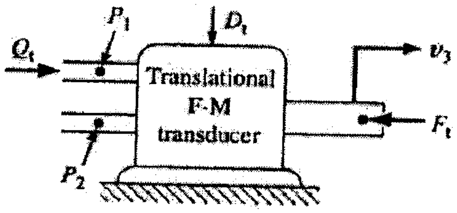
Name ______________________

MAE 340: Test II

March 25, 2003



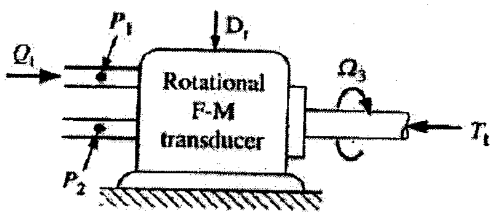
$F_t = D_t P_{12}$

$v_{3g} = \left(\frac{1}{D_t}\right) Q_t$

$P_{fluid} = P_{mech.}$

$P_{12} Q_t = v_{3g} F_t$

(a)



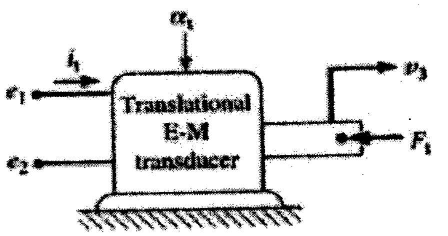
$T_t = D_t P_{12}$

$\Omega_{3g} = \left(\frac{1}{D_t}\right) Q_t$

$P_{fluid} = P_{mech.}$

$P_{12} Q_t = \Omega_{3g} T_t$

(b)



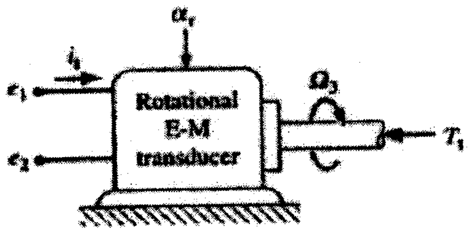
$v_{3g} = \alpha_t e_{12}$

$F_t = \left(\frac{1}{\alpha_t}\right) i_t$

$P_{elect.} = P_{mech.}$

$e_{12} i_t = v_{3g} F_t$

(a)



$\Omega_{3g} = \alpha_t e_{12}$

$T_t = \left(\frac{1}{\alpha_t}\right) i_t$

$P_{elect.} = P_{mech.}$

$e_{12} i_t = \Omega_{3g} T_t$

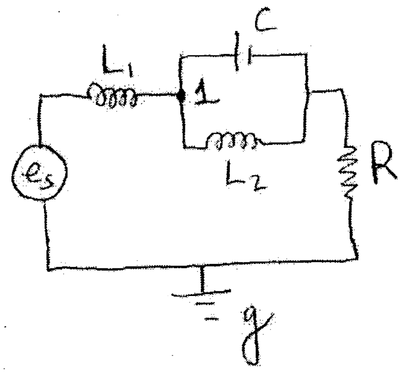
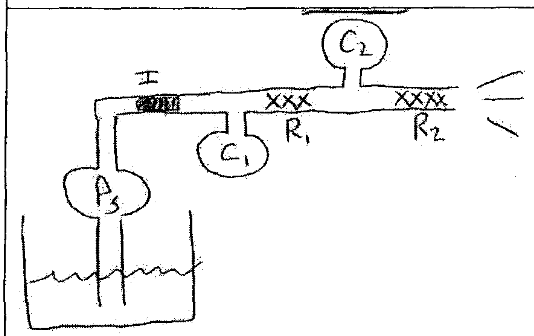
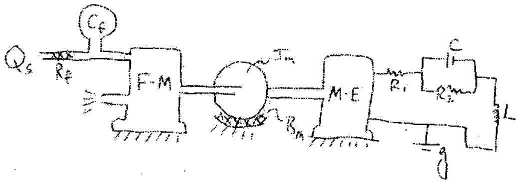
(b)

Number 1-3 are 25 points each.

For each system shown, derive a state-space model in the standard form:

$$\dot{\underline{q}} = A\underline{q} + B\underline{u} \quad , \quad \underline{y} = C\underline{q} + D\underline{u}$$

You must define the vectors $\underline{q}$, $\underline{u}$, and $\underline{y}$, and the matrices A, B, C, D.

