CE427 - CHEMICAL ENGINEERING LABORATORY III
FALL 2005

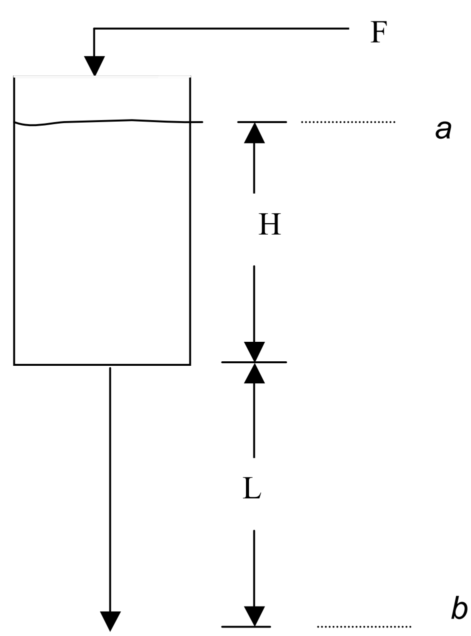
MATHEMATICAL MODELLING OF TANK DRAINING

Objectives:
Develop three mathematical models of varying complexity to predict the time required to drain a vertical cylindrical tank and compare models with experimental data.

System:
Two tanks located in 121 Jarvis have diameters of 8.375 inches and 4.0 inches. The two tanks have exit lines that are replaceable with 0.49 inch diameter tubes having different lengths. The lengths available are approximately 6, 12, 24, 36, 48, 60, and 72 inches long. There are two types of contraction assemblies placed on the tanks. The larger-diameter tank has a sharp-edged contraction and the smaller-diameter tank has rounded contraction. The tanks can be filled with water using a pump and plugs for the drain lines are available. A pressure transducer, located at the tank exit, transmits data to a PC running a LabView program which records the water level as a function of time as the tank drains.

Model development

If a tank system is operated as shown in the figure below, the flow in, F, is sufficient to maintain the level in the tank at H inches while the tank is being drained through a pipe of length L.



Model Level 1:

In the simplest approach to the problem, the Bernoulli equation without friction can be written for the situation described above choosing station a at the surface of the liquid in the tank and station b at the point of discharge of the drain pipe. Since the pressure at both stations is atmospheric, p_a and p_b are equal, and $p_a/\rho = p_b/\rho$. At the surface of the liquid, the velocity V_a is negligible, and the term $V_a^2/2$ is dropped. Thus, the remaining terms in the Bernoulli equation without friction are:

$$g(L+H)=\frac{V_b^2}{2} \qquad (1)$$

where
g=acceleration of free fall
L=length of drain pipe
H= liquid level.

From Eqn. (1), it can be seen that the velocity at the discharge will vary with the height of liquid in the tank. If the flow, F, is stopped and the tank is allowed to drain, an unsteady state material balance on the tank and exit pipe gives:

$$S_b V_b = -S_a \frac{dH}{dt} \qquad (2)$$

(note that the minus sign is needed because the derivative is negative) S_a and S_b are the cross-sectional areas of the tank and the drain pipe, respectively.
Substitution for V_b from Eqn.(1) into Eqn.(2) (quasi-steady state approximation), rearrangement, and integration yield:

$$\int_0^{t_e} dt = -\frac{D^2}{d^2\sqrt{2g}}\int_{H_i}^{H_e}\sqrt{\frac{1}{H+L}}dH \qquad (3)$$

where te =efflux (or drain) time
Hi = initial liquid level
He= final liquid level
d= drain tube diameter
D= tank diameter

Thus time required to drain the tank from an initial liquid level to a final liquid level is predicted by Model 1 to be:

$$t_{e1} = \frac{2D^2}{d^2\sqrt{2g}}\{\sqrt{H_i + L} - \sqrt{H_e + L}\} \quad (4)$$

Model level 2:

Obviously, Model 1 does not account for the effects of the solid boundaries of the tank and drain pipe. The Bernoulli equation is extended to account for the existence of fluid friction by adding a term, h_f, representing all the friction generated per unit mass of fluid that occurs between stations a and b. Therefore, adding this term to Eqn. (1) would yield:

$$g(L + H) = \frac{V_b^2}{2} + h_f \qquad (5)$$

It can be argued that the major source of friction loss in our tank draining system is the sudden contraction of cross section from the tank to the drain pipe. The friction loss from sudden contraction is proportional to the velocity head in the drain pipe and is given by the expression

$$h_f \approx h_{fc} = K_c \frac{V_b^2}{2} \qquad (6)$$

where K_c is called the contraction loss coefficient. The value of the contraction loss coefficient varies with the geometry of the contraction.

