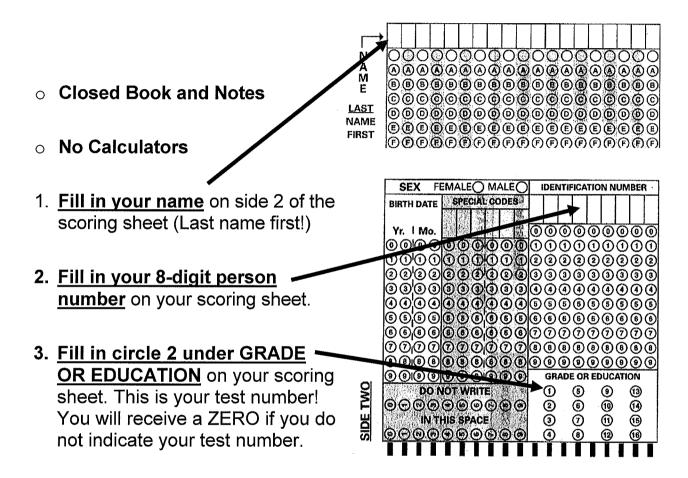
MAE334 - Introduction to Instrumentation and Computers

Final Exam

December 11, 2006



- For each question, choose <u>THE BEST ANSWER</u> and mark the corresponding answer on the scoring sheet.
- There is only 1 best answer per question.

The last page of the exam has the "Student-t Distribution Table" and the "Probability Values for Normal Error Function $z_1 = \frac{x_1 - x'}{\sigma}$ "

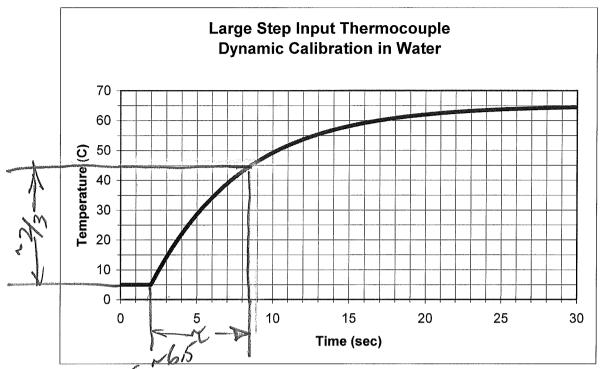


Figure 1. Data set from Lab 2 dynamic calibration.

The approximate time constant, τ , of the thermocouple response plotted in

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- a. 24 seconds
- b. 5 seconds
- c. 6 seconds
 - d. 10 seconds
- e. 15 seconds
- 2. If a thermocouple is more sensitive (the static sensitivity is larger) the dynamic response would be faster (the time constant would be smaller).
 - a. True
 - (b) False
- 3. The ADC used in the lab would output what binary value corresponding to -4?
 - a. 00000000100
 - (b) 111111111100
 - c. 100000000100
 - d. 111111111011
 - e. None of the above
- 4. The ADC used in our lab has what type of architecture?
 - a. Flash
 - **(b)** Successive approximation
 - c. Pipelined
 - d. Sigma-delta

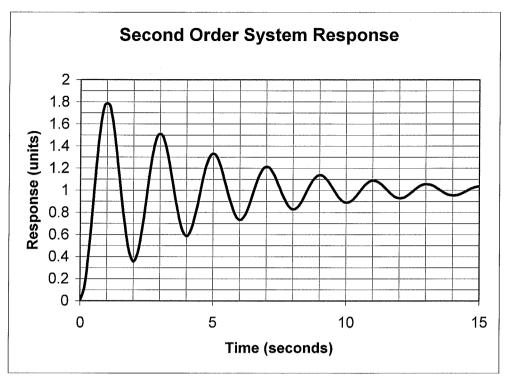


Figure 2. Pressure transducer time response to a step input function.

- 5. The rise time in seconds of the pressure transducer plotted in Figure 2 is approximately
 - a. 10.
 - **(**b) 0.5
 - c. 1.0
 - d. None of the above
- 6. The natural frequency of the pressure transducer plotted in Figure 2 is very close to
 - a. 10 Hz
 - **√**0 0.5 Hz
 - c. 1.0 Hz
 - d. None of the above
- 7. The ADC architecture normally associated with the best resolution is
 - a. Successive approximation
 - b. Flash
 - c. Pipelined
 - (d) Sigma-delta
- 8. A 55 Hz sine wave sampled at 100 Hz will result in a sampled data set with what frequency
 - a. 5 Hz
 - (b) 45 Hz
 - c. 55 Hz
 - d. none of the above