	Input is the voltage source e_S Outputs are the voltage drop from node 1 to ground, and the current through the capacitor.
	Input is the pump pressure source P_S Outputs are the flow rate out the pipe end, and the flow rate through R_1
	Input is the fluid flow rate Q_S Outputs are the voltage through the electrical inductor, and the fluid flow rate through the F-M transducer

(15 pts) Find the conditions on the design variable k to insure that the settling time is less than 1 second, if the system's characteristic equation is given by

$$s^3 + ks^2 + 10s + 4 = 0$$

(10 pts): Find the transfer function for a system modeled by $\ddot{x} + 3\dot{x} + x = 2t + 13$, if the output is given by $4\dot{x}$

$$q = \begin{Bmatrix} i_{L_1} \\ e_c \\ i_{L_2} \end{Bmatrix} \qquad \underline{u} = \begin{Bmatrix} e_s \end{Bmatrix} \qquad \underline{y} = \begin{Bmatrix} e_c + e_R \\ i_c \end{Bmatrix}$$

$$(\dot{i}_{L_1}) = \frac{1}{L_1} e_{L_1} = \frac{1}{L_1}\left(e_s - e_c - e_R\right) = \frac{1}{L_1}\left(e_s - e_c - R i_{L_1}\right)$$

$$\dot{e}_c = \frac{1}{C} i_c = \frac{1}{C}\left(i_{L_1} - i_{L_2}\right)$$

$$(\dot{i}_{L_2}) = \frac{1}{L_2} e_{L_2} = \frac{1}{L_2}\left(e_c\right)$$

$$\underset{3\times3}{A} = \begin{bmatrix} -\frac{R}{L_1} & -\frac{1}{L_1} & 0 \\ \frac{1}{C} & 0 & -\frac{1}{C} \\ 0 & \frac{1}{L_2} & 0 \end{bmatrix} \qquad \underset{3\times1}{B} = \begin{Bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{Bmatrix}$$

$$\underset{2\times3}{C} = \begin{bmatrix} R & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \qquad \underset{2\times1}{D} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$q = \left\{ \begin{matrix} Q_I \\ P_{c_1} \\ P_{c_2} \end{matrix} \right\} \qquad \underline{u} = \left\{ P_s \right\} \qquad \underline{y} = \left\{ \begin{matrix} Q_{R_2} \\ Q_{R_1} \end{matrix} \right\}$$

$$\dot{Q}_I = \frac{1}{I}\left[P_I\right] = \frac{1}{I}\left(P_s - P_{c_1}\right)$$

$$\dot{P}_{c_1} = \frac{1}{C_1} Q_{c_1} = \frac{1}{C_1}\left(Q_I - Q_{R_1}\right) = \frac{1}{C_1}\left(Q_I - \frac{P_{c_1} - P_{c_2}}{R_1}\right)$$

$$\dot{P}_{c_2} = \frac{1}{C_2} Q_{c_2} = \frac{1}{C_2}\left(Q_{R_1} - Q_{R_2}\right) = \frac{1}{C_2}\left[\frac{P_{c_1} - P_{c_2}}{R_1} - \frac{P_{c_2}}{R_2}\right]$$

$$\underset{3\times 3}{A} = \begin{bmatrix} 0 & -\frac{1}{I} & 0 \\ \frac{1}{C_1} & -\frac{1}{C_1 R_1} & \frac{1}{C_1 R_1} \\ 0 & \frac{1}{C_2 R_1} & \left(-\frac{1}{C_2 R_1} - \frac{1}{C_2 R_2}\right) \end{bmatrix} \qquad \underset{3\times 1}{B} = \left\{ \begin{matrix} \frac{1}{I} \\ 0 \\ 0 \end{matrix} \right\}$$

$$\underset{2\times 3}{C} = \begin{bmatrix} 0 & 0 & -\frac{1}{R_2} \\ 0 & \frac{1}{R_1} & \frac{1}{R_2} \end{bmatrix} \qquad \underset{2\times 1}{D} = \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

$$\underline{q} = \begin{Bmatrix} P_C \\ \Omega \\ e_C \\ i_L \end{Bmatrix} \qquad \underline{u} = \{ Q_s \} \qquad \underline{y} = \begin{Bmatrix} e_L \\ Q_{EM} \end{Bmatrix}$$

$$\dot{P}_C = \frac{1}{C_f} Q_C = \frac{1}{C_f}\left(Q_s - Q_{EM}\right) = \frac{1}{C_f}\left(Q_s - D\Omega\right)$$

