Name

MAE 340: Test I **February 11, 2003**

Number 1-20 are 4 points each; Questions 21 and 22 are 10 points each

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Answer questions 1-10 based on the following problem statement:

A system is modeled by the differential equations

$$\ddot{x}_1 = 3(\dot{x}_2 - \dot{x}_1) + 11(x_2 - x_1) + 2F_1(t) + 5F_2(t)$$
$$\ddot{x}_2 = -4(\dot{x}_2 - \dot{x}_1) - 25(x_2 - x_1) + 5F_2(t) + F_1(t)$$

where $F_1(t)$ and $F_2(t)$ are inputs. The outputs for this system are defined as

$$y_1 = 6(x_2 - x_1) + 2(\dot{x}_2 - \dot{x}_1) \quad , \quad y_2 = 3\dot{x}_1 - x_1 + 4F_1(t) + 2F_2(t)$$

Write the state-space and output equations for this system in the standard form

$$\underline{\dot{q}} = A\underline{q} + B\underline{u} \quad , \quad \underline{y} = C\underline{q} + D\underline{u}$$

1. The size of the A matrix is:
   (a) 4 x 2 (b) 2 x 4 (c) 2 x 2 (d) 4 x 4 (e) Other

2. The size of the B matrix is:
   (a) 4 x 2 (b) 4 x 4 (c) 2 x 2 (d) 2 x 4 (e) Other

3. The size of the C matrix is:
   (a) 4 x 2 (b) 2 x 4 (c) 2 x 2 (d) 4 x 4 (e) Other

4. The size of the D matrix is:
   (a) 4 x 4 (b) 4 x 2 (c) 2 x 2 (d) 2 x 4 (e) Other

5. Write a state vector for this system: $\begin{Bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}$ or $\begin{Bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{Bmatrix}$ etc.

6. Write the input vector for this system: $\underline{u} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$

7. Write the A matrix for this system:

* depends on q definition

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -11 & 11 & -3 & 3 \\ 25 & -25 & 4 & -4 \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -11 & -3 & 11 & 3 \\ 0 & 0 & 0 & 1 \\ 25 & 4 & -25 & -4 \end{bmatrix}$$

8. Write the B matrix for this system:

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2 & 5 \\ 1 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 2 & 5 \\ 0 & 0 \\ 1 & 5 \end{bmatrix}$$

9. Write the C matrix for this system:

$$C = \begin{bmatrix} -6 & 6 & -2 & 2 \\ -1 & 0 & 3 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} -6 & -2 & 6 & 2 \\ -1 & 3 & 0 & 0 \end{bmatrix}$$

10. Write the D matrix for this system:

$$D = \begin{bmatrix} 0 & 0 \\ 4 & 2 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 4 & 2 \end{bmatrix}$$

A system's characteristic equation is given by: $s^2 + 2s + 5 = 0$

$$s_{1,2} = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm 2i$$

11. Write the general expression for the homogeneous solution:

$$x_H(t) = C_1 e^{-t} \sin(2t + \phi)$$

12. The time constant of the homogeneous solution is:

(a) 1 sec (b) π sec (c) ½ sec (d) Other (e) Not enough information

13. The settling time of the homogeneous solution is:

a) 1 sec (b) 2 sec (c) 4/3 sec (d) Other (e) Not enough information

14. The ~~period of the~~ homogeneous solution is:

a) 1 sec (b) π sec (c) ½ sec (d) Other (e) Not enough information

15. The value of the solution at t = 1 sec is:

a) 0.37 (b) 0.02 (c) 0.63 (d) Other (e) Not enough information

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16. Assume ~~that a~~ system is modeled by $\ddot{x} + 3\dot{x} + 10x = 4\sin 3t$. The system is:

