CE407 SEPARATIONS

Lecture 03

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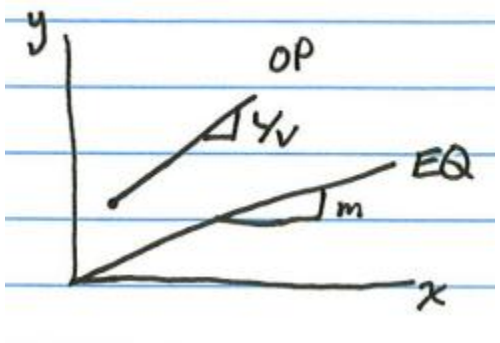
Absorption – Kremser Equation (aka Absorption Factor Method)

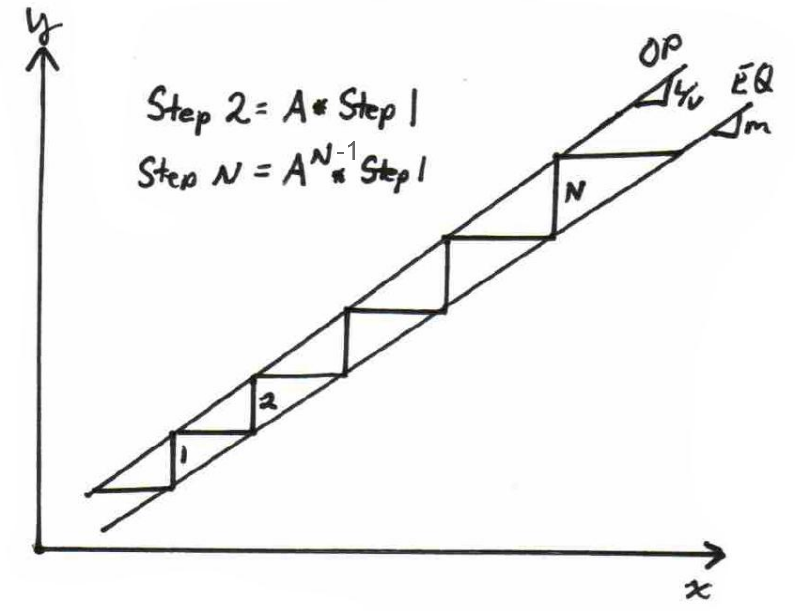
- Pages 653-659 of McCabe/Smith/Harriott cover the mathematical derivation. These notes cover the concepts
- 2 Main Conditions for the Kremser Method to be valid
  - Operating Line must be a straight line
    - This is a reasonable assumption if x and y <<1 (~ < 0.1)
  - Equilibrium Curve is a straight line
    - Raoult's or Henry's Law applies
    - Mixture of similar chemicals
    - As long as EQ curve approximates a straight line over the region of interest



- Slope of OP line ≈ $\frac{L}{V}$, this is approximately constant for dilute system
- Slope of EQ line is given by equilibrium relationship (Raoult, etc) and is noted as **m**
- Absorption Factor **A**: $A = \frac{L/V}{m} = \frac{L}{mV}$ this is the ratio of the OP line slope vs the EQ line slope

Kremser Equation

- Observe: If the two lines have different slopes, then each successive step will increase in size relative to the previous step.
- The increase is proportional to **A**
- If you take N steps, then that step is **A^{N-1}** times the original step



- Take logarithm **$\ln(A^N) = N * \ln(A)$** this all leads to the Kremser Equation

Kremser Equation

- Various Forms, all are equivalent

$$N = \frac{ln\left[(y_b - y_b^*)/(y_a - y_a^*)\right]}{\ln\left[(y_b - y_a)/(y_b^* - y_a^*)\right]} \qquad \text{eq 20.24}$$

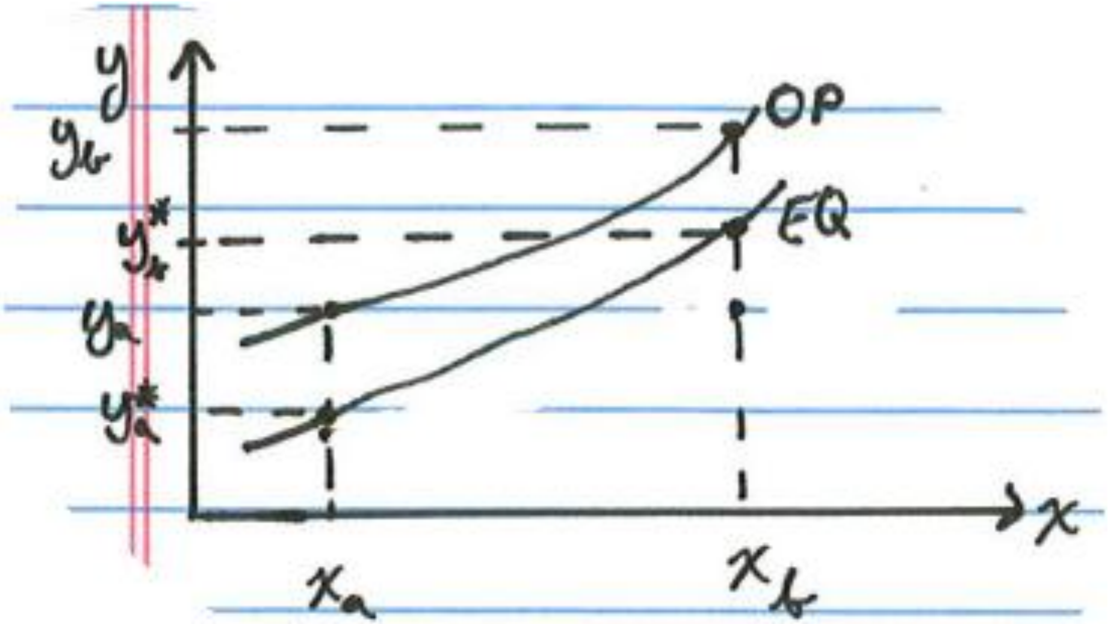
$$N = \frac{ln\left[(y_a - y_a^*)/(y_b - y_b^*)\right]}{\ln\left[(y_b^* - y_a^*)/(y_b - y_a)\right]} \qquad \text{eq 20.27}$$

