CE407
SEPARATIONS

Lecture 02

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Counter Current Absorption

- See notes titled "Counter Current Absorption" for demonstration that counter current contact is the most efficient manner to clean up a mixture
- Absorbers – 3 components considered:
  - Fixed Gas / Inert Gas — typically Air
  - Solute — the material we are removing
  - Absorbing Liquid — typically water or oil
- The "Usual Assumptions" state that the Fixed gas does not transfer into the Absorbing Liquid and the Absorbing Liquid does not evaporate into the Fixed Gas

Counter Current Absorption

- **a** refers to top of tower, **b** refers to bottom
- **<u>Entering Gas</u>**

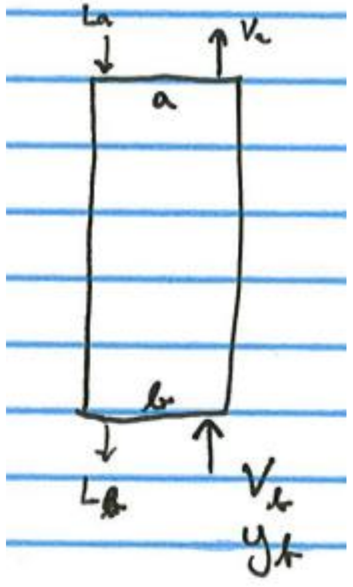
  $\mathbf{V_b}$ = the # of moles of total gas entering at b end of column

  $\mathbf{y_b}$ = the mole fraction of solute in gas entering at b end

  Note: No subscript i is needed for the mole fraction. It is understood that the y_b refers to the solute.

  $\mathbf{(V_i)_b = y_b * V_b}$ = the # of moles of SOLUTE entering at b end

  $\mathbf{V_c = (1 - y_b) * V_b}$ = the # of moles of INERT GAS entering at b end
- Note that V_c is constant. (That's what the "c" stands for…)
- If entering gas is presented as a volumetric flow, use the Ideal Gas Law to convert to molar flow

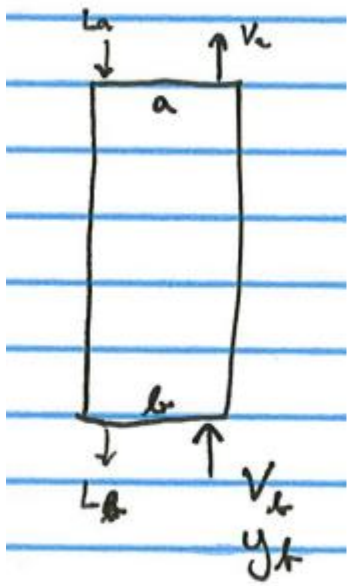


Counter Current Absorption

- **<u>Exiting Gas</u>**
  - Often specified by mole fraction of solute in exiting vapor stream
  - $\mathbf{V_a}$ = the # of moles of total gas exiting at a end of column
  - $\mathbf{y_a}$ = the mole fraction of solute in gas exiting at a end
  - Note: No subscript i is needed for the mole fraction. It is understood that the y_a refers to the solute.
  - $\mathbf{(V_i)_a = y_a * V_a = y_a * ((V_i)_a + V_c)}$ = the # of moles of SOLUTE exiting at **a** end
  - This can be rearranged to obtain

$$(V_i)_a = \frac{y_a}{1 - y_a} V_c$$

- Note that V_c is constant and therefore equal to value calculated at b end of tower.



Counter Current Absorption

- Exiting gas may be specified by % of solute removed
  - "R % of incoming solute is removed"
  - Therefore the rest of the solute remains in the exiting stream and the number of moles of solute in the exiting vapor is

$$(V_i)_a = \left(\frac{100 - R}{100}\right)(V_i)_b$$

  - The mole fraction of the exiting vapor can be calculated as

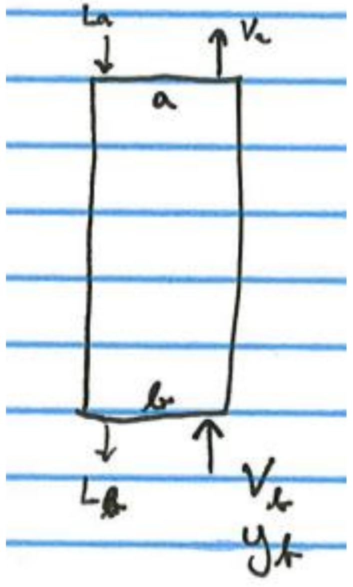
$$y_a = \frac{(V_i)_a}{(V_i)_a + V_c}$$

Counter Current Absorption

- L_a = total # of moles of liquid entering at **a** end
  - This is a given (for now)
- x_a = mole fraction of solute in entering liquid at **a** end
  - This will be given (and is often equal to 0)
- L_c = # of moles of pure absorbing liquid

$$L_c = (1 - x_a) * L_a$$

  - This is a constant through the tower under the “Usual Assumptions”