(a) Unstable (b) Stable (c) Marginally stable (d) Not enough information to know

17. Assume that a system is modeled by $\ddot{x} + 18x = 0$. The system is:

(a) Unstable (b) Stable (c) Marginally stable (d) Not enough information to know

18. Assume that a system is modeled by $\ddot{x} - 2\dot{x} - 12x = e^{-2t}$. The system is:

(a) Unstable (b) Stable (c) Marginally stable (d) Not enough information to know

19. A system is modeled by $\ddot{x}+7\dot{x}+12x=0$. The settling time of the system is:

(a) 3 secs (b) 1 sec (c) 1/3 sec (d) 4 sec (e) Other

$s_{1,2} = \frac{-7 \pm \sqrt{49-48}}{2} = -\frac{7}{2} \pm \frac{1}{2} = -3, -4$

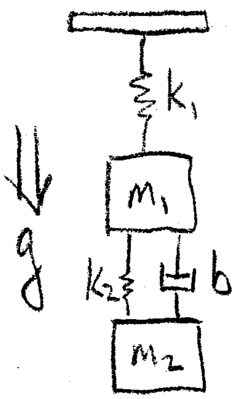
20. The particular solution for $\ddot{x}+2\dot{x}+x=2t+5$ is:

(a) $x_p = 6t+7$ (b) $x_p = 2t+1$ (c) $x_p = 2t+5$ (d) Not enough info

(e) None of the above

$x_p = c_1 t + c_2$, $\dot{x}_p = c_1$ $\Big\}$ $c_1 = 2$, $c_1 = 2$; $c_2 = 5-4 = 1$

21. For the system shown, find the ordinary differential equation model.



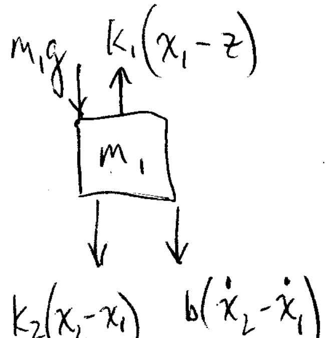
z = input displacement

$x_1 = 0$ at $z=0$
unstretched position k_1

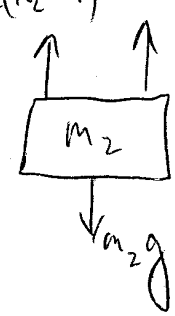
$x_2 = 0$ at $z=0$
unstretched position k_1 & k_2

z = position of base (input)

x_1, x_2 = position of masses m_1, m_2



$$m_1\ddot{x}_1 = m_1 g + k_2(x_2-x_1) + b(\dot{x}_2-\dot{x}_1) - k_1(x_1-z)$$



$$m_2\ddot{x}_2 = m_2 g - k_2(x_2-x_1) - b(\dot{x}_2-\dot{x}_1)$$

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22. Sketch the solution of $4\ddot{x}+8\dot{x}+104x=8$ for the time period $t=0$ to the settling time, with $x(0)=10$ and $\dot{x}(0)=0$.

Settling time =

$\ddot{x}+2\dot{x}+26x=2 \Rightarrow x_p = \frac{2}{26}$

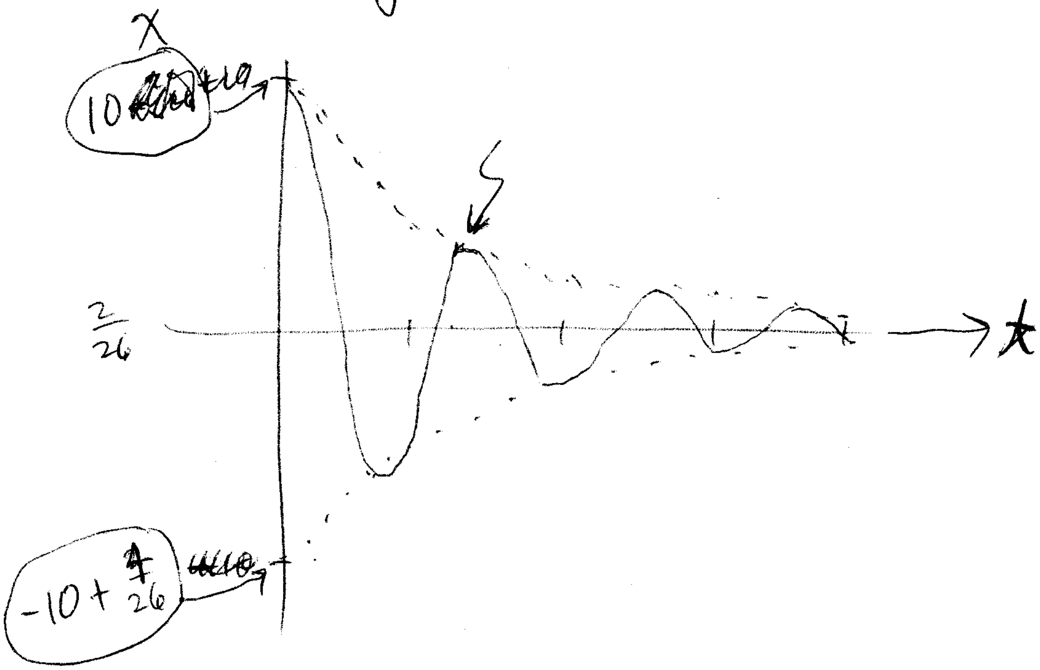
$s_{1,2} = \frac{-2 \pm \sqrt{4-104}}{2} = -1 \pm 5i$