$$N = \frac{ln\left[(x_a - x_a^*)/(x_b - x_b^*)\right]}{\ln\left[(x_a - x_b)/(x_a^* - x_b^*)\right]} \qquad \text{eq 20.28}$$

$$N = \frac{ln\left[(x_b - x_b^*)/(x_a - x_a^*)\right]}{\ln\left[(x_a^* - x_b^*)/(x_a - x_b)\right]}$$

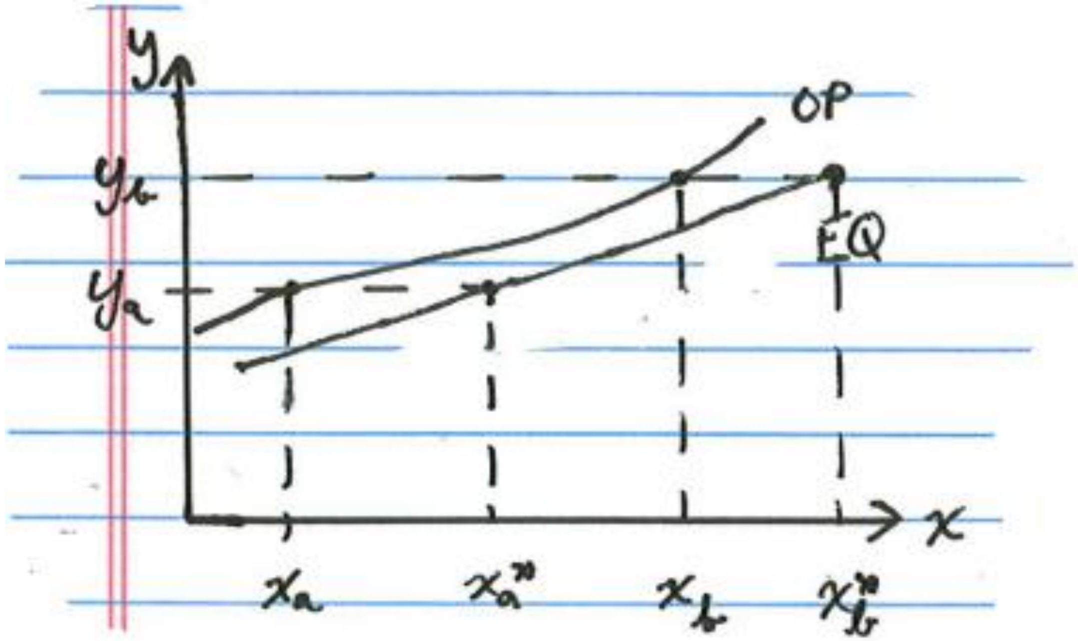
Okay, what are those *s?

- y_a^* is the value of vapor mole fraction which is in equilibrium with liquid mole fraction x_a
- y_b^* is the value of vapor mole fraction which is in equilibrium with liquid mole fraction x_b



Okay, what are those *s?

- x_a* is the value of liquid mole fraction which is in equilibrium with vapor mole fraction y_a
- x_b* is the value of liquid mole fraction which is in equilibrium with vapor mole fraction y_b