- $(L_i)_a$ = # of moles of solute entering with the liquid stream at **a** end

$$(L_i)_a = x_a * L_a$$

Counter Current Absorption

- Process specifications define the three streams vapor in, vapor out, and liquid in
- We will use a mass balance on the solute to determine the liquid out stream conditions

Solute In = Solute Out

$$(V_i)_b + (L_i)_a = (V_i)_a + (L_i)_b$$

- From this we see that: $(L_i)_b = (V_i)_b - (V_i)_a + (L_i)_a$

- We can compute exiting liquid mole fraction as:

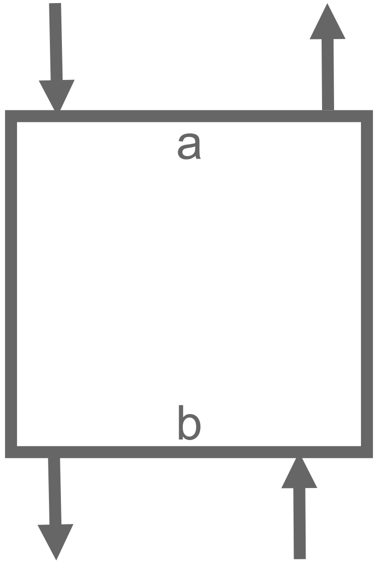
$$x_b = \frac{(L_i)_b}{(L_i)_b + L_c} = \frac{moles\ solute\ exiting\ in\ liquid}{total\ moles\ exiting\ in\ liquid}$$

- Note: This mass balance is telling us what the exiting liquid stream must be if we are to achieve the specified change to the vapor stream – we have not looked at what tower design is required to accomplish this change!

Counter Current Absorption Example

- **Contaminated air 100 mol/minute**
- **Composition 20 mole percent contaminant, 80 mole percent air**
- **Exiting vapor to have contaminant mole fraction of 0.05**
- **Pure Water enters the tower at the top with a flow rate of 10 mole/minute**
- **What is the contaminant mole fraction in the exiting liquid?**

L_a = 10 mol water
$x_a = 0$
L_c = 10 mol water
$L_{i\,a} = 0$



V_c = 80 mol air
$y_a = 0.05$
$V_{i\,a} = ?$

L_c = same = 10
$x_b = ?$
$L_{i\,b} = ?$

V_b = 100 mol
V_c = 0.80 *100 = 80 mol air
V_{ib} = 0.20 *100 = 20 mol contaminant
$y_b = 0.20$

Counter Current Absorption Example

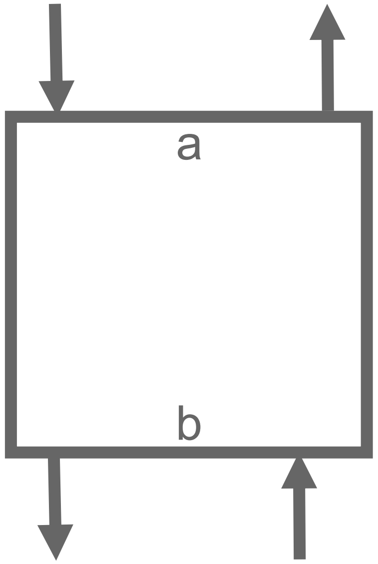
- Let's look at upper right-hand corner (exiting vapor)
- $y_a = 0.05 = \frac{V_{i\,a}}{V_{i\,a} + V_c} = \frac{V_{i\,a}}{V_{i\,a} + 80}$
- $0.05 * (V_{i\,a} + 80) = V_{i\,a}$
- $0.05 * V_{i\,a} + 0.05 * 80 = V_{i\,a}$
- $4.00 = 0.95 * V_{i\,a}$
- $V_{i\,a} = 4.2105\, mol\ contaminant$

- So now we know how many moles of contaminant exit with the vapor

Counter Current Absorption Example

- Let's go to lower left-hand corner
- Mole balance on contaminant:
- Out = In
- $V_{i\,a} + L_{i\,b} = V_{i\,b} + L_{i\,a}$
- $4.2105 + L_{i\,b} = 20 + 0$
- $L_{i\,b} = 15.7895$

L_c = 10 mol water
x_a = 0
$L_{i\,a}$= 0



V_c = 80 mol air
y_a = 0.05
$V_{i\,a}$ = 4.2105

L_c = 10
x_b = ?
$L_{i\,b}$ = ?

