, the fluid is said to be non-Newtonian. Polymer solutions and melts are usually non-Newtonian such that η decreases with increasing $\dot{\gamma}$ (known as shear thinning or pseudoplasticity) (Fig. 3). The situation in which η increases with increasing $\dot{\gamma}$ is less common and is called shear thickening or dilatancy. These situations are illustrated in Fig. 4.

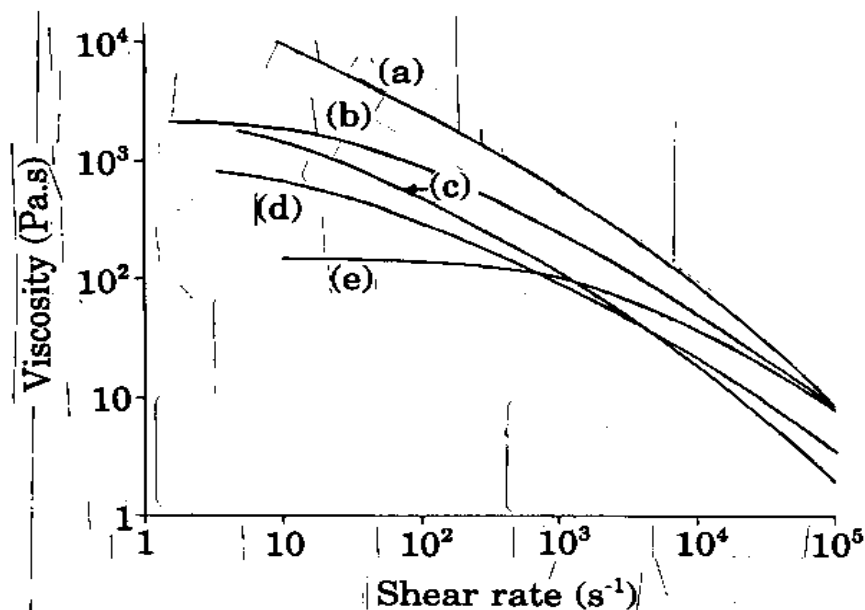


Fig. 3 Variation of the viscosity η with the shear rate $\dot{\gamma}$ for various polymeric non-Newtonian fluids that exhibit shear thinning (or pseudoplasticity). (a) Polyvinyl chloride. (b) Polymethylmethacrylate. (c) Polypropylene. (d) Polyethylene. (e) Nylon

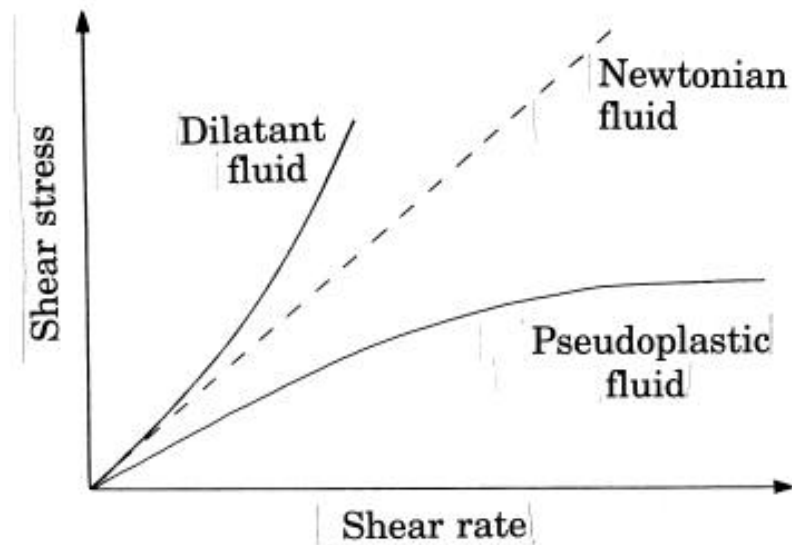


Fig. 4 Relationship between shear stress τ and shear rate $\dot{\gamma}$ for Newtonian and non-Newtonian fluids.