Kremser Equation

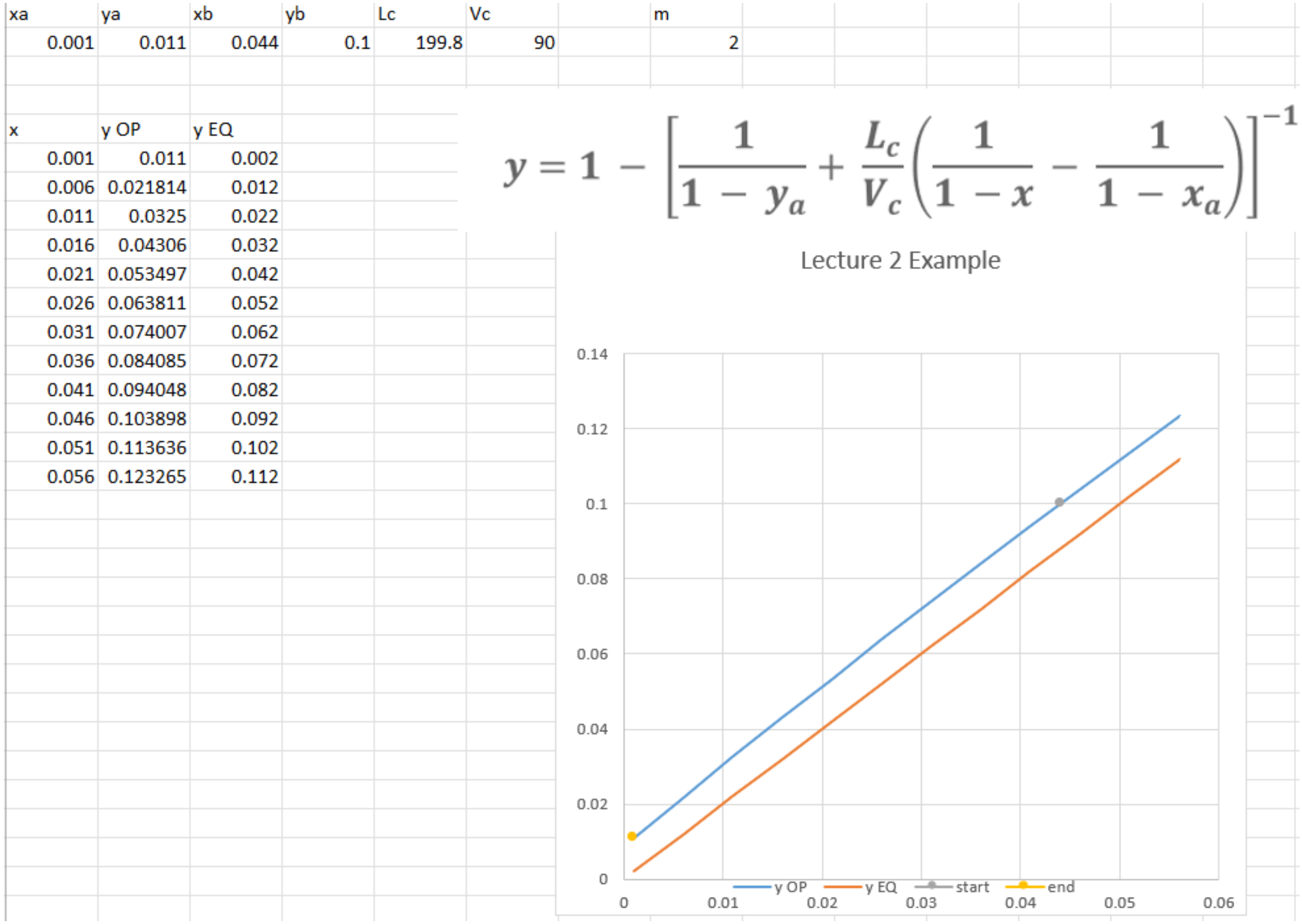
$$N = \frac{ln[(y_b - y_b^*)/(y_a - y_a^*)]}{ln[(y_b - y_a)/(y_b^* - y_a^*)]}$$ eq 20.24

- **y_b – y_b*** is the difference between entering vapor mole fraction and the mole fraction that could be achieved IF we were to reach equilibrium – this is a driving force
- **y_a – y_a*** is the difference between exiting vapor mole fraction and the mole fraction that could be achieved IF we were to reach equilibrium – this is a driving force
- **y_b – y_a** is the change that we are trying to achieve
- **y_b* - y_a*** is the potential for change if the entering and exiting vapors both achieved equilibrium with their respective liquid counterparts

- Note: **N** will most likely NOT be an integer – Round Up!

Kremser Equation Example

- Example Problem from Lecture 2
- Notice that Operating Line is quite straight
- Equilibrium Curve was perfectly straight
- Good candidate for Kremser Method!



xa	ya	xb	yb	Lc	Vc		m
0.001	0.011	0.044	0.1	199.8	90		2

x	y OP	y EQ
0.001	0.011	0.002
0.006	0.021814	0.012
0.011	0.0325	0.022
0.016	0.04306	0.032
0.021	0.053497	0.042
0.026	0.063811	0.052
0.031	0.074007	0.062
0.036	0.084085	0.072
0.041	0.094048	0.082
0.046	0.103898	0.092
0.051	0.113636	0.102
0.056	0.123265	0.112

$$y = 1 - \left[\frac{1}{1-y_a} + \frac{L_c}{V_c}\left(\frac{1}{1-x} - \frac{1}{1-x_a}\right)\right]^{-1}$$

Kremser Equation Example

$$N = \frac{ln[(y_b - y_b^*)/(y_a - y_a^*)]}{ln[(y_b - y_a)/(y_b^* - y_a^*)]} \qquad \text{eq 20.24}$$