$x(t) = C_1 e^{-t}\sin(5t+\phi) + \frac{2}{26}$

$x(0)=10 \Rightarrow C_1 + \frac{2}{26}$; $\dot{x}(0)=0 \Rightarrow \phi = 0$

$$x(t) = C_1 e^{-t} \sin(5t + \phi) + 2/26 \quad , \quad x(0) = 10$$
$$\dot{x}(0) = 0$$

Settling time = 4 secs ; period = $\frac{2\pi}{5}$ secs = 1.2566 secs



Approx 3.2 cycles to settle.

Name

MAE 340: Test I **February 11, 2003**

Number 1-20 are 4 points each; Questions 21 and 22 are 10 points each

Answer questions 1-10 based on the following problem statement:

A system is modeled by the differential equations

$$\ddot{x}_1 = 11(\dot{x}_2 - \dot{x}_1) + 3(x_2 - x_1) + 5F_1(t) + 2F_2(t)$$
$$\ddot{x}_2 = -25(\dot{x}_2 - \dot{x}_1) - 4(x_2 - x_1) + F_2(t) + 5F_1(t)$$

where $F_1(t)$ and $F_2(t)$ are inputs. The outputs for this system are defined as

$$y_1 = 2(x_2 - x_1) + 6(\dot{x}_2 - \dot{x}_1) \quad , \quad y_2 = 4\dot{x}_1 - x_1 + 2F_1(t) + 3F_2(t)$$

Write the state-space and output equations for this system in the standard form

$$\dot{\underline{q}} = A\underline{q} + B\underline{u} \quad , \quad \underline{y} = C\underline{q} + D\underline{u}$$

1. The size of the A matrix is:
   (a) 2 x 4 (b) 4 x 4 (c) 2 x 2 (d) 4 x 2 (e) Other

2. The size of the B matrix is:
   (a) 2 x 4 (b) 4 x 2 (c) 2 x 2 (d) 4 x 4 (e) Other

3. The size of the C matrix is:
   (a) 4 x 2 (b) 2 x 2 (c) 2 x 4 (d) 4 x 4 (e) Other

4. The size of the D matrix is:
   (a) 2 x 2 (b) 2 x 4 (c) 4 x 2 (d) 4 x 4 (e) Other

5. Write a state vector for this system: $\underline{q} = \begin{Bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{Bmatrix}$ or $\underline{q} = \begin{Bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}$

6. Write the input vector for this system: $\underline{u} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$

7. Write the A matrix for this system:

Depends on $\underline{q}$ definition

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -3 & -11 & 3 & 11 \\ 0 & 0 & 0 & 1 \\ 4 & 25 & -4 & -25 \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 3 & -11 & 11 \\ 4 & -4 & 25 & -25 \end{bmatrix}$$

8. Write the B matrix for this system:

$$B = \begin{bmatrix} 0 & 0 \\ 5 & 2 \\ 0 & 0 \\ 5 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 5 & 2 \\ 5 & 1 \end{bmatrix}$$