V = 100 mol
V_c = 80 mol air
$V_{i\,b}$ = 20 mol contaminant
y_b = 0.20

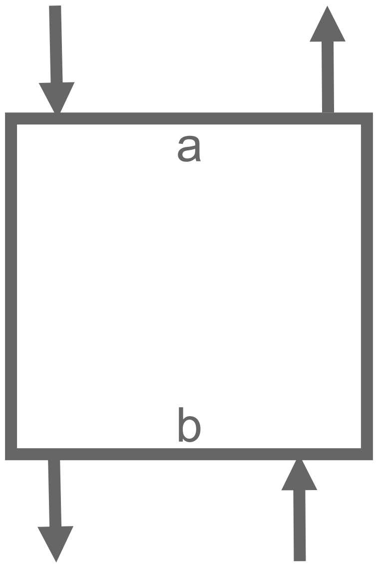
- Now we can calculate mole fraction of exiting liquid:
- $$x_b = \frac{L_{i\,b}}{L_{i\,b}+L_c} = \frac{15.7895}{15.7895+10} = \mathbf{0.612}$$

Counter Current Absorption Example

- Contaminated air 100 mol/minute
- Composition 20 mole percent contaminant, 80 mole percent air
- 95% of entering contaminant is to be removed from vapor (i.e. absorbed into the liquid)
- Pure Water enters the tower at the top with a flow rate of 10 mole/minute
- What is the contaminant mole fraction in the exiting liquid and vapor?

L_c = 10 mol water
$x_a = 0$

V_c = 80 mol air
y_a = ?
$V_{i\,a}$ = ?



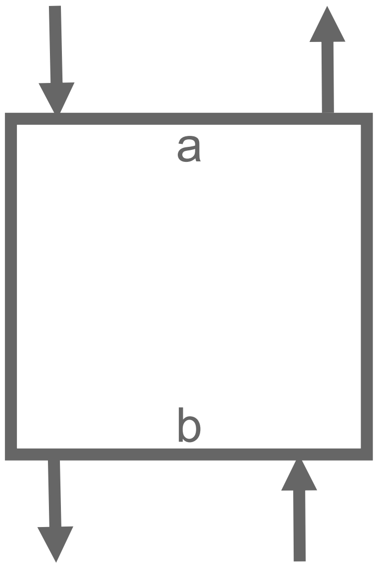
L_c = same = 10
x_b = ?
$L_{i\,b}$ = ?

V = 100 mol
V_c = 0.80 *100 = 80 mol air
$V_{i\,b}$ = 0.20 *100 = 20 mol contaminant
$y_b = 0.20$

Counter Current Absorption Example

- Let's go to lower left-hand corner
- $L_{i\,b} = 0.95 * V_{i\,b} = 0.95 * 20 = 19\,mol$

L_c = 10 mol water
x_a = 0



V_c = 80 mol air

$V_{i\,a}$ = 1.00 mol

- Mole balance on contaminant:
- Out = In
- $V_{i\,a} + L_{i\,b} = V_{i\,b} + L_{i\,a}$
- $V_{i\,a} + 19 = 20 + 0$
- $V_{i\,a} = 1.00$

L_c = 10
x_b = ?
$L_{i\,b}$= 19

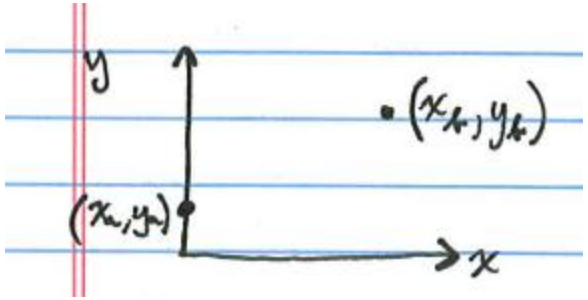
V = 100 mol
V_c = 80 mol air
$V_{i\,b}$ = 20 mol contaminant
y_b = 0.20