- $y_a = 0.011$
- $y_b = 0.10$
- Equilibrium relationship: $y = 2x$
- $x_a = 0.001$ $\therefore y_a^* = 0.002$
- $x_b = 0.044$ $\therefore y_b^* = 0.088$

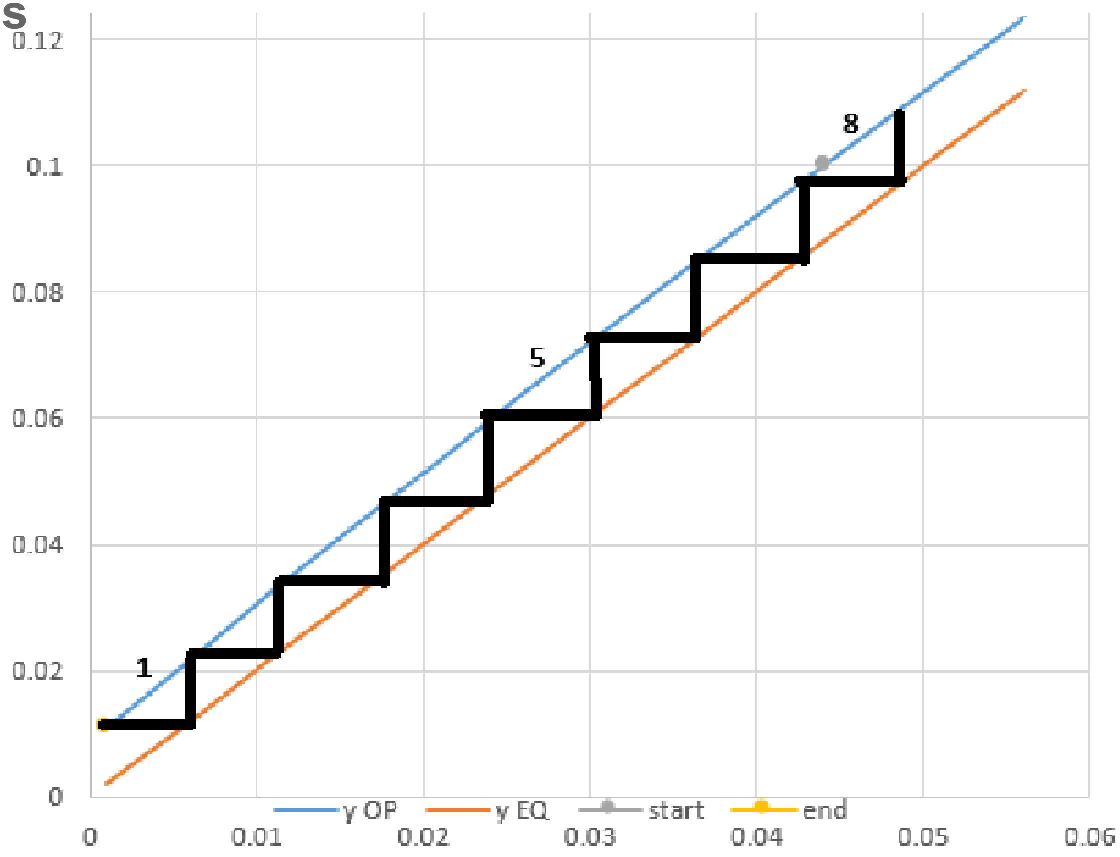
Kremser Equation Example

$$N = \frac{ln[(y_b - y_b^*)/(y_a - y_a^*)]}{ln[(y_b - y_a)/(y_b^* - y_a^*)]}$$

$$N = \frac{ln[(0.10 - 0.088)/(0.011 - 0.002)]}{ln[(0.10 - 0.011)/(0.088 - 0.002)]} = 8.39$$

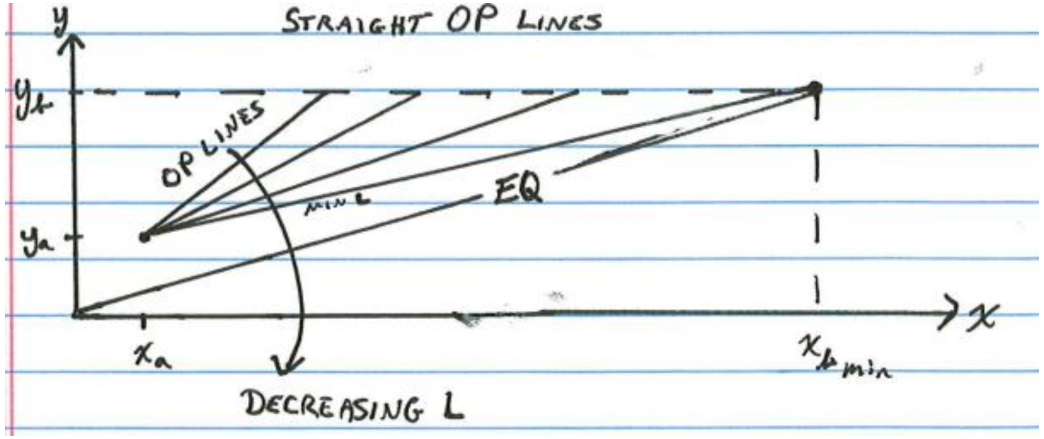
Kremser Equation Example

McCabe – Thiele gave a little more than 7 stages



Absorption Tower Minimum Liquid Flow McSH pp 577-578

- Minimum liquid flow is the value which can just achieve the desired removal of solute
  - It unfortunately leads to an infinitely tall tower…
  - However, it’s calculation is used as a basis to determine a reasonable liquid flow