9. Write the C matrix for this system:

$$C = \begin{bmatrix} -2 & -6 & 2 & 6 \\ -4 & 4 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & 2 & -6 & 6 \\ -4 & 0 & 4 & 0 \end{bmatrix}$$

10. Write the D matrix for this system:

$$D = \begin{bmatrix} 0 & 0 \\ 2 & 3 \end{bmatrix}$$

A system's characteristic equation is given by: $s^2 + 4s + 13 = 0$

$$s_{1,2} = \frac{-4 \pm \sqrt{16-52}}{2} = \frac{-4 \pm 6i}{2} = -2 \pm 3i$$

11. Write the general expression for the homogeneous solution:

$$x_H(t) = C_1 e^{-2t} \sin(3t + \phi)$$

12. The time constant of the homogeneous solution is:
(a) 1 sec (b) π sec (c) ½ sec (d) Other (e) Not enough information

13. The settling time of the homogeneous solution is:
a) 1 sec (b) 2 sec (c) 4/3 sec (d) Other (e) Not enough information

14. The period of the homogeneous solution is:
a) 3 sec (b) π sec (c) ½ sec (d) Other (e) Not enough information

15. The value of the solution at t = 1 sec is:
a) 0.37 (b) 0.02 (c) 0.63 (d) Other (e) Not enough information

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16. Assume that a system is modeled by $\ddot{x} + 3\dot{x} - 10x = e^{-2t}$. The system is:
(a) Unstable (b) Stable (c) Marginally stable (d) Not enough information to know

17. Assume that a system is modeled by $\ddot{x} + 6\dot{x} + 18x = 4\sin 3t$. The system is:
(a) Unstable (b) Stable (c) Marginally stable (d) Not enough information to know

18. Assume that a system is modeled by $\ddot{x} + 12x = 0$. The system is:
(a) Unstable (b) Stable (c) Marginally stable (d) Not enough information to know

19. A system is modeled by $\ddot{x}+5\dot{x}+6x=0$. The settling time of the system is:

(a) 2 secs (b) 1 sec (c) 1/2 sec (d) 4 sec (e) Other

$$s_{1,2} = \frac{-5 \pm \sqrt{25-24}}{2} = \frac{-5 \pm 1}{2} = -2, -3$$

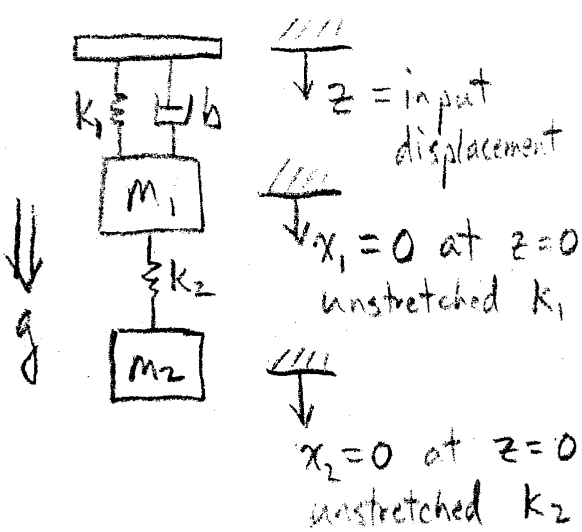
20. The particular solution for $\ddot{x}+\dot{x}+x=6t+13$ is:

(a) $x_p = 6t+7$ (b) $x_p = 2t+1$ (c) $x_p = 6t+13$ (d) Not enough info

(e) None of the above

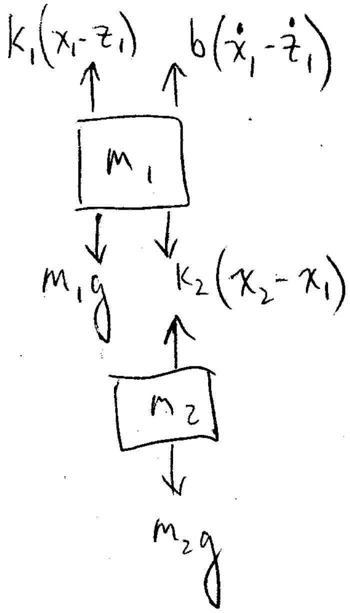
$$\left.\begin{aligned} x_p &= c_1 t + c_2 \\ \dot{x}_p &= c_1 \end{aligned}\right\} \quad \begin{aligned} c_1 &= 6 \\ c_2 &= 13-6 = 7 \end{aligned}$$