$$\dot{\Omega} = \frac{1}{J_M} T = \frac{1}{J_M}\left[T_{FM} - T_{B_m} - T_{ME}\right] = \frac{1}{J_M}\Bigg[D P_{FM} - B_m \Omega - \frac{1}{\alpha} i_{ME}$$

$$= \frac{1}{J_m}\left[D P_{C_f} - B_m \Omega - \frac{1}{\alpha} i_L\right]$$

$$\dot{e}_C = \frac{1}{C} i_C = \frac{1}{C}\left(i_L - i_{R_2}\right) = \frac{1}{C}\left(i_L - \frac{e_C}{R_2}\right)$$

$$\dot{i}_L = \frac{1}{L} e_L = \frac{1}{L}\left(e_{ME} - e_{R_1} - e_C\right) = \frac{1}{L}\left(\frac{1}{\alpha}\Omega - R_1 i_L - e_C\right)$$

$$\underset{4\times4}{A} = \begin{bmatrix} 0 & -D/C_f & 0 & 0 \\ D/J_m & -B_m/J_m & 0 & -\frac{1}{J_m \alpha} \\ 0 & 0 & -\frac{1}{C R_2} & \frac{1}{C} \\ 0 & \frac{1}{L\alpha} & -\frac{1}{L} & -\frac{R_1}{L} \end{bmatrix} \qquad \underset{4\times1}{B} = \begin{Bmatrix} \frac{1}{C_f} \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\underset{2\times4}{C} = \begin{bmatrix} 0 & \frac{1}{\alpha} & -1 & -R_1 \\ 0 & -D & 0 & 0 \end{bmatrix} \qquad \underset{2\times1}{D} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$s^3 + ks^2 + 10s + 4$ Settling time = 1 second

$\Rightarrow \tau = 1/4$ sec

$\therefore$ Axis-shift to -4

$s-4$

$(s-4)^2 = s^2 - 8s + 16$

$(s-4)^3 = s^3 - 12s^2 + 48s - 64$

$$\begin{aligned} &s^3 - 12s^2 + 48s - 64 \\ &\quad + ks^2 - 8ks + 16k \\ &\quad\quad + 10s - 40 \\ &\quad\quad\quad + 4 \end{aligned}$$

$$s^3 + \underbrace{(k-12)}_{k>12} s^2 + (58-8k)s + (16k-100) \qquad k < \frac{58}{8}$$

$\therefore$ Impossible

For 1-second shift

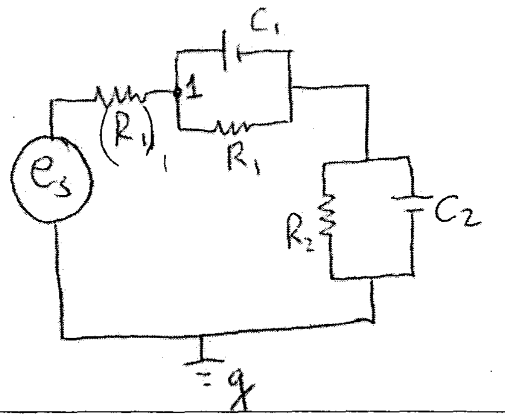
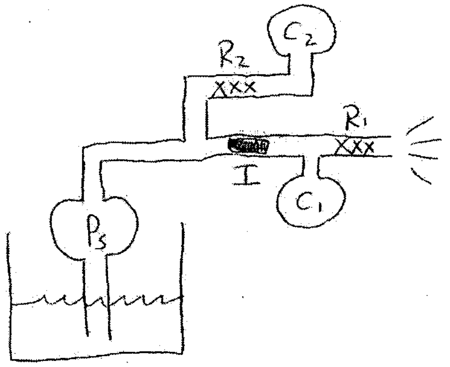
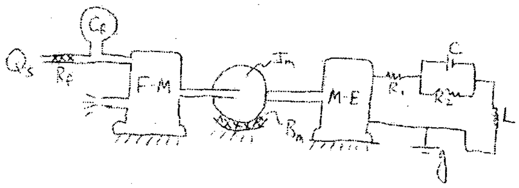
9 pts (from 15)

$\ddot{x} + 3\dot{x} + x = 2t + 13 = f(t)$

Take $\mathcal{L}(\ )$: $s^2X(s) + 3sX(s) + X(s) = F(s)$

$$\frac{X(s)}{F(s)} = \frac{1}{s^2+3s+1}$$

$$\frac{\mathcal{L}(4\dot{x})}{\mathcal{L}(f(t))} = \frac{4sX(s)}{F(s)} = \frac{4s}{s^2+3s+1}$$