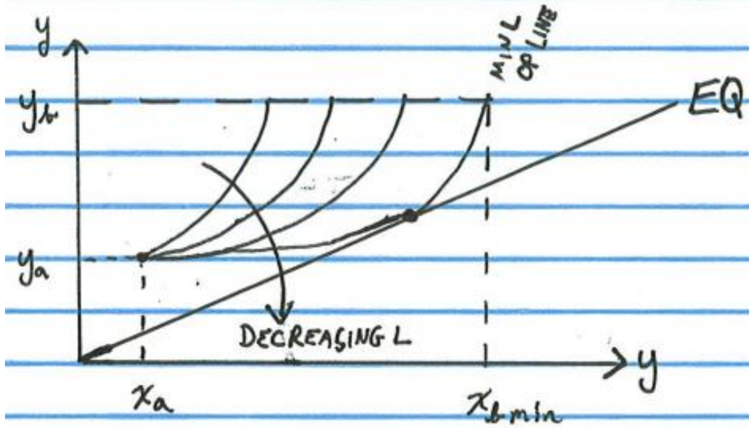


  - Decreasing liquid flow leads to lower L/V and decreases the slope of OP Line
  - At minimum flow the OP Line intersects the EQ Line at ($x_{b,\ min}$, y_b) and the steps between the lines become infinitely small leading to an infinite number of steps
  - With a lower liquid flow you can NEVER reach y_b (where y* indicates equilibrium)

$$y_b = y^*(x_b)$$

Minimum Liquid Flow with Curved OP Line

- Minimum liquid flow is that for which OP Line first contacts (but does not cross) the EQ Curve



- $\mathbf{x}_{\mathbf{b,\,min}}$ is the value of **x** that corresponds to $\mathbf{y}_\mathbf{b}$ on the OP Line for minimum flow
- Also note that $(x_b)_{min} = \dfrac{(L_i)_b}{(L_i)_b + (L_c)_{min}}$
- $\mathbf{(L_i)_b}$ is the # of moles of solute exiting b end of tower – calculated by mass balance on solute
- $\mathbf{(L_c)_{min}}$ is the flow of pure liquid that leads to first contact of OP Line and EQ Line
  - i.e. minimum liquid flow of pure absorbing liquid

Recycled Absorbing Liquid

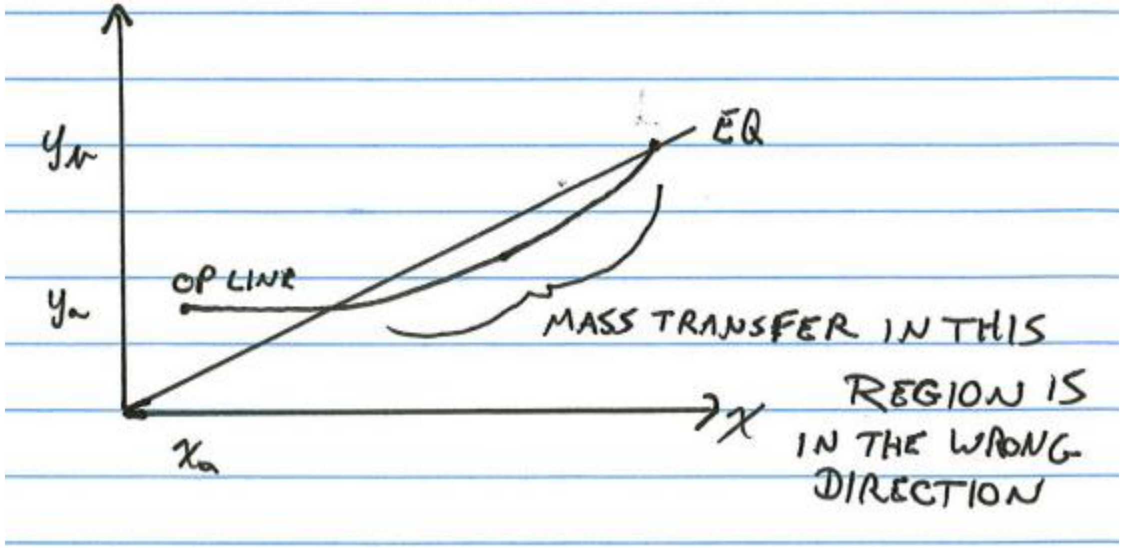
- When using recycled absorbing liquid $x_a \neq 0$

$$(L_a)_{min} * (1 - x_a) = (L_c)_{min}$$

$$(L_a)_{min} = \frac{(L_c)_{min}}{1 - x_a}$$

Do **Not** Assume that $y_b = y^*(x_b)$

- If there is curvature in the Operating Line that assumption leads to the following situation:

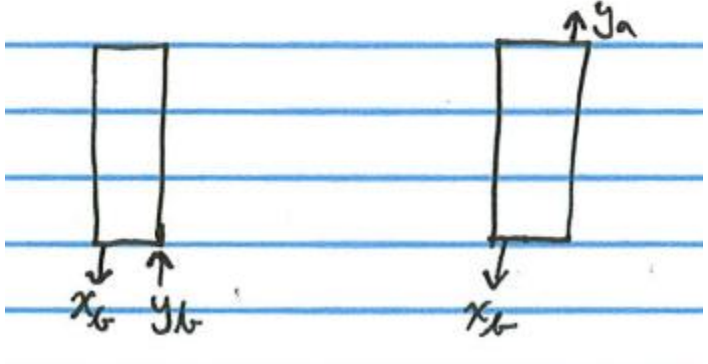


When Op Line is straight…

- When we have a dilute system with straight operating lines and can therefore use

$$x_b = x^*(y_b)$$

- it is important to note

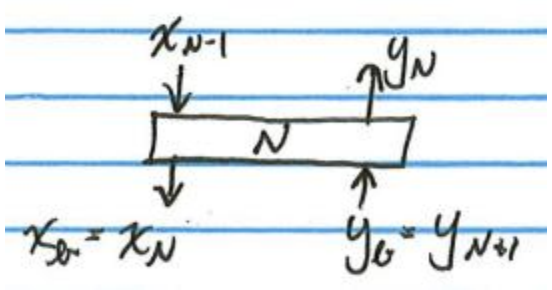


In equilibrium NOT in equilibrium

- **x_b** and **y_a** are separated by many, many stages and are never in contact!

Comment

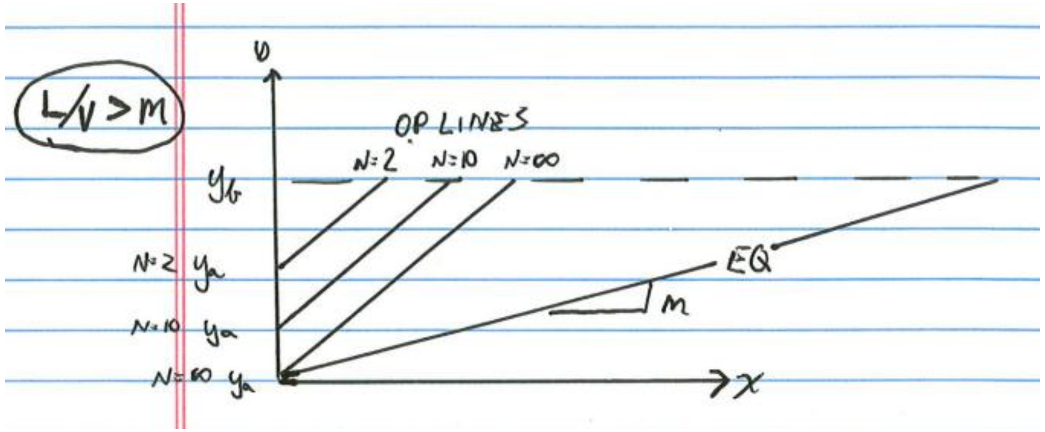
- Actually $\mathbf{x_b}$ and $\mathbf{y_b}$ are not technically in equilibrium…



- $\mathbf{x_b}$ is in equilibrium with $\mathbf{y_N}$, not $\mathbf{y_b = y_{N+1}}$
- However, for minimum liquid flow $\mathbf{y_N}$ and $\mathbf{y_{N+1}}$ are essentially the same due to the infinitely small steps

How clean can you make the vapor with a set value of L/V? L/V > m

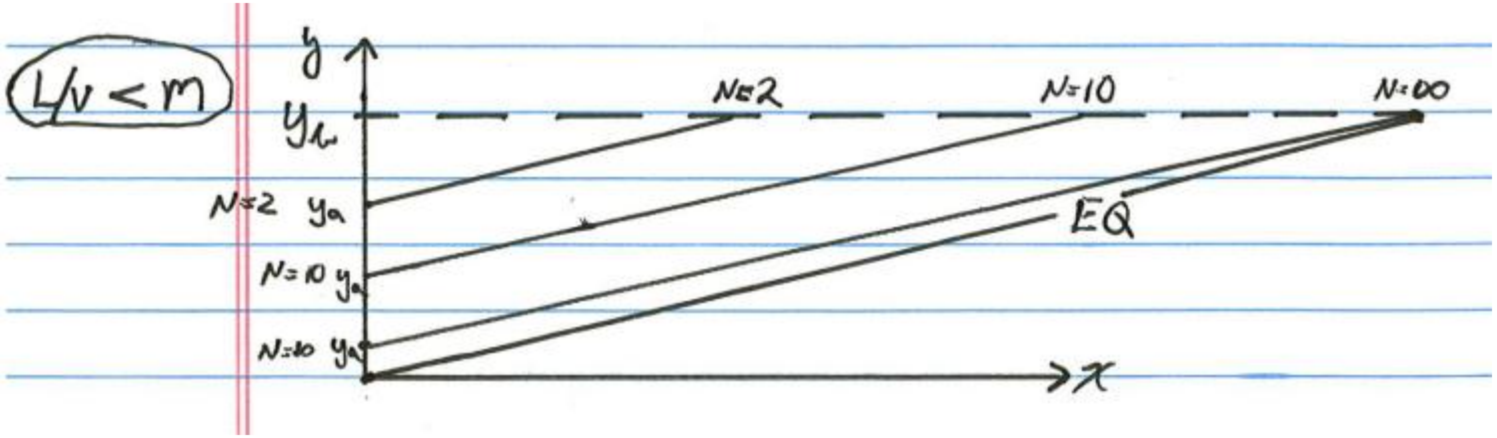
- OP Line Slope = L/V (approximately)
- EQ Curve Slope = m
  - y = mx with m being supplied perhaps by Raoult’s or Henry’s Laws
- N = # of stages