21. For the system shown, find the ordinary differential equation model.



z = position of base (input)

x_1, x_2 = position of masses m_1, m_2



$$m_1\ddot{x}_1 = m_1 g + k_2(x_2-x_1) - k_1(x_1-z_1) - b(\dot{x}_1-\dot{z}_1)$$

$$m_2\ddot{x}_2 = m_2 g - k_2(x_2-x_1)$$

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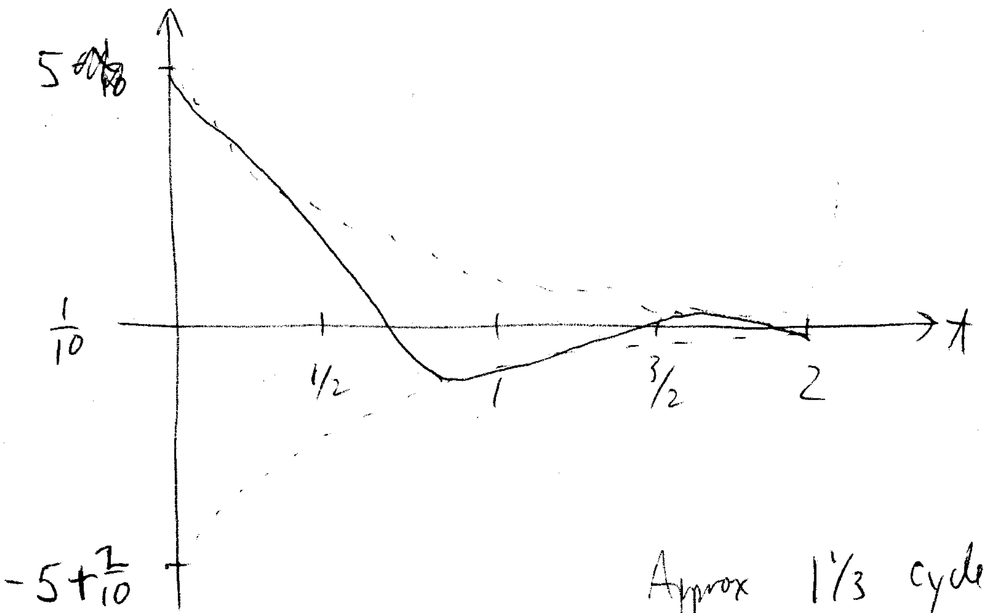
22. Sketch the solution of $4\ddot{x}+16\dot{x}+80x=8$ for the time period $t=0$ to the settling time, with $x(0)=5$ and $\dot{x}(0)=0$.

$$\ddot{x} + 4\dot{x} + 20x = 2 \Rightarrow x_p = \frac{1}{10}$$

$$s_{1,2} = \frac{-4 \pm \sqrt{16-80}}{2} = -2 \pm 4i$$

$$\left.\begin{aligned} &\text{Settling time} = 2 \text{ secs} \\ &\text{Period} = \frac{2\pi}{4} \approx 1.57 \text{ secs} \end{aligned}\right\} \rightarrow$$



Approx 1 1/3 cycles to settle

Name

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Answer questions 1-10 based on the following problem statement:

A system is modeled by the differential equations

$$\ddot{x}_1 = -25(\dot{x}_2 - \dot{x}_1) - 4(x_2 - x_1) + F_1(t) + 5F_2(t)$$
$$\ddot{x}_2 = 11(\dot{x}_2 - \dot{x}_1) + 3(x_2 - x_1) + 5F_2(t) + 2F_1(t)$$

where $F_1(t)$ and $F_2(t)$ are inputs. The outputs for this system are defined as

$$y_1 = 4(x_2 - x_1) - (\dot{x}_2 - \dot{x}_1) + 2F_1(t) + 3F_2(t) \quad , \quad y_2 = 2\dot{x}_2 + 6x_1$$