	Input is the voltage source e_S Outputs are the voltage drop from node 1 to ground, and the current through the capacitor.
	Input is the pump pressure source P_S Outputs are the flow rate out the pipe end, and the flow rate through R_1
	Input is the fluid flow rate Q_S Outputs are the voltage through the electrical inductor, and the fluid flow rate through the F-M transducer

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$$\boxed{\underline{q} = \begin{Bmatrix} e_{c_1} \\ e_{c_2} \end{Bmatrix} \qquad \underline{u} = \begin{Bmatrix} e_s \end{Bmatrix} \qquad \underline{y} = \begin{Bmatrix} e_{c_1} + e_{c_2} \\ \dot{e}_{c_1} \end{Bmatrix}}$$

$$\dot{e}_{c_1} = \frac{1}{C_1} i_{c_1} = \frac{1}{C_1}\left[i_s - (i_{R_1})_{\substack{\text{Parallel} \\ \text{loop}}} \right] = \frac{1}{C_1}\left[\frac{\overset{\swarrow \text{Outside loop}}{e_{R_1}}}{R_1} - \frac{e_{c_1}}{R_1} \right]$$

$$= \frac{1}{C_1}\left[\frac{e_s - e_{c_1} - e_{c_2}}{R_1} - \frac{e_{c_1}}{R_1} \right] = \frac{1}{C_1}\left[\frac{e_s - e_{c_2}}{R_1} \right]$$

$$\dot{e}_{c_2} = \frac{1}{C_2} i_{c_2} = \frac{1}{C_2}\left[i_s - i_{R_2} \right] = \frac{1}{C_2}\left[\frac{e_s - e_{c_1} - e_{c_2}}{R_1} - \frac{e_{c_2}}{R_2} \right]$$

$$= \frac{1}{C_2}\left[\frac{e_s - e_{c_1} - e_{c_2}}{R_1} - \frac{e_{c_2}}{R_2} \right]$$

$$\underset{2\times 2}{A} = \begin{bmatrix} 0 & -\frac{1}{C_1 R_1} \\ -\frac{1}{C_2 R_1} & \left(-\frac{1}{C_2 R_1} - \frac{1}{C_2 R_2} \right) \end{bmatrix} \qquad \underset{2\times 1}{B} = \begin{Bmatrix} \frac{1}{C_1 R_1} \\ \frac{1}{C_2 R_1} \end{Bmatrix}$$

$$\underset{2\times 2}{C} = \begin{bmatrix} 1 & 1 \\ 0 & -\frac{1}{C_1 R_1} \end{bmatrix} \qquad \underset{2\times 1}{D} = \begin{Bmatrix} 0 \\ \frac{1}{C_1 R_1} \end{Bmatrix}$$

$$\underline{q} = \begin{Bmatrix} P_{C_1} \\ Q_I \\ P_{C_2} \end{Bmatrix} \qquad \underline{u} = \begin{Bmatrix} P_S \end{Bmatrix} \qquad \underline{y} = \begin{Bmatrix} Q_{R_1} \\ Q_{R_1} \end{Bmatrix}$$

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$$\underset{3\times3}{A} = \begin{bmatrix} -\frac{1}{C_1 R_1} & \frac{1}{C_1} & 0 \\ -\frac{1}{I} & 0 & 0 \\ 0 & 0 & -\frac{1}{C_2 R_2} \end{bmatrix} \qquad \underset{3\times1}{B} = \begin{bmatrix} 0 \\ \frac{1}{I} \\ \frac{1}{C_2 R_2} \end{bmatrix}$$

$$\underset{2\times3}{C} = \begin{bmatrix} -\frac{1}{R_1} & 0 & 0 \\ -\frac{1}{R_1} & 0 & 0 \end{bmatrix} \qquad \underset{2\times1}{D} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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$$\underset{4\times 4}{A} = \begin{bmatrix} 0 & -D/C_f & 0 & 0 \\ D/J_m & -B_m/J_m & 0 & -\frac{1}{J_m \alpha} \\ 0 & 0 & -\frac{1}{C R_2} & \frac{1}{C} \\ 0 & \frac{1}{L\alpha} & -\frac{1}{L} & -\frac{R_1}{L} \end{bmatrix} \qquad \underset{4\times 1}{B} = \begin{Bmatrix} \frac{1}{C_f} \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\underset{2\times 4}{C} = \begin{bmatrix} 0 & \frac{1}{\alpha} & -1 & -R_1 \\ 0 & -D & 0 & 0 \end{bmatrix} \qquad \underset{2\times 1}{D} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