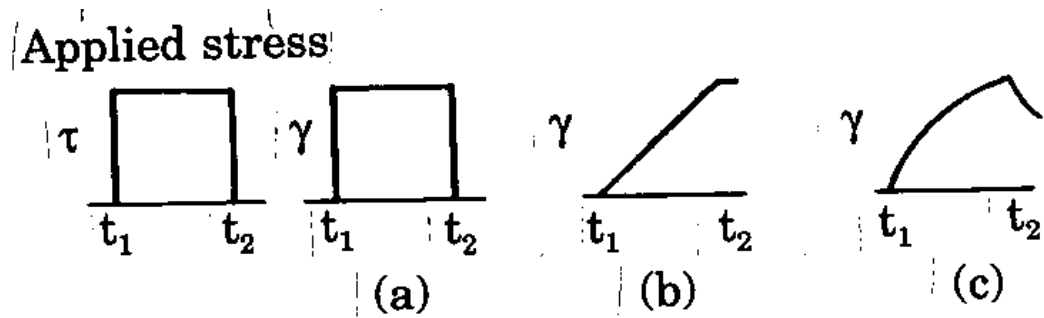


Fig. 5 Deformation and recovery behavior of a material subjected to a constant shear stress. (a) Hookean behavior. (b) Newtonian behavior. (c) Viscoelastic behavior.

A material which responds to stress in a manner which combines elastic and viscous behavior is said to exhibit viscoelastic behavior. Fig. 5 illustrates schematically the shear strain γ in response to a constant shear stress τ for Hookean, Newtonian and viscoelastic cases. In the Hookean case, γ is constant when τ is constant. In the Newtonian case, $\dot{\gamma}$ is constant when τ is constant. The viscoelastic behavior is intermediate between the Hookean and Newtonian behavior. Moreover, in the Hookean case, the strain γ totally and immediately recovers when the shear stress τ is removed. However, in the Newtonian case, γ does not recover (i.e., the fluid stays deformed after the stress is removed). In the viscoelastic case, recovery is partial and time-dependent. Fig. 6 illustrates the stress response to an applied sinusoidal strain for Hookean, Newtonian and viscoelastic cases. In the Hookean case, the stress does not lag the applied strain. In the Newtonian case, the stress lags the strain. The curve for the viscoelastic case is between those for the Hookean and Newtonian cases.

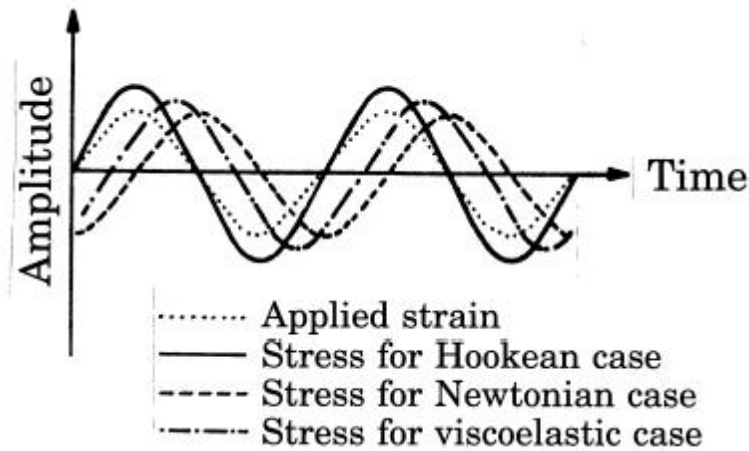


Fig. 6 Schematic stress response to an applied sinusoidal strain for Hookean, Newtonian and viscoelastic cases.

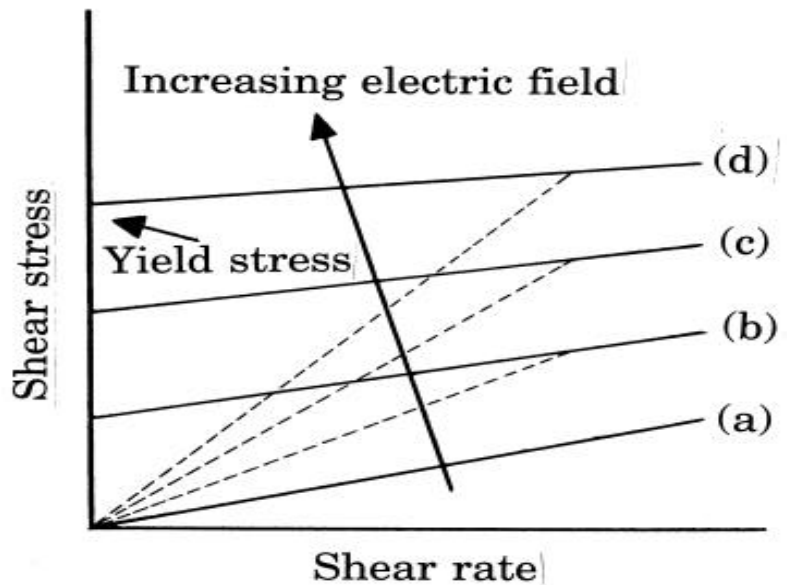


Fig. 7 Variation of shear stress with shear rate for an electrorheological fluid (a) in the absence of electric field; (b), (c) and (d): in the presence of an electric field of increasing magnitude. The slopes of the dashed lines are the apparent viscosity at a particular shear rate.