Write the state-space and output equations for this system in the standard form

$$\dot{\underline{q}} = A\underline{q} + B\underline{u} \quad , \quad \underline{y} = C\underline{q} + D\underline{u}$$

1. The size of the A matrix is:
   (a) 4 x 4 (b) 2 x 4 (c) 2 x 2 (d) 4 x 2 (e) Other

2. The size of the B matrix is:
   (a) 2 x 4 (b) 2 x 2 (c) 4 x 2 (d) 4 x 4 (e) Other

3. The size of the C matrix is:
   (a) 2 x 4 (b) 4 x 4 (c) 4 x 2 (d) 2 x 2 (e) Other

4. The size of the D matrix is:
   (a) 4 x 4 (b) 2 x 4 (c) 4 x 2 (d) 2 x 2 (e) Other

5. Write a state vector for this system: $\underline{q} = \begin{Bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}$ or $\underline{q} = \begin{Bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{Bmatrix}$

6. Write the input vector for this system: $\underline{u} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$

7. Write the A matrix for this system:

Depends on q definition

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 4 & -4 & 25 & -25 \\ -3 & 3 & -11 & 11 \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 4 & 25 & -4 & -25 \\ 0 & 0 & 0 & 1 \\ -3 & -11 & 3 & 11 \end{bmatrix}$$

8. Write the B matrix for this system:

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 5 \\ 2 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 1 & 5 \\ 0 & 0 \\ 2 & 5 \end{bmatrix}$$

9. Write the C matrix for this system:

$$C = \begin{bmatrix} -4 & 4 & 1 & -1 \\ 6 & 0 & 0 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} -4 & 1 & 4 & -1 \\ 6 & 0 & 0 & 2 \end{bmatrix}$$

10. Write the D matrix for this system:

$$D = \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}$$

A system's characteristic equation is given by: $s^2 + 6s + 10 = 0$

$$s_{1,2} = \frac{-6 \pm \sqrt{36-40}}{2} = -3 \pm 1i$$

11. Write the general expression for the homogeneous solution:

$$x_H(t) = C_1 e^{-3t} \sin(t + \phi)$$

12. The time constant of the homogeneous ~~solution~~ is:

(a) 1 sec (b) π sec (c) ½ sec (d) Other (e) Not enough information

13. The settling time of the homogeneous solution is:

a) 1 sec (b) 2 sec (c) 4/3 sec (d) Other (e) Not enough information

14. The period of the homogeneous solution is:

a) 3 sec (b) π sec (c) 2π sec (d) Other (e) Not enough information

15. The value of the solution at t = 1 sec is:

a) 0.37 (b) 0.02 (c) 0.63 (d) Other (e) Not enough information

---

16. Assume that a system is modeled by $\ddot{x} + 4x = 0$. The system is:

(a) Unstable (b) Stable (c) Marginally stable (d) Not enough information to know

17. Assume that a system is modeled by $\ddot{x} - \dot{x} + 12x = e^{-2t}$. The system is:

(a) Unstable (b) Stable (c) Marginally stable (d) Not enough information to know

18. Assume that a system is modeled by $\ddot{x} + 2\dot{x} + 12x = 6\sin 2t$. The system is:

(a) Unstable (b) Stable (c) Marginally stable (d) Not enough information to know

19. A system is modeled by $\ddot{x}+4\dot{x}+3x=0$. The settling time of the system is:

(a) 2 secs (b) 1 sec (c) 3 sec (d) 4 sec (e) Other

$s_{1,2} = \frac{-4 \pm \sqrt{16-12}}{2} = \frac{-4 \pm 2}{2} = -1, -3$

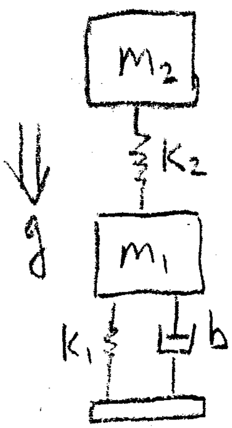
20. The particular solution for $\ddot{x}+3\dot{x}+x=2t+13$ is:

(a) $x_p = 6t+7$ (b) $x_p = 2t+1$ (c) $x_p = 2t+13$ (d) Not enough info

(e) None of the above

$x_p = c_1 t + c_2$, $\dot{x}_p = c_1$; $c_1 = 2$, $c_2 = 13 - 6 = 7$

21. For the system shown, find the ordinary differential equation model.

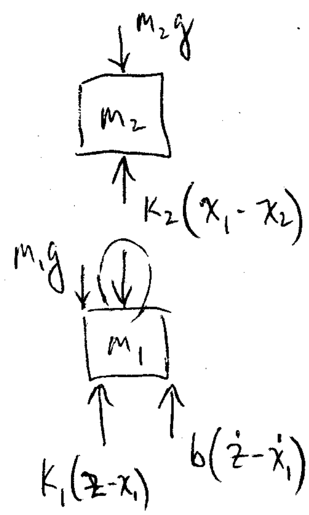


$x_2 = 0$ at $z = 0$
at unstretched k_2

z = position of base (input)
x_1, x_2 = position of masses m_1, m_2

$x_1 = 0$ at $z = 0$
at unstretched k_1

z = input displacement



$$m_2 \ddot{x}_2 = k_2(x_1 - x_2) - m_2 g$$

$$m_1 \ddot{x}_1 = k_1(z - x_1) + b(\dot{z} - \dot{x}_1) - m_1 g - k_2(x_1 - x_2)$$

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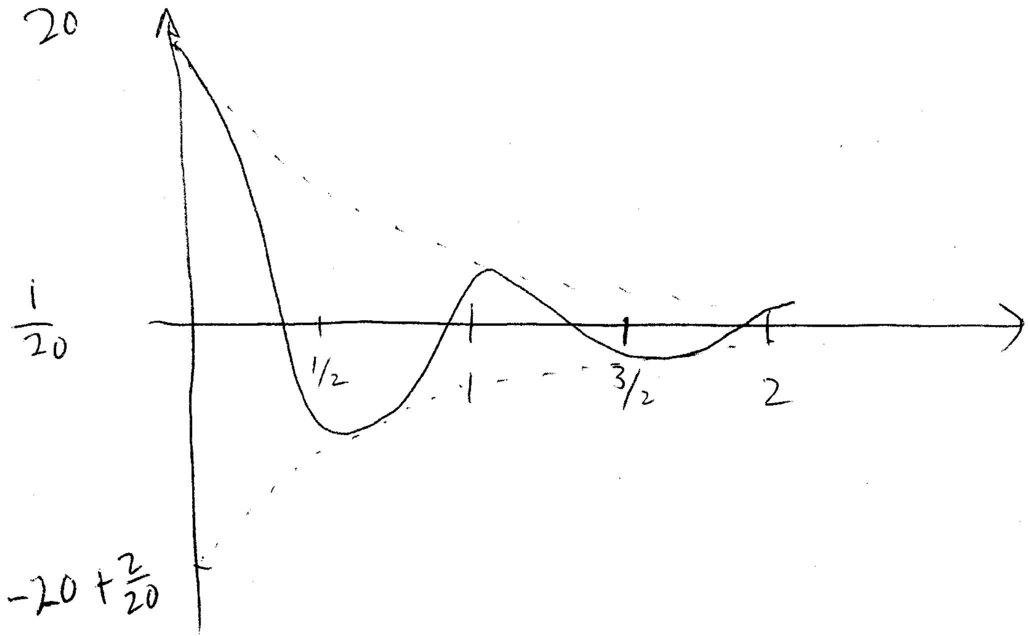
22. Sketch the solution of $3\ddot{x}+12\dot{x}+120x=6$ for the time period $t=0$ to the settling time, with $x(0)=20$ and $\dot{x}(0)=0$.

$\ddot{x} + 4\dot{x} + 40x = 2 \Rightarrow x_p = 1/20$

$s_{1,2} = \frac{-4 \pm \sqrt{16-160}}{2} = -2 \pm 6i$

Settling time 2 secs
Period $= \frac{2\pi}{6} = 1.05$ secs



20
$\frac{1}{20}$
$-20+\frac{2}{20}$
1/2
1
3/2
2