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Settling time = 1 second

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$$\left.\begin{aligned} & s-4 \\ & (s-4)^2 = s^2 - 8s + 16 \\ & (s-4)^3 = s^3 - 12s^2 + 48s - 64 \end{aligned}\right\}$$

$$\begin{aligned} s^3 - 12s^2 + 48s - 64 \\ + ks^2 - 8ks + 16k \\ + 10s - 40 \\ + 4 \\ \hline \end{aligned}$$

$$s^3 + \underbrace{(k-12)}_{k>12}s^2 + (58-8k)s + (16k-100)$$

$$\underbrace{k > 12 \qquad K < \frac{58}{8}}$$

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$\ddot{x} + 3\dot{x} + x = 2t + 13 = f(t)$

Take $\mathcal{L}(\ )$: $s^2X(s) + 3sX(s) + X(s) = F(s)$

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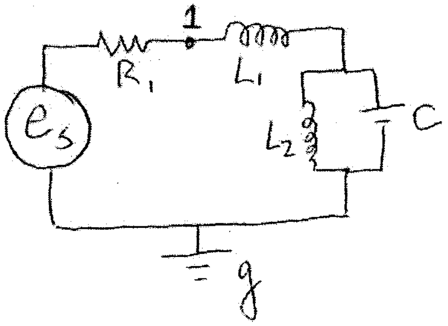
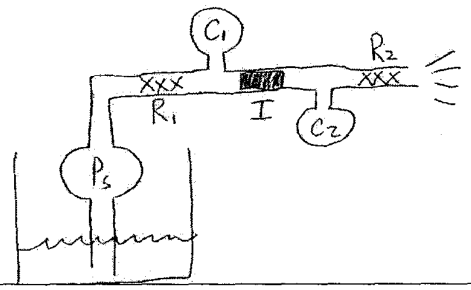
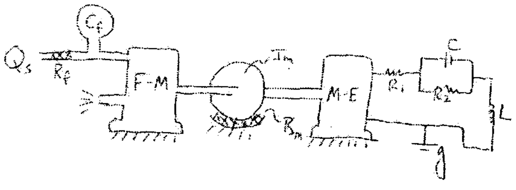
	Input is the voltage source e_S Outputs are the voltage drop from node 1 to ground, and the current through the capacitor.
	Input is the pump pressure source P_S Outputs are the flow rate out the pipe end, and the flow rate through R_1
	Input is the fluid flow rate Q_S Outputs are the voltage through the electrical inductor, and the fluid flow rate through the F-M transducer

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$$\underline{q} = \left\{ \begin{matrix} i_{L_1} \\ i_{L_2} \\ e_c \end{matrix} \right\} \qquad \underline{u} = \left\{ e_s \right\} \qquad \underline{y} = \left\{ \begin{matrix} e_s - e_{R_1} \\ i_c \end{matrix} \right\}$$

$$\dot{i}_{L_1} = \frac{1}{L_1} e_{L_1} = \frac{1}{L_1}\left(e_s - e_{R_1} - e_c\right) = \frac{1}{L_1}\left(e_s - R_1 i_{L_1} - e_c\right)$$

$$\dot{i}_{L_2} = \frac{1}{L_2} e_{L_2} = \frac{1}{L_2}\left(e_c\right)$$

$$\dot{e}_c = \frac{1}{C} i_c = \frac{1}{C}\left(i_{L_1} - i_{L_2}\right)$$

$$\underset{3\times3}{A} = \begin{bmatrix} -\frac{R_1}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & 0 & \frac{1}{L_2} \\ \frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix} \qquad \underset{3\times1}{B} = \left\{ \begin{matrix} \frac{1}{L_1} \\ 0 \\ 0 \end{matrix} \right\}$$

$$\underset{2\times3}{C} = \begin{bmatrix} -R_1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \qquad \underset{2\times1}{D} = \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\}$$

$$q = \left\{ \begin{matrix} P_{C_1} \\ P_{C_2} \\ Q_I \end{matrix} \right\} \qquad \underline{u} = \left\{ P_s \right\} \qquad y = \left\{ \begin{matrix} Q_{R_2} \\ Q_{R_1} \end{matrix} \right\}$$