Electrorheology refers to the phenomenon in which the rheological behavior changes reversibly upon application of an electric field. In the absence of an electric field, the shear stress is zero when the shear rate is zero (Fig. 7(a)), so that there is no minimum shear stress for the shear rate to be non-zero (i.e., for the fluid to flow). The behavior is essentially Newtonian. On the other hand, in the presence of an electric field, the shear stress has to reach a minimum (the yield stress) in order for the fluid to flow (Fig. 7(b)). The rheological behavior in Fig. 7(b) is referred to as that of a Bingham fluid. The higher the electric field, the higher the yield stress (Fig. 7(b), (c) and (d)). For practical application, a high yield stress at a low electric field is desirable. A typical field is 1 kV/mm. The yield

stress (also called the static yield stress due to its being the shear stress at zero shear rate) is typically > 10 kPa. Once the fluid is in motion, i.e., shear rate > 0 , the shear stress may decrease to > 5 kPa. At a sufficiently high shear rate, shear thinning occurs and the shear stress decreases (not shown in Fig. 7). The increase in yield stress is also reflected in the shear stress-strain curve (Fig. 8). It is accompanied by an increase in the shear modulus G (slope of the stress-strain curve before yielding) (Fig. 4.33). Moreover, it is accompanied by a small decrease

in the viscosity above the yield stress (called plastic viscosity), as shown by the decrease in slope of the curves in Fig. 7 as the electric field increases. A low value of the plastic viscosity is desirable for decreasing the amount of fluid needed. More importantly, it is accompanied by a large increase (e.g., from 10 to 300 Pa.s) in the apparent viscosity, which is indicated by the slopes of the dashed lines in Fig. 7 for a particular shear rate.

The electrorheological behavior described in Fig. 7 causes the material to apparently change from a liquid state to a solid state (below the yield stress) upon the application of an electric field. In other words, the mechanical properties of the material are greatly changed upon application of an electric field. Thus, a structural material with an embedded pocket of electrorheological fluid has mechanical properties that can be varied by an electric field. This characteristic is particularly attractive for vibration damping of structures. A colloidal dispersion (or a colloid or a sol) is a suspension of fine solid particles of diameter 10 to 1000 Å in a liquid medium. The solid may be attracted to the liquid, so that the solid-liquid interface is preferred energetically and the solid particles will not coagulate (will not clump together). When this is the case, the dispersion is said to be lyophilic (or hydrophilic if the liquid is water). On the other hand, the solid may repel the liquid, so that the solid-liquid interface is not preferred and the solid particles will tend to coagulate. When this is the case, the dispersion is said to be lyophobic (or hydrophobic if the liquid is water).

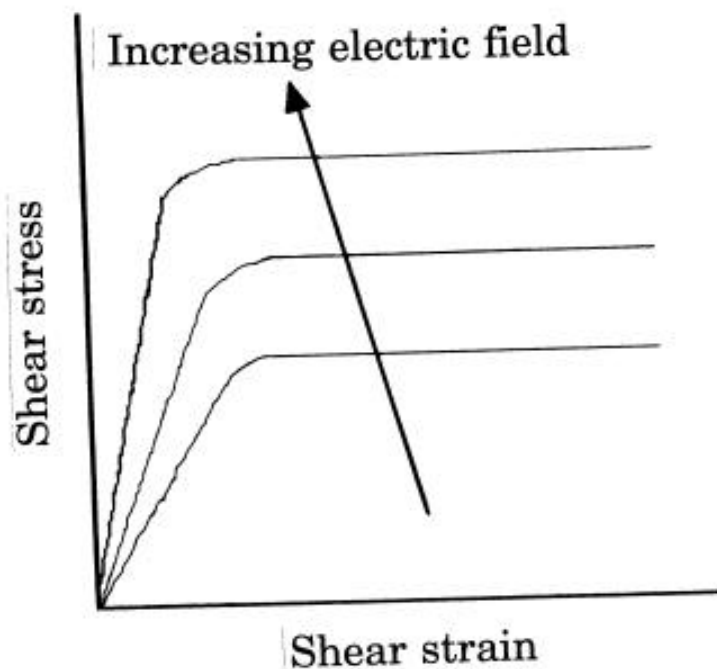


Fig. 8 Shear stress-strain curves for an electrorheological fluid at various electric fields.

