The above frequency-selective filtering is equivalent to the following operation in the time domain:

$$p(t) = \sum_{n=-\infty}^{\infty} p(n\Delta_t) \operatorname{sinc}[\frac{\pi(t - n\Delta_t)}{\Delta_t}].$$

This is called ***reconstruction*** or ***interpolation*** of the original signal from its delta-sampled signal.

The success of this operation hinges on the constraint

$$\omega_0 \leq \frac{2\pi}{2\Delta_t},$$

which is known as the ***Nyquist*** sampling criterion for *aliasing-free* or *error-free* reconstruction of $p(t)$ from the sampled data $p(n\Delta_t)$'s. The linear harmonic components $P(\omega - \frac{2\pi n}{\Delta_t})$, $n = \pm 1, \pm 2, \ldots$ (or $n \neq 0$), result in aliasing if the Nyquist criterion is not satisfied.

The issue of sampling and its dual form, that is, sampling in the frequency domain, and aliasing-free processing and reconstruction from the sampled signal will be extensively encountered in array imaging systems.

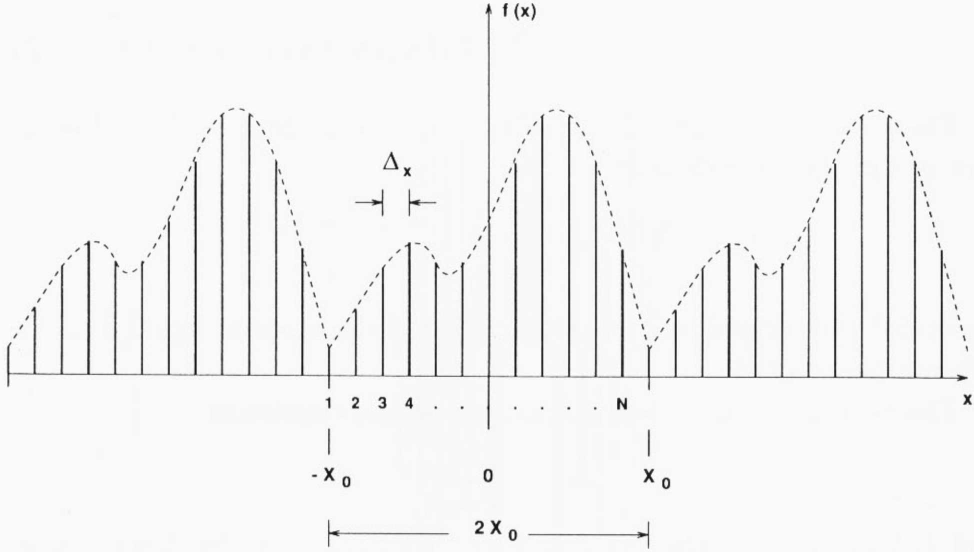
1.3 DISCRETE FOURIER TRANSFORM

Let $f(x)$ be a periodic signal composed of evenly spaced delta functions, for example,

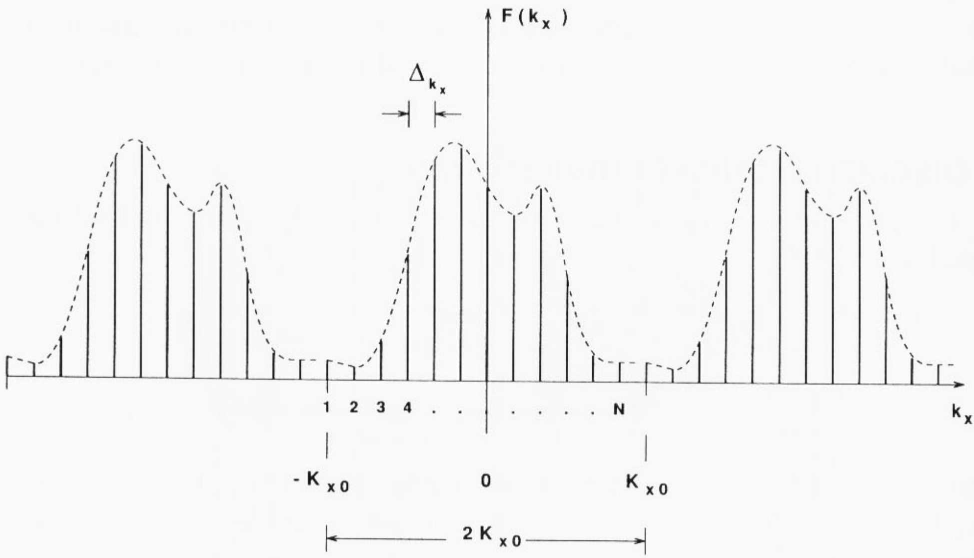
$$f(x) = \sum_{\ell=-\infty}^{\infty} \underbrace{\sum_{n=-N/2}^{N/2-1} f_n \, \delta[x - (n + N\ell)\Delta_x],}_{\textit{One period}}$$

where Δ_x is the spacing between two consecutive delta functions and $N\Delta_x$ is the period (see Figure 1.7). This model provides a link between a discrete sequence and evenly spaced samples of a continuous signal [6]. Using the forward and inverse Fourier integrals, one can show that

$$F(k_x) = 2\pi \sum_{\ell=-\infty}^{\infty} \sum_{m=-N/2}^{N/2-1} F_m \, \delta[k_x - (m + N\ell)\Delta_{k_x}],$$



Depiction of a Discrete Fourier Transform Pair:
The Spatial Domain Signal



Depiction of a Discrete Fourier Transform Pair:
The Spatial Frequency Domain Signal

Figure 1.7 A discrete Fourier transform pair.

where

$$F_m = \sum_{n=-N/2}^{N/2-1} f_n \exp(-j\frac{2\pi}{N}mn)$$
$$f_n = \frac{1}{N}\sum_{m=-N/2}^{N/2-1} F_m \exp(j\frac{2\pi}{N}mn) \tag{1.1}$$

and

$$N\ \Delta_x\ \Delta_{k_x} = 2\pi. \tag{1.2}$$

Equations (1.1) and (1.2) are called the Discrete Fourier Transform (DFT) equations. It should be noted $F(k_x)$ *is also a periodic signal* that is composed of evenly spaced delta functions (see Figure 1.7).

- *In practice, we deal with continuous functions that are not band-limited in space/time and frequency domains. For storage and processing purposes in a computer, we represent (approximate) them via periodic functions that are made up of a finite number of evenly spaced delta functions (N sampled data). The main period of $f(x)$ [or $F(k_x)$] is assumed to be the region in the x (or k_x) domain, for example, $x \in [-X_0, X_0)$ [or $k_x \in [-K_{x0}, K_{x0})$], where most, for example, 95 percent, of its energy is concentrated (X_0 and K_{x0} are known constants). $[-X_0, X_0)$ and $[-K_{x0}, K_{x0})$ are called the* ***effective*** *support bandwidths of $f(x)$ and $F(k_x)$, respectively.*

  Noting the fact that

$$2X_0 = N\ \Delta_x$$
$$2K_{x0} = N\ \Delta k_x,$$

we can write from (1.2) the following equations that are the Nyquist sampling rate (constraint) for representing/recovering a lowpass signal from its sampled data:

$$\Delta_x = \frac{\pi}{K_{x0}}$$
$$\Delta_{k_x} = \frac{\pi}{X_0}.$$