$$\dot{P}_{C_1} = \frac{1}{C_1} Q_{C_1} = \frac{1}{C_1}\left(Q_{R_1} - Q_I\right) = \frac{1}{C_1}\left(\frac{P_s - P_{C_1}}{R_1} - Q_I\right)$$

$$\dot{P}_{C_2} = \frac{1}{C_2} Q_{C_2} = \frac{1}{C_2}\left(Q_I - Q_{R_2}\right) = \frac{1}{C_2}\left(Q_I - \frac{P_{C_2}}{R_2}\right)$$

$$\dot{Q}_I = \frac{1}{I} P_I = \frac{1}{I}\left(P_{C_1} - P_{C_2}\right)$$

$$\underset{3\times3}{A} = \begin{bmatrix} -\frac{1}{C_1R_1} & 0 & -\frac{1}{C_1} \\ 0 & -\frac{1}{C_2R_2} & \frac{1}{C_2} \\ \frac{1}{I} & -\frac{1}{I} & 0 \end{bmatrix} \qquad \underset{3\times1}{B} = \left\{ \begin{matrix} \frac{1}{C_1R_1} \\ 0 \\ 0 \end{matrix} \right\}$$

$$\underset{2\times3}{C} = \begin{bmatrix} 0 & \frac{1}{R_2} & 0 \\ -\frac{1}{R_1} & 0 & 0 \end{bmatrix} \qquad \underset{2\times1}{D} = \left\{ \begin{matrix} 0 \\ \frac{1}{R_1} \end{matrix} \right\}$$

$$q = \begin{Bmatrix} P_c \\ \Omega \\ e_c \\ i_L \end{Bmatrix} \qquad \underline{u} = \{ Q_s \} \qquad y = \begin{Bmatrix} e_L \\ Q_{EM} \end{Bmatrix}$$

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$$= \frac{1}{J_m}\left[D P_{C_f} - B_m \Omega - \frac{1}{\alpha} i_L\right]$$

$$\dot{e}_c = \frac{1}{C} i_c = \frac{1}{C}\left(i_L - i_{R_2}\right) = \frac{1}{C}\left(i_L - \frac{e_c}{R_2}\right)$$

$$\dot{i}_L = \frac{1}{L} e_L = \frac{1}{L}\left(e_{ME} - e_{R_1} - e_c\right) = \frac{1}{L}\left(\frac{1}{\alpha}\Omega - R_1 i_L - e_c\right)$$

$$\underset{4\times 4}{A} = \begin{bmatrix} 0 & -D/C_f & 0 & 0 \\ D/J_m & -B_m/J_m & 0 & -\frac{1}{J_m \alpha} \\ 0 & 0 & -\frac{1}{CR_2} & \frac{1}{C} \\ 0 & \frac{1}{L\alpha} & -\frac{1}{L} & -\frac{R_1}{L} \end{bmatrix} \qquad \underset{4\times 1}{B} = \begin{Bmatrix} \frac{1}{C_f} \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\underset{2\times 4}{C} = \begin{bmatrix} 0 & \frac{1}{\alpha} & -1 & -R_1 \\ 0 & -D & 0 & 0 \end{bmatrix} \qquad \underset{2\times 1}{D} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$s^3 + ks^2 + 10s + 4$

Settling time = 1 second

$\Rightarrow \tau = 1/4$ sec

$\therefore$ Axis shift to -4

$$\left.\begin{aligned} &s-4 \\ &(s-4)^2 = s^2 - 8s + 16 \\ &(s-4)^3 = s^3 - 12s^2 + 48s - 64 \end{aligned}\right\}$$

$$\begin{aligned} s^3 - 12s^2 &+ 48s - 64 \\ + ks^2 &- 8ks + 16k \\ &+ 10s - 40 \\ &+ 4 \end{aligned}$$

$$s^3 + \underbrace{(k-12)}_{k>12} s^2 + \underbrace{(58-8k)}_{k<\frac{58}{8}} s + (16k-100)$$

$\therefore$ Impossible

$\ddot{x} + 3\dot{x} + x = 2t + 13 = f(t)$

Take $\mathcal{L}(\ )$: $s^2 X(s) + 3s X(s) + X(s) = F(s)$

$$\frac{X(s)}{F(s)} = \frac{1}{s^2+3s+1}$$

$$\frac{\mathcal{L}(4\dot{x})}{\mathcal{L}(f(t))} = \frac{4s\,X(s)}{F(s)} = \frac{4s}{s^2+3s+1}$$