A lyophobic dispersion may be changed to a lyophilic dispersion by treating the surface of the particles (prior to colloid formation) so as to form surface functional groups that cause the particles to attract the liquid. A lyophobic suspension may be stabilized (i.e., diminishing the tendency for coagulation) by having surface electrical charges on the particles, so that these charges attract ions of opposite charge in the liquid. Thus each particle is associated with two layers of charges (called the electric double layer), which help to prevent the particles from coagulation. An additive commonly used in a dispersion for stabilization is a surfactant (like a detergent), in which each molecule has an end which is lyophilic and an end which is lyophobic. The lyophobic end is attracted to the solid particles (which are lyophobic), while the lyophilic end is attracted to the liquid. In this way, the solid is attracted to the liquid via the surfactant.

An electrorheological fluid (abbreviated ER fluid) is a non-colloidal suspension, in which the solid particles are a nonconducting or semiconducting material that is polarizable (with relative dielectric constant from 2 to 40), with particle size typically around 10^{-6} m (larger than that of a colloid), such as corn starch, silica and titania. The particle surface is frequently coated with an organic activator compound (water or other polar fluids such as ethanol) to enhance polarization. The liquid is commonly an oil, such as silicone oil. The volume fraction of the solid ranges from 0.1 to 0.4, typically around 0.3. Under an applied electric field, each solid particle may be polarized due to charge migration within the particle. As a result, each particle is an electric dipole and particles are attracted to one another by dipole-dipole interaction to form columns in the direction of the electric field, as illustrated in Fig. 9. The column formation is also called particle fibrillation. The columns resist shear, but cannot explain the rapid response time observed. In another mechanism, the solid particles are not polarized, but each particle is associated with an electric double layer (positive surface charges on a particle surrounded by fixed negative charges), as illustrated in Fig. 10. Mobile charges (with net positive charge) between adjacent particles allow attraction between the particles. In the presence of an electric field, the particles align in the field direction, forming columns, as shown in Fig. 10. The mechanism in Fig. 10 is more common than that in Fig. 9. The formation of columns is responsible for the change in rheological behavior upon application of an electric field, since the columns resist shear in the direction perpendicular to the columns. The zero field viscosity of an electrorheological fluid is typically around 100 mPa.s. The shear yield stress is typically around 4 kPa at 3.5 kV/mm. The current density is typically 0.1 A/m^2 at 3.5 kV/mm. The response time is typically 2 ms. Both shear yield stress and viscosity increase with increasing volume fraction of solid in the electrorheological fluid.

Applications of ER fluids include vibration isolation devices (e.g., shock absorbers), active feedback control damper systems, electrically triggered clutches, pumps, hydraulic valves, miniature robotic joints and robotic control systems. ER fluids are attractive in the relatively large forces produced with comparatively small electric fields and the design flexibility associated with