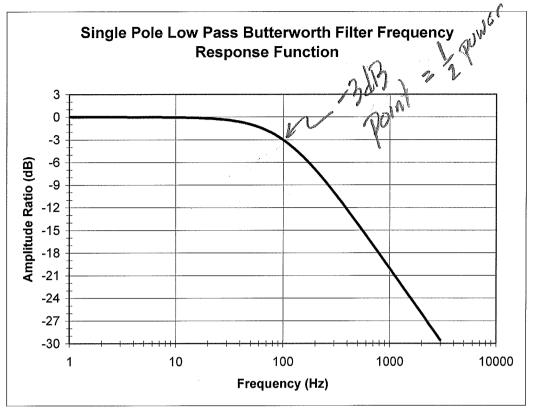
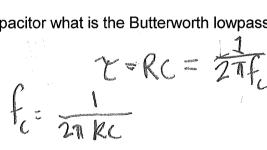


Figure 3. Filter amplitude ratio, M(f), of a low pass filter.

9. What is the cut-off frequency of the filter whose response function is plotted in Figure 3?

@ -3 dB point

- ິ a.) 100 Hz
 - б. 9 Hz
 - c. 50 Hz
 - d. 200 Hz
 - e. None of the above
- 10. Given a 5000 Ω resistor and 1 μ F capacitor what is the Butterworth lowpass filter corner frequency in Hertz?
 - a. 5000 * 10⁻⁶
 - b. $2\pi * 5000 * 10^{-6}$
 - C.) $1/(2\pi * 5000 * 10^{-6})$
 - d. 1/(5000 * 10⁻⁶)
 - e. None of the above



- 11. What is the input impedance of an ideal op amp?
 - a. Zero
 - /b). Infinity
 - c. None of the above
- 12. What is the output impedance of an ideal op amp?
 - (a) Zero
 - b. Infinity
 - c. None of the above

13. The static sensitivity of a thermistor is normally considered to be a constant over the temperature range it was design to be used in. a. True constant over limited range (designed range) 14. In lab 2 the thermocouple response was linearized by taking the natural log of the function, $\ln\left[\frac{(T_0-T(t))}{(T_0-T_\infty)}\right]$, where t is time and T, is temperature.

a. True
b) False

15. Repeated measurements of a static temperature reading will a. Can be used to determine the measurement system precision b. Have a normal distribution c. Show the bias error ∕d. All of the above e. None of the above 16. An 8 bit ADC with an ±12.8 volt input signal range subjected to a 2.26 volt signal will output a value. (a.)23 b. 2 c. 22 d. 46 e. None of the above $Q = \frac{t_{rse}}{2^{m}} = \frac{t_{12.8}}{2^{s}} = \frac{25.6}{2^{s}} = 0,1$ A temperature sensor is to be selected to measure the fluctuating temperature within a cylinder of an internal combustion engine. It is suspected that the temperature will behave as a periodic waveform with a frequency around 180 radians/second. (Rotating at 1800 rpm). Several size sensors are available, each with a known time constant. 17. What percent reduction in output/input signal magnitude would you expect at the 1800 cycle/minute frequency from a thermocouple with a 1/9 of a second time constant? (assume $\pi = 3$ and static sensitivity, K=1) a. 70% b. 5% c. 30% (d) 95% e. None of the above 18. If you were required to maintain a dynamic error of less than 29.3% $(M(\omega) \ge 70.7\% = 1/\sqrt{2})$ for the internal combustion engine temperature measurement described above what would be an acceptable thermocouple time constant?

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a. 1/60 seconds
b. 1/180 seconds
c. 1/90 seconds
d. 1/30 seconds
e. None of the above

19. Thermistors are normally not as sensitive as RTDs, but are much less expensive to manufacture.	
a. True MINE SENSTIVE, NOT AS ALCHEME, LESS	expensive
20. An inclined manometer with an indicating leg at 30° containing colored water (specific weight, $\gamma = 1.0$) is used to measure pressure. What is the static sensitivity of the manometer in (Inches of Water/Inches of deflection)? a. 1.5 b) 0.5 c. 1.0 d. 2.0 e. None of the above	
21. A strain-gauge equipped diaphragm pressure transducer is a null device with a dynamic behavior described as a second-order system.	
a. True (b) False defloction	
22. An under damped second order system will <i>always</i> oscillate with a greater amplitude than the forcing when the input forcing is at the natural frequency. a. True (b) False NUT ALWAYS	
23. The precision error associated with the ADC used in our lab can not be less	
than a. $20/2^{12}$ Volts b. $10/200/2^{12}$ Volts c) $20/200/2^{12}$ Volts d. None of the above Range ± 10 MAX GAIN 2200 2 BIT	
24. It is known that the statistics of a normally distributed temperature signal are x' = 20 °C and σ^2 = 4 °C ² . What is the probability that a measurement will yield a	
value outside the range of 16 to 24 °C?	
(b) 5% \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	
c. 32% d. 48% e. 52%	24-10 = 2
25. The input impedance of a deflection device such as a Bourdon Tube pressure gauge is inversely proportional to static sensitivity.	1971 m 70-1
True 5. False	47.72% m 20-20
26. An extraneous variable in an experiment usually refers to all possible unaccounted for or uncontrollable variables that can affect the value of the measured variable.	95.44% M16-24
a. True	5% outside
(b.) False	16-24