- Now we can calculate mole fraction of exiting liquid and vapor:
- $$x_b = \frac{L_{i\,b}}{L_{i\,b}+L_c} = \frac{19}{19+10} = 0.655$$

$$y_a = \frac{V_{i\,a}}{V_{i\,a}+V_c} = \frac{1}{1+80} = 0.0124$$

Operating Line

- A plot of mole fraction of vapor versus mole fraction of liquid throughout the tower



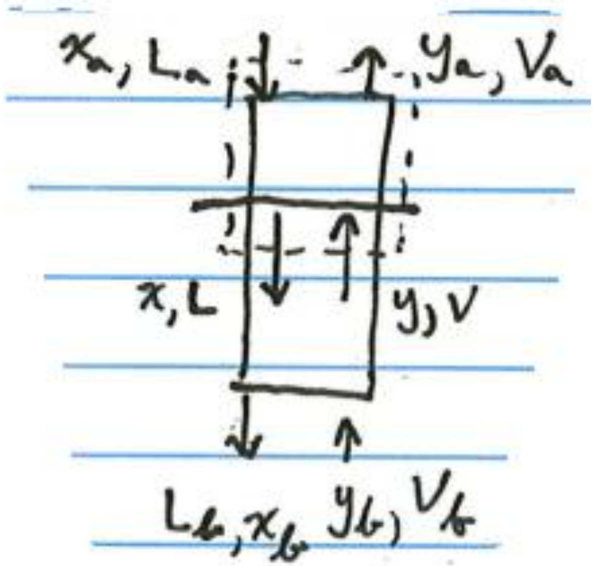
- Flow rate of vapor decreases as we go up higher in the tower (we are removing solute…)
- Flow rate of liquid increases as we go lower in the tower (we are absorbing solute…)
- V_c and L_c are assumed constant
- We know the points at the **a** and **b** ends of the tower – how do we determine the intermediate points?

Operating Line

- In order to determine the relationship of y to x at any given height in the tower, we establish a control volume from the top of the tower to an arbitrary height
- At the top:

$$L_c = (1 - x_a) * L_a \text{ therefore } L_a = \frac{L_c}{1 - x_a}$$

$$V_c = (1 - y_a) * V_a \text{ therefore } V_a = \frac{V_c}{1 - y_a}$$



- At a generic height:

$$L_c = (1 - x) * L \quad \text{therefore} \quad L = \frac{L_c}{1 - x}$$

$$V_c = (1 - y) * V \quad \text{therefore} \quad V = \frac{V_c}{1 - y}$$

- We have now defined the total molar flow of vapor and liquid at both ends of the control volume…