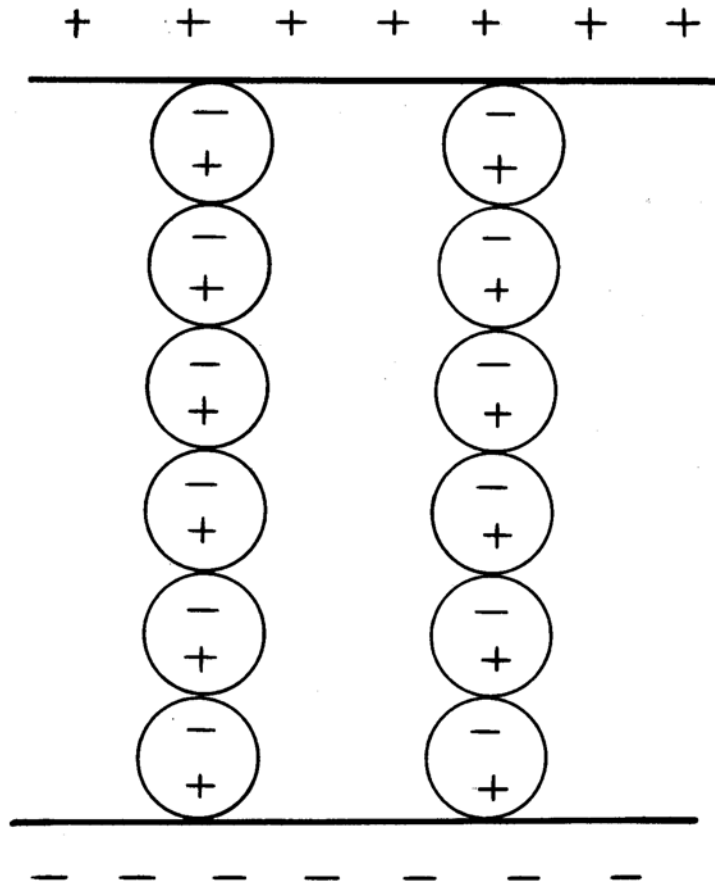


Fig. 9 Columns of polarized solid particles in an electrorheological fluid in the presence of an electric field in the direction of the columns.

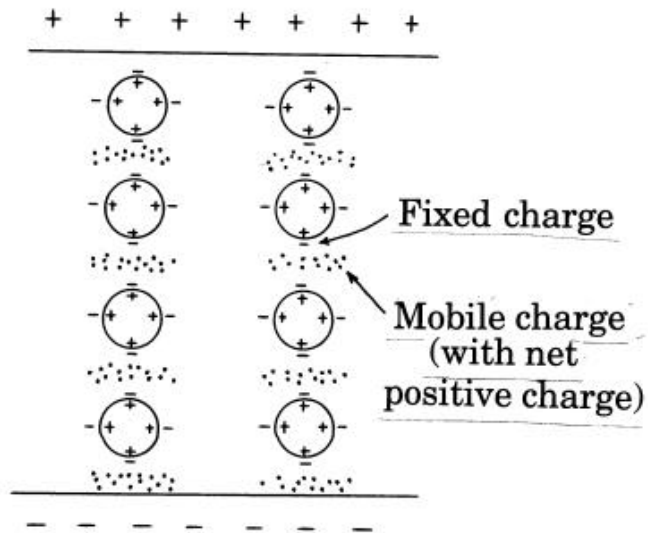


Fig. 10 Columns of solid particles with an electric double layer in an electrorheological fluid in the presence of an electric field in the direction of the columns.

the wide variety of dispersions. However, they suffer from their temperature dependence and insufficient long-term stability of the ER response.

Example problem

2. A Newtonian fluid of viscosity 1.21 Pa.s is subjected to a shear stress of 5.5 MPa. What is the resulting shear rate?

Solution:

From Eq. (4), the shear rate is

$$\dot{\gamma} = \tau/\eta = \frac{5.5 \times 10^{-6}}{1.21 \text{ Pa.s}}$$

$$= 4.5 \times 10^6 \text{ s}^{-1}$$

3. (a) The zero electric field viscosity of an electrorheological fluid is 1.0 Pa.s. Assuming Newtonian behavior, what shear rate is needed in order to attain a shear stress of 0.4 kPa?
 (b) At an electric field of 3.5 kV/mm, the shear yield stress of an electrorheological fluid is 4.0 kPa and the plastic viscosity is 0.7 Pa.s. What is the apparent viscosity at a shear rate of 100 s^{-1} ? What shear rate is needed in order to attain a shear stress of 4.5 kPa?
 (a) At an electric field of 3.5 kV/mm, the current density in the field direction is 0.1 A/m^2 for an electrorheological fluid. What is the apparent electrical conductivity of this fluid?

Solution:

(a) $\tau = \eta \dot{\gamma}$

$$\dot{\gamma} = \tau/\eta = \frac{4 \times 10^2 \text{ Pa}}{1 \text{ Pa.s}} = 4 \times 10^2 \text{ s}^{-1}$$

(b) Apparent viscosity = $\frac{4 \times 10^3 \text{ Pa} + (0.7 \text{ Pa.s})(100 \text{ s}^{-1})}{100 \text{ s}^{-1}} = 41 \text{ Pa.s}$

$$\frac{0.5 \times 10^3 \text{ Pa}}{\dot{\gamma}} = 0.7 \text{ Pa.s}$$

$$\dot{\gamma} = \frac{0.5 \times 10^3 \text{ Pa}}{0.7 \text{ Pa.s}} = 7.1 \times 10^2 \text{ s}^{-1}$$

(c) $\sigma = \bar{\mathbf{J}}/\Sigma = \frac{0.1 \text{ A/m}^2}{3.5 \times 10^6 \text{ V/m}} = 2.9 \times 10^{-8} \Omega^{-1} \text{ m}^{-1}$

Note: $\frac{kV}{mm} = \frac{10^3 V}{10^{-3} m} = 10^6 \frac{V}{m}$