Model 2 is developed by proceeding as in Model 1 to obtain:

$$t_{e2} = \frac{2D^2\sqrt{(1 + K_c)}}{d^2\sqrt{2g}}\{\sqrt{H_i + L} - \sqrt{H_e + L}\} \qquad (7)$$

Model level 3:

While Model 2 took into account the major source of friction loss, the friction loss due to the sudden contraction of cross section as the flow enters the drain pipe is not the only source of friction loss in our system. Model 3 includes the friction loss due to friction between the wall of the drain pipe and the fluid stream (the friction between the fluid and the wall of the tank is neglected). In this case, the friction term in the Bernoulli equation with friction (Eqn. (5)) would include h_{fs}, denoting the skin friction:

$$g(L + H) = \frac{V_b^2}{2} + h_{fc} + h_{fs} \qquad (8)$$

where $h_{fs} = 4f\frac{L}{d}\frac{V_b^2}{2}$

f = Fanning friction factor
d = drain pipe diameter

The loss coefficients and the Fanning friction factor are discussed in Chapter 5 of McCabe, Smith, and Harriott (M,S,&H) and it is expected that the experimenter will consult the text for further information.

Proceeding as in the development of the previous models would yield:

$$t_{e3} = \frac{2D^2\sqrt{(1+K_c+\frac{4fL}{d})}}{d^2\sqrt{2g}}\{\sqrt{H_i+L}-\sqrt{H_e+L}\} \qquad (9)$$

(In the integration that led to Eqn. (9) ,it was assumed that the friction factor, f, is a constant throughout the draining process. **Explain why one can assume this is valid**)

<u>Comments and Items to Consider</u>

<u>Drain Time Measurements</u>:

For measurements of drain time, initially fill the tanks to the overflow line while plugging the drain tube. Then remove the plug while simultaneously starting the data acquisition system to record the drain time.

1) Use both tanks and at least 5 drain tubes. Obtain at least three readings for each tube.

2) Because it is difficult to determine exactly when the tank is empty, record the time to drain to a consistent depth (1 or 2 inches) above the bottom of the tank.

3) Determine the efflux times predicted by all three models. Note that for model 3 you will need to estimate the friction factor. To do this, you will need to calculate the "average" velocity of the water leaving the tube based on the volume of water in the tank at the beginning and how long it took for it to reach the final liquid level.

4) How do the measured values compare to those predicted? Which model more accurately predicted the drain time? Include uncertainty limits for your measured drain times.

5) Plot the drain times as a function of the length of drain pipe length. Include the models' predictions.

6) Does changing the pipe length “significantly” alter the efflux time measured? Does it depend on the contraction type?
7) What further refinements would you suggest for the mathematical model?

References

1. McCabe, Smith, and Harriott, *Unit Operations of Chemical Engineering,* 6th Edition, McGraw-Hill, 2001.
2. Perry, R.H. and D.W. Green (eds.), Chemical Engineer’s Handbook, 7th ed., McGraw-Hill, 1997.

Pre-lab Homework for Tank Draining Experiment**- to be completed individually**

1. Show the steps to obtain Eqn.(7), beginning with Eqn.(2) and Eqn.(5).

2. Find an expression or value for the contraction loss coefficient for a sharp-edged contraction (List the reference).

3. Repeat problem 2 for a rounded contraction (List the reference).

4. During your experiment, you found that the small tank in the lab(D=4 in.) takes 32.2 seconds to drain water at 25 C from a liquid level of 77 inches to a liquid level of 2 inches through a 24 inch drain pipe (d=0.49 inch). Based on your data, what is the average velocity at the discharge of the drain pipe during the draining process?

5. Determine the Fanning friction factor for the experimental data in problem 4 assuming the drain pipe is smooth.