Operating Line

- Total Molar Balance Across the Control Volume

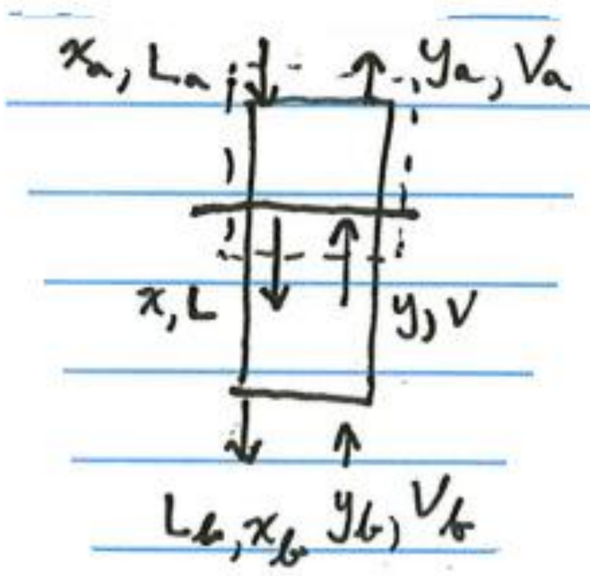
Total Moles In = Total Moles Out

$$V + L_a = V_a + L$$

$$\frac{V_c}{1-y} + \frac{L_c}{1-x_a} = \frac{V_c}{1-y_a} + \frac{L_c}{1-x}$$

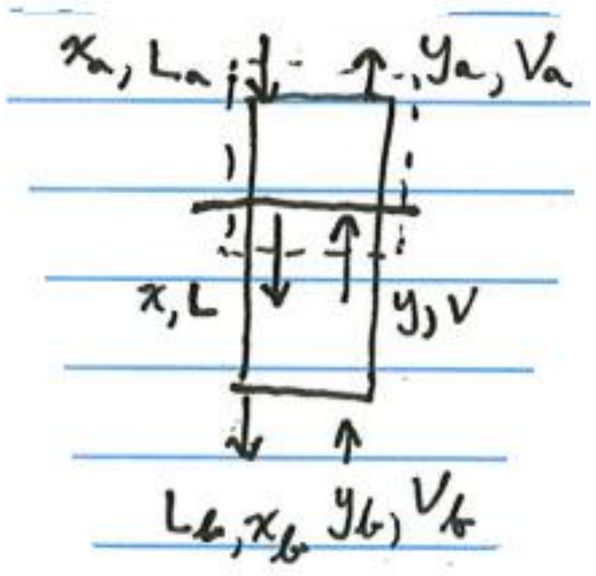
- Rearrange algebraically:

$$y = 1 - \left[\frac{1}{1-y_a} + \frac{L_c}{V_c}\left(\frac{1}{1-x} - \frac{1}{1-x_a}\right)\right]^{-1}$$



- This is called the **Operating Line**. It is actually somewhat curved. If the system is dilute, it is fairly straight.

Operating Line



- What if we had done a solute balance instead?

Solute In = Solute out

$$V * y + L_a * x_a = V_a * y_a + L * x$$

- If the system is dilute (say, x and y < 0.1 or 0.05)
  - Then $V_a \approx V_b \approx V \; and \; L_a \approx L_b \approx L$
- Now solute balance is approximately

$$V * y + L * x_a = V * y_a + L * x$$

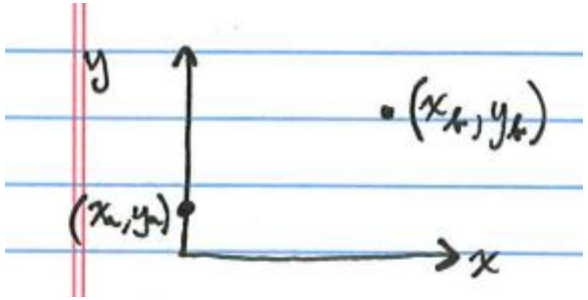
and therefore

$$y = y_a + \frac{L}{V} * (x - x_a)$$

  - This is an approximation – USE THE BOXED EQUATION ON PREVIOUS SLIDE!!!

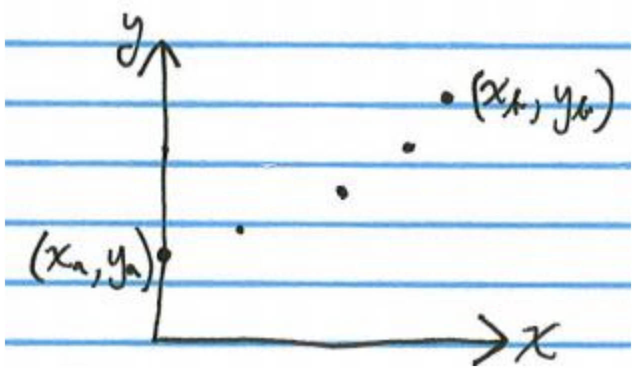
L / V

- L / V is a common parameter when describing an absorption tower.
- For a dilute system it is relatively constant, but what value do we use?
- 3 Reasonable Choices:
  - $\frac{L}{V} = \frac{\frac{1}{2}(L_a + L_b)}{\frac{1}{2}(V_a + V_b)}$
  - $\frac{L}{V} = \frac{1}{2}\left(\frac{L_a}{V_a} + \frac{L_b}{V_b}\right)$
  - $\frac{L}{V} = \frac{y_b - y_a}{x_b - x_a}$ Preferred Method
    - $y = y_a + \frac{L}{V} * (x - x_a)$ - Approximate OP Line for dilute cases



Operating Line

- Choose several values of x between x_a and x_b
- Use operating line equation (boxed equation on slide 10) to calculate the corresponding values of y
- Plot:



- The Operating Line is just a MOLE BALANCE (I will sometimes use the term mass balance, as well…) (If there are no chemical reactions taking place then a mole balance and a mass balance are equivalent.)

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