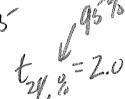
Table 1. Sample data set with a normal distribution, a mean value of 1.0 and a standard deviation of 0.15 and a plot of the data set with a linear curve fit added.

olai	luaru	uc	viation	01 0. 1	is and a plot of the data set with a linear curve it adde
i	Xi	j	X _i	1.40	
1	0.98	14	1.02		
2	1.07	15	0.94	1.30 -	Y
3	0.86	16	1.11		ϕ $y = -0.0019x + 1.0276$
4	1.16	17	0.99	1.20 -	$R^2 = 0.008$
5	0.96	18	0.78		♦
6	0.68	19	1.06	1.10 -	│
7	1.34	20	0.96		
8	1.04	21	0.99	1.00 -	
တ	1.21	22	1.02		\
10	0.86	23	1.10	0.90 -	♦
11	1.02	24	0.98		
12	1.26	25	0.97	0.80 -	
				0.70 -	•
				0.60	0 5 10 15 20 25
13	1.08				0 5 10 15 20 25

27. Given the data set in Table 1, what is the probability of recording a value within the range of 1.0 ± 0.30

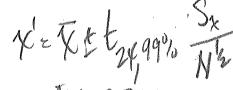
- a 95%
- b. 50%
- c. 90%
- d. 99%
- e. None of the above





28. Given the data set in Table 1, give an estimate of the true mean value of the measurand at 99% probability

- a. $x' = \overline{x} \pm (2.787 \times 0.15)$
- b. $x' = \overline{x} \pm (2.787 \times 0.03)$
- c. $x' = \overline{x} \pm (2.797 \times 0.03)$
- d. $x' = \overline{x} \pm (2.797 \times 0.15)$
- e. None of the above



29. The line fit to the data set in Table 1 has how many degrees of freedom?

- c. 2 22



30. The correlation coefficient, R² value of 0.008, indicates a high quality fit to the data in Table 1.

- a. True

Poor quarry

31. The frequency spectrum of a data set sampled at 100 samples/second for 2 seconds will have a frequency spacing or resolution in frequency space of

- e. 100 Hz
- 32. The frequency spectrum of a data set sampled at 200 samples/second for 10 seconds will have a maximum frequency of

- Forma = Nyanist = SAMPLEPATE = 100 HZ
- 33. A tachometer has an analog display dial graduated in 5 revolutions per minute (rpm) increments. Estimate the zero order uncertainty in this instrument

- 14Q = 145 = 125 pm
- 34. The users manual states an accuracy of 1% of the reading for a tachometer with an analog display dial graduated in 5 revolutions per minute (rpm) increments. Estimate the design stage uncertainty at 5000 rpm.

d.
$$\pm (2525)^{\frac{1}{2}}$$

- 1502 + 2,52 = \$\frac{12506.25}{} = 50.
- 35. The cooling of a thermometer can be modeled as a first-order system with $\Gamma = e^{-t/\tau}$. If Γ can be measured within 2% and time within 1%, what is the uncertainty, u_{τ} , in the ability to determine the time constant, τ . Remember $\tau = -t/\ln\Gamma$.

a.
$$u_{\tau} = \pm \left[\left(\frac{\partial \tau}{\partial t} u_{t} \right)^{2} + \left(\frac{\partial \tau}{\partial \Gamma} u_{\Gamma} \right)^{2} \right]^{1/2}$$

b.
$$u_{\tau} = \pm \left[\left(\frac{u_{t}}{\ln \Gamma} \right)^{2} + \left(\frac{u_{\Gamma}}{\Gamma(\ln \Gamma)^{2}} \right)^{2} \right]^{1/2}$$

d. None of the above

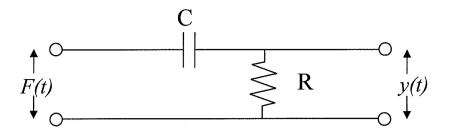


Figure 4. Filter Circuit

- 36. The filter depicted in Figure 4 is a
 - a. High-pass Bessel filter
 - (b) High-pass Butterworth filter
 - c. Low-pass Bessel filter
 - d. Low-pass Butterworth filter

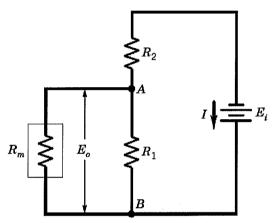


Figure 5. Instruments in parallel to signal path form an equivalent voltage dividing circuit.

37. Referring to Figure 5. If, R_m , is the resistance of the meter used to measure a thermistor whose resistance is R_1 what is the equivalent resistance, R_L , of the parallel loop formed by R_m and R_1 ?

a.
$$R_L = R_1 + R_m$$

$$\begin{array}{c}
\text{b.} R_L = \frac{R_1 R_m}{R_1 + R_m}
\end{array}$$

$$c. \quad R_L = \frac{R_1 + R_m}{R_1 R_m}$$

- 38. Referring to Figure 5. The loading error goes to zero as R_m goes to zero.
 - a. True /b,/ False



- 39. Referring to Figure 5. The impedance of the meter used to measure the resistance R_1 is
 - (a) R_m
 - ້b. 1/*R_m*

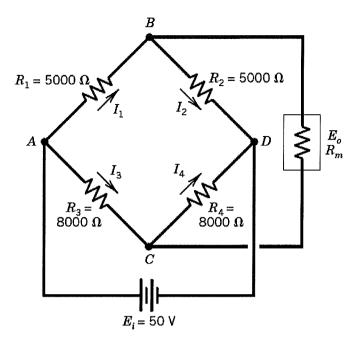


Figure 6. Wheatstone bridge circuit for question 40.

40. If the meter has infinite impedance the bridge in Figure 6 has an output voltage, E_o , of

a.
$$E_o = E_i \left(\frac{(R_1 + R_2)(R_3 + R_4)}{R_1 + R_2 + R_3 + R_4} \right)$$

(b.)
$$E_o = E_i \left(\frac{R_1}{R_1 + R_2} + \frac{R_3}{R_3 + R_4} \right)$$

c.
$$E_o = E_i \left(\frac{R_1 R_3}{R_1 + R_2 + R_3 + R_4} \right)$$

d.
$$E_o = E_i \left(\frac{(R_1 + R_3)(R_2 + R_4)}{R_1 + R_2 + R_3 + R_4} \right)$$

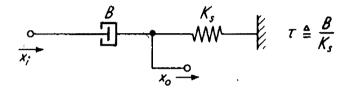
41. At resistances given for the bridge in Figure 6 this bridge can be said to be balanced.

42. The error function of a thermocouple subjected to a step input will vary from

a.
$$T_0$$
 to T_∞

- e. None of the above
- 43. Flow variables should be measured with a sensor with very high input impedance.
 - a. True
 - √b.) False

- 44. The output impedance of a thermistor circuit should be much greater than the input impedance of the volt meter used to record the output.
 - a. True
- 45. If we want to maximize the power output of an audio speaker system the output impedance of the amplifier driving the speakers should be the same as the input impedance of the speakers.
 - (a) True
 - b. False
- 46. The impedance of a thermometer is inversely proportional to its heat capacity.
 - a. True b. False



Mechanical Figure 7. A mechanical filter schematic

- 47. The mechanical filter schematic in Figure 7 is of a low-pass filter
 - a. True
 - (b) False
- 48. A sensor which can be modeled as a first-order system will induce a linear phase shift of the input signal.
 - a. True
 - √b False
- 49. A linear potentiometer can be modeled as a first-order system.
 - a. True
 - √b.) False
- Zeno onoen
- 50. An interferometer has a precision which is proportional to the wavelength of the laser light used by the instrument.
 - ⓐ True
 - b. False
- 51. A linear variable differential transformer is based on a variable resistor or potentiometer to sense position.
 - a. True
 - ⟨b⊋ False

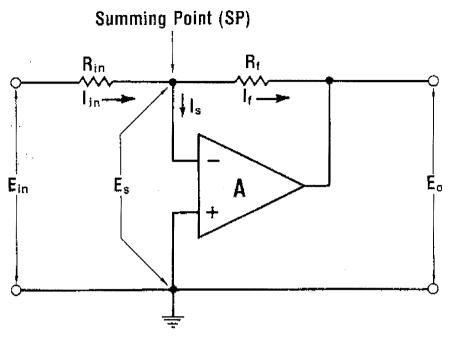


Figure 8. Basic operational amplifier circuit.

- 52. The op amp circuit in Figure 8 is a
 - (a) Inverting amp
 - b. Summing amp
 - c. Integrating amp
 - d. Non-inverting amp
- 53. The voltage E_s in Figure 8 is
 - a. $E_s = I_{in} R_{in}$
 - b. $E_s = E_o$
 - © $E_s = 0$
 - d. None of the above
- 54. The gain of the op amp circuit in Figure 8 is
 - a. R_{in}/R_f
 - b. R_f/R_{in}
 - (c) -R_f/R_{in}
 - d. -R_{in}/R_f
 - e. None of the above
- 55. The current Is in Figure 8 is
 - (a) $I_s = 0$
 - $b. l_s = l_{in}$
 - c. $I_s = I_f$
 - d. $-E_o/R_f$
 - e. None of the above

the

- 56. If the op amp in Figure 8 is assumed to be ideal the input impedance circuit would be infinite.
 - a. True
 - ♠ False
- 2 = RIN

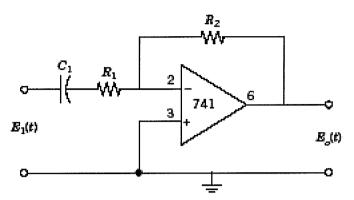


Figure 9. Active filter circuit.

57.	The	active	filter	in	Figure	9	is	а	low-pass	filter.

- a. True
- 的 False

HIGH-PASS

58. As the number of stages of a filter increases the filter cut off point, f_c, decreases.

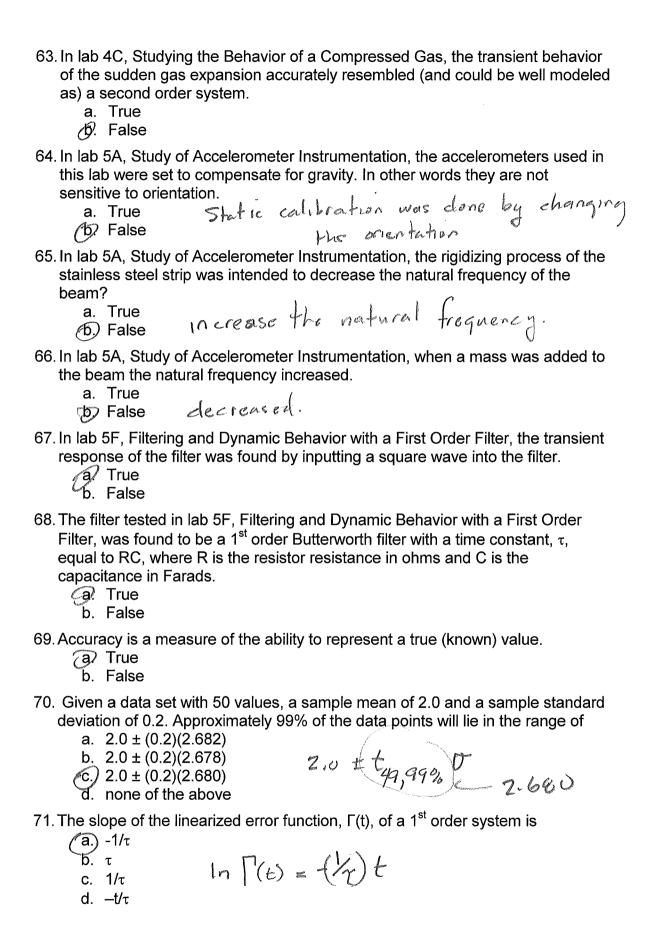
a. True

Remains constant, roll-off is steeper

- ැති False
- 59. In lab 3 (Transient Thermal Behavior with Work and Heat Loss) the rate of increase in temperature, dT/dt, of the calorimeter is linearly proportional to rate at which you turned the calorimeter handle.
 - (a) Approximately true when the calorimeter is at the lab air temperature
 - ந். True in all cases
 - c. True when the calorimeter is significantly hotter than the lab air temperature
 - d. Never true
- 60. In lab 3 the cooling studies were used to
 - a. Determine the overall convection factor, H.
 - b. Determine the time constant, τ , of the calorimeter
 - c. All of the above
 - d. None of the above
- 61. In lab 4s, Strain Gage Experiences, the first two unloaded beam natural frequencies could be found. The second or higher natural frequency of the beam was of similar magnitude to the fundamental frequency.
 - a. True
 - ∂b⊅ False

much lower magnitude

- 62. In lab 4s, Strain Gage Experiences, the experimental deflection data was predicted with reasonable accuracy by the experimental deflection calculation.
 - √a,) True
 - b. False



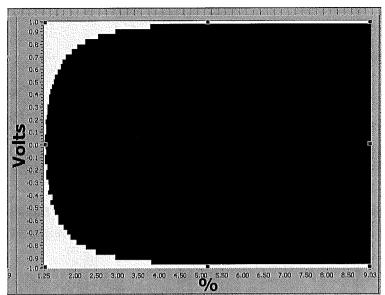
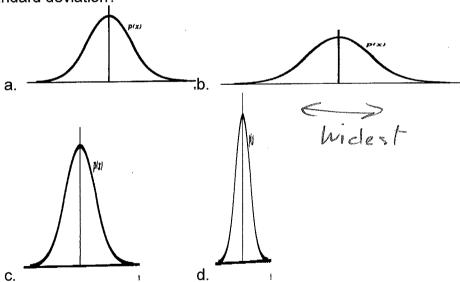


Figure 10. Signal histogram

72. The signal whose histogram is plotted in Figure 10 spends approximately how much of its time below -0.9 volts.

9.03 + 3.75 = 12.78%

- (a) 12.78%
 - b. 3.75 %
 - c. 9.03 %
 - d. None of the above
- 73. Given the following probability density functions. Which signal has the largest standard deviation?



- 74. The slope of the static calibration curve is known as the:
 - a. Static sensitivity
 - b. Response function
 - c. Time constant
 - d. None of the above

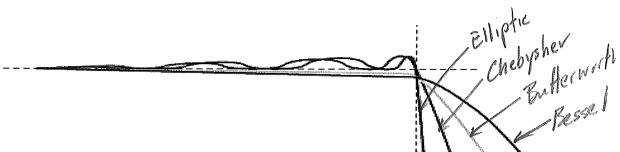


Figure 11. Low pass filter magnitude response in dB versus log of frequency of four different filter types, Butterworth, Bessel, Elliptic and Chebyshey, all with the same cut off frequency.

- 75. Of all the filter response characteristics plotted in Figure 11, the Elliptic filter is most likely to have the steepest magnitude roll off as the frequency increases.
 - (a) True
 - ซึ. False
- 76. Of all the filter response characteristics plotted in Figure 11, the Bessel filter is most likely to have the most gradual magnitude roll off as the frequency increases.
 - √a. True
 - b. False
- 77. Heat flux is a flow variable.
 - /a/ True
 - b. False
- 78. The time constant, τ , of a thermocouple
 - (a) Is not effected by the input signal
 - b. Is smaller when subjected to a larger step input
 - c. Is larger when subjected to a larger step input
 - d. Is constant as long as the mass of the thermocouple does not change
- media (air hanges)

 schanges

 change

 relation 79. If you would like to resolve the daily, weekly, monthly and annual temperature fluctuations at your home what is the least amount of data you must collect?
 - a. twice a day for 6 months
 - b. once a day for 12 months
 - c. once a day for 6 months

 - (d.) twice a day for 12 months
- daily fluctuation requires at least 2 samples per day.
- 80. To estimate the 95% confidence interval for a linear curve fit of 15 data points you would use the formula
 - a. $C.I. = \pm t_{13.95}S_{r}$
 - b. $C.I. = \pm t_{13.95} S_{\overline{x}}$
 - **c.** $C.I. = \pm t_{15.05} S_{yy}$
 - (d) $C.I. = \pm t_{13.95} S_{xy}$
 - e. None of the above

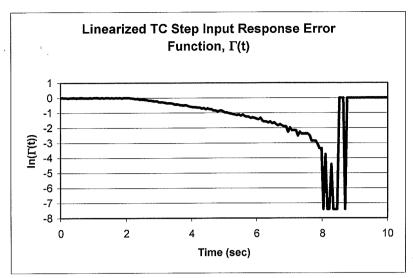


Figure 12. Linearized step input response error function.

- 81. While linearizing the dynamic calibration data taken during lab #2 report using equation: $\ln\{[(T(t)-T_0)/(T_f-T_0)]\}=-t/\tau$ you obtain a curve like the one in Figure 12. This plot is indicative of
 - a. a data set with too much noise to properly analyze.

b. a well collected, properly processed data set with quantization error at the end of the record.

c. a truncated data set or a poor estimation of T_f. Wrong T_f causes the none of the above

None straight line plot

A None T_f causes the none of the above the none of the above the none straight line plot

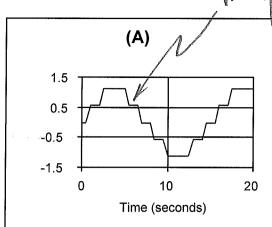
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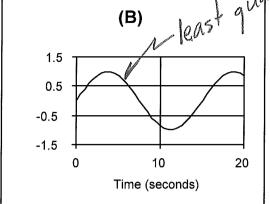
Figure 13. A plot of the response of a second order system of various damping ratios to a step input function.

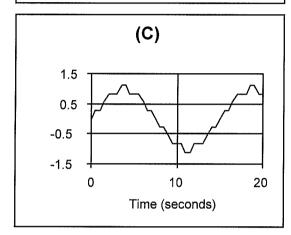
82. In the figure above (Figure 13) which curve has a damping ratio, ξ , of 1 Ω 83. In the figure above (Figure 13) which curve has a damping ratio, ξ , of 0.25

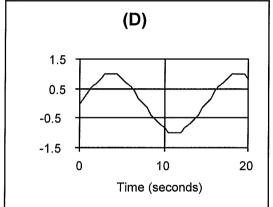
mortquantization

(B) least quantitation







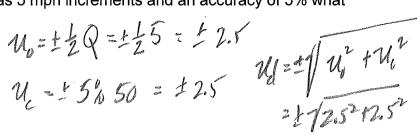


- 84. If the input signal is a true sine wave which of the above graphs shows the largest quantization error.
- 85. The sine wave in the above graphs has a frequency of
 - a) 2π/15 Radians/Second
 - b. 15 Hertz
 - c. .15 Hertz
 - d. 30π Radians/Second
 - e. 15π Radians/Second

- 86. Which of the following temperature sensors is the least expensive to purchase and implement?
 - a. Infra Red Detector
 - b. Thermocouple
 - C Thermistor
 - d. RTD
- 87. When an instrument manufacturer lists an accuracy (of for example 1 minute/month for my stopwatch) what is the assumed uncertainty percentage?
 - a. 99%
 - b. 90 %
 - ©.) 95%
 - d. None of the above

- 88. It the speedometer in a car has 5 mph increments and an accuracy of 5% what is the uncertainty at 50 mph
 - a. $\sqrt{5+2.5}$
 - b. $\sqrt{5^2 + 2.5^2}$
 - (c) $\sqrt{2.5^2 + 2.5^2}$

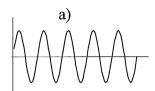


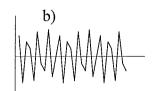


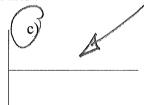
89. The following equation is used to simulate a digitized sinusoidal signal.

$$Y=sin(2\pi fn/f_s)$$

Where f is signal frequency; f_s is sampling frequency; n=1,2,3,...500. If f=/100Hz and f_s = 200 Hz what will the output signal look like?







- 90. The time constant (τ) of a thermocouple can be effected by the following factor(s)?
 - (a) the medium around the thermocouple
 - b. size of the temperature step
 - c. the direction of the temperature change \ NO \
 - d. all of the above
- 91. After performing a linear regression on the temperature vs. time data collected while turning the calorimeter drum lab #3, you found that the quality of the fit improved as the lab progressed (the temperature of the calorimeter drum increased).

 - a. True As calorimete temperature increused more convection (6) False to lab air occurred.
- 92. Amplitude ambiguity will occur in a Fourier transformation of a periodic signal if the period chosen for transformation does not contain an integral multiple of the frequency of the periodic signal.
 - a) True ზ. False
- 93. The only filter covered in class that has a linear phase shift (does not scamble the phase and could be used for audio signal filtering) is a
 - a. Chebyshev
 - /b/ Bessel
 - c. Butterworth
 - d. Elliptic

Design Stage Uncertainty Problem

An ADC is to be used to measure the output from a thermocouple. The nominal temperature expected will be about 20 °C. Estimate the design-stage uncertainty in this combination. The following information is available:

ADC

Gain:

Range:

±1 volt

Resolution:

10 bits

Accuracy:

within 0.001% of reading

Thermocouple

Sensitivity:

10⁻⁴ V/°C

Linearity:

within 1 mVolt/°C over range

Repeatability: Resolution:

within 2 mV/°C over range negligible

94. The voltage measurement design stage uncertainty of the ADC is

a.
$$u_0 = \pm \frac{1}{2} \left(\frac{2}{2^{10}} \right)$$

(b.)
$$(u_d)_E = \pm \sqrt{\left(\frac{1}{2}\left(\frac{2}{2^{10}}\right)\right)_E^2 + \left(2 \times 10^{-8}\right)_E^2}$$

c.
$$(u_d)_E = \pm \frac{1}{2} \sqrt{\left(\frac{2}{2^{10}}\right)_E^2 + \left(2 \times 10^{-8}\right)_E^2}$$

d. None of the above

a.
$$u_0 = \pm \frac{1}{2} \left(\frac{2}{2^{10}} \right)$$

(b.) $(u_d)_E = \pm \sqrt{\left(\frac{1}{2} \left(\frac{2}{2^{10}} \right) \right)_E^2 + \left(2 \times 10^{-8} \right)_E^2}$
c. $(u_d)_E = \pm \frac{1}{2} \sqrt{\left(\frac{2}{2^{10}} \right)^2 + \left(2 \times 10^{-8} \right)_E^2}$

95. The design stage uncertainty of the thermocouple can be assumed to be

(a.)
$$\pm\sqrt{\left[1\text{mV}/^{\circ}\text{C}\times20\ ^{\circ}\text{C}\right]^{2}+\left[2\text{mV}/^{\circ}\text{C}\times20\ ^{\circ}\text{C}\right]^{2}}$$

(a.)
$$\pm \sqrt{[1\text{mV/°C} \times 20 \text{ °C}]^2 + [2\text{mV/°C} \times 20 \text{ °C}]^2}$$

b. $\pm \frac{1}{2} \sqrt{[1\text{mV/°C} \times 20 \text{ °C}]^2 + [2\text{mV/°C} \times 20 \text{ °C}]^2}$

c.
$$\pm \sqrt{[1mV]^2 + [2mV]^2}$$

d. None of the above

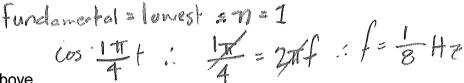
- 96. The delta function integral $\int \delta(t)dt$ is defined to be equal to:
 - (b) 1

 - e. None of the above
- 97. Velocity and Voltage are both effort variables.
 - a. True
 - b. False

- 98. A linear potentiometer as used in the fourth lab in combination with the ADC used to acquire the output voltage has a position measurement zero order uncertainty of _____ where K is the static sensitivity of the position vs. voltage calibration and Q is the resolution of the ADC.
 - a. $\pm \frac{1}{2}Q$
 - (b) ± ½ (K Q)
 - c. ± (K Q)
 - $d. \pm Q$
 - e. None of the above
- 99. For the Fourier series given by $y(t) = 4 + \sum_{n=1}^{\infty} \frac{2n\pi}{10} \cos \frac{n\pi}{4} t + \frac{120n\pi}{30} \sin \frac{n\pi}{4} t$ where t

is time in seconds. What is the fundamental frequency in hertz?

e. None of the above



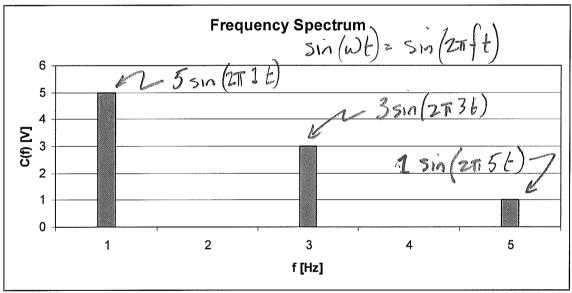


Figure 14. Spectrum for problem 31

- 100. Which of the following describes the frequency spectrum plotted in Figure 14.
 - a. $5\sin(t) + 3\sin(3t) + \sin(5t)$

$$5\sin(2\pi t) + 3\sin(6\pi t) + \sin(10\pi t)$$

c.
$$5\sin(\pi t) + 3\sin(3\pi t) + \sin(5\pi t)$$

d.
$$5\sin\left(\frac{t}{2\pi}\right) + 3\sin\left(\frac{3t}{2\pi}\right) + \sin\left(\frac{5t}{2\pi}\right)$$

Table 2. Student-t Distribution

ν	t ₅₀	t ₉₀	t ₉₅	t ₉₉
15	0.691	1.753	2.063	2.947
16	0.690	1.746	2.052	2.921
17	0.689	1.740	2.043	2.898
18	0.688	1.734	2.035	2.878
19	0.688	1.729	2.027	2.861
20	0.687	1.725	2.021	2.845
21	0.686	1.721	2.015	2.831
22	0.686	1.717	2.010	2.819
23	0.685	1.714	2.005	2.807
24	0.685	1.711	2.000	2.797
25	0.684	1.708	1.996	2.787
46	0.680	1.679	1.953	2.687
47	0.680	1.678	1.952	2.685
48	0.680	1.677	1.951	2.682
49	0.680	1.677	1.950	2.680
50	0.679	1.676	1.949	2.678

Table 3. Probability Values for Normal Error Function $z_1 = \frac{x_1 - x'}{\sigma}$

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990