- By increasing the number of stages one can theoretically obtain as low a value of $\mathbf{y_a}$ as desired
- You do reach a point of diminishing returns…

How clean can you make the vapor with a set value of L/V? L/V < m

- When L/V < m, eventually the Operating Line contacts the Equilibrium Line at **y** = $\mathbf{y_b}$ and leads to a pinch point



- Continuing to add stages will NOT lead to a lower value of $\mathbf{y_a}$

What Flow Should We Use?

- Using $\mathbf{L_{min}}$ leads to an infinite number of stages and an infinite capital cost
- Using an **L** that is extremely high reduces capital cost but leads to very high expense for absorbing liquid

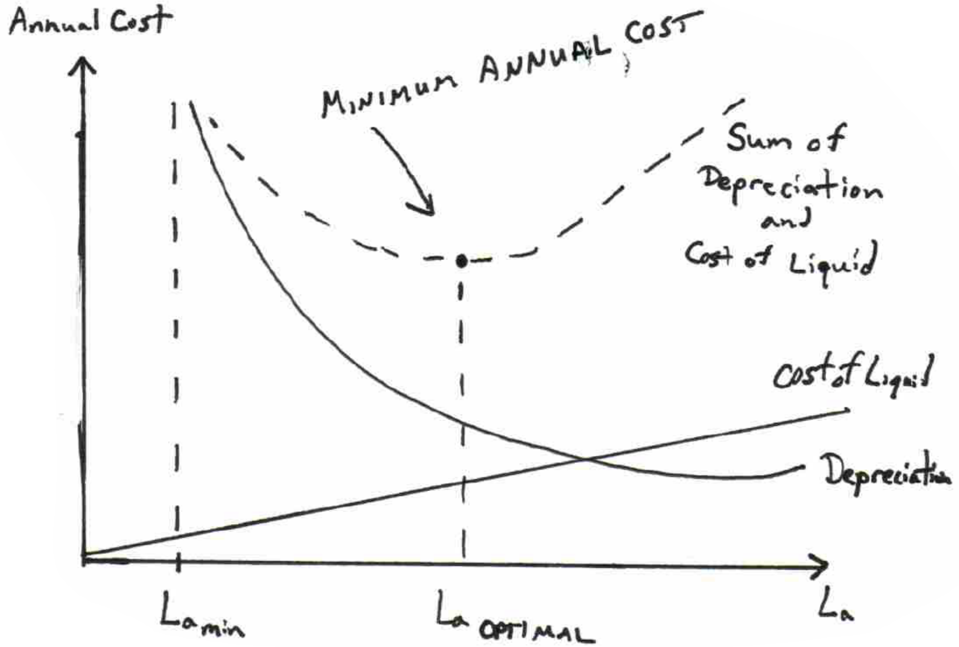
Optimize Cost

- Capital cost of building tower is spread out over the useful life of the equipment and is called depreciation

$$Simple\ Depreciation = \frac{Capital\ Cost\ of\ Equipment}{\#\ of\ years\ in\ use}$$

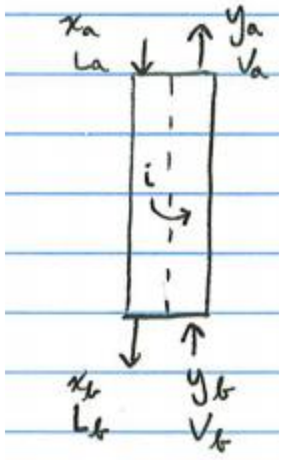
Optimized Cost

- At **$L_{a\ min}$** the capital cost (and therefore annual depreciation) is infinite
- As liquid flow increases the capital cost drops asymptotically
- The cost of liquid is approximately directly proportional to the amount used
- Look for minimum of the sum of capital and liquid cost
- This value is typically $L_{a,optimal} \approx 1.3 * L_{a,min}$



Stripping

- The same equipment can be used to remove solute from a liquid!
- This can be done when the conditions are such that equilibrium drives the mass transfer from liquid to vapor
- It is the reverse of absorption
- The analysis is the same…



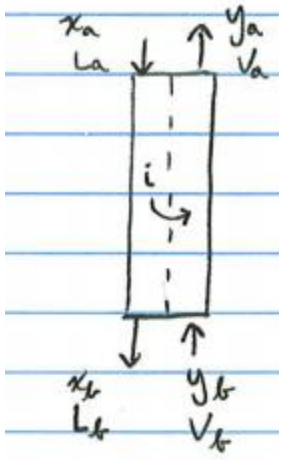
Now we know all about the entering liquid (it is what we want to treat)

- **$L_c = (1 - x_a) * L_a$** where **L_c** is the moles of pure liquid and is constant
- **$(L_i)_a = x_a * L_a$** where **$(L_i)_a$** is moles of solute entering in liquid stream
- We will know both **L_a** and **x_a**

Stripping

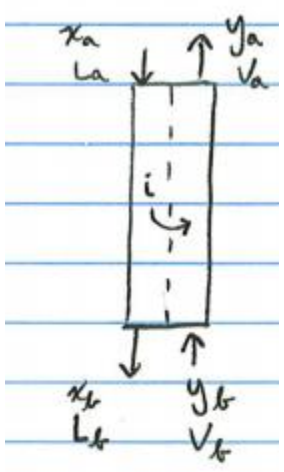
Exiting Liquid

- We will either know: Required $\mathbf{x_b}$
  - Then $\mathbf{(L_i)_b = x_b * [(L_i)_b + L_c]}$
  - And therefore $$(L_i)_b = \frac{x_b}{1 - x_b} L_C$$
- Or we will know that we are required to remove some fraction, **R**, of incoming molar solute flow
- Then $\mathbf{(L_i)_b = (1 - R) * (L_i)_a}$
- And therefore $$x_b = \frac{(L_i)_b}{(L_i)_b + L_c}$$



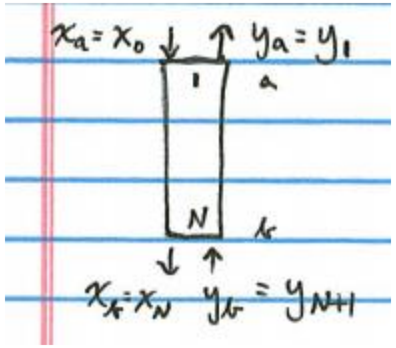
Stripping

- Entering Vapor: we will know $\mathbf{y_b}$ and $\mathbf{V_b}$ (for now…)
  - $\mathbf{(V_i)_b = y_b * V_b}$ and $\mathbf{V_c = (1 - y_b) * V_b}$
  - $\mathbf{V_c}$ is constant
- Exiting Vapor: we need to do material balance (similar to the analysis of the exiting liquid stream in an absorption process)
  - Moles of solute in exiting vapor $\mathbf{(V_i)_a = (L_i)_a + (V_i)_b - (L_i)_b}$

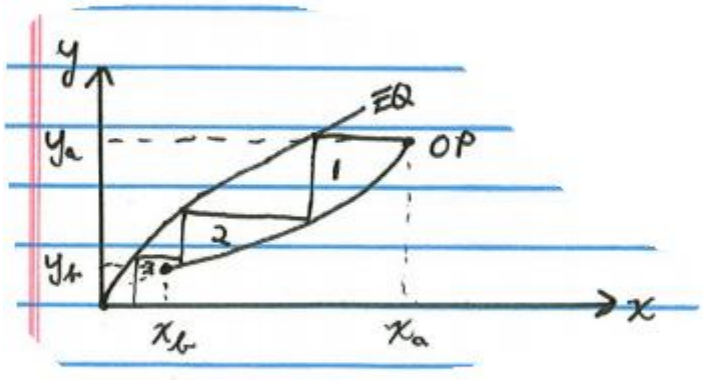


$$y_a = \frac{(V_i)_a}{(V_i)_a + V_c}$$

McCabe-Thiele and Stripping



- Now the top of the tower is the “dirty” section and bottom of tower is the “clean” section, which is the opposite of an absorption tower
- OP Line is the same equation as for absorption
- EQ Curve is the same as for absorption
- For Stripping the EQ curve is ABOVE the OP Line
  - We are transferring mass in the opposite direction as for absorption
- Now counting from top to bottom means starting in the upper right portion



Kremser Equation and Stripping

- The same equations apply
- Some consider the **x** forms to be more appropriate for stripping and the **y** forms more appropriate for absorption
- Can use any of the forms of Kremser as long as OP and EQ are straight lines in the region of interest

Minimum Gas Flow and Stripping

- This is a similar analysis as for absorption
- Straight OP Lines
  - **$y_a = y^*(x_a)$**
- Curved OP Lines
  - $y_{a,min} = \frac{(V_i)_a}{(V_i)_a + (V_c)_{min}}$
- If entering gas is not pure $(V_b)_{min} = \frac{(V_c)_{min}}{1 - y_b}$
- If absorbing vapor is steam then $V_b \approx 1.3 * (V_b)_{min}$
- If absorbing gas is air then $V_b \approx (3\ to\ 5) * (V_b)_{min}$
  - (The cost of steam >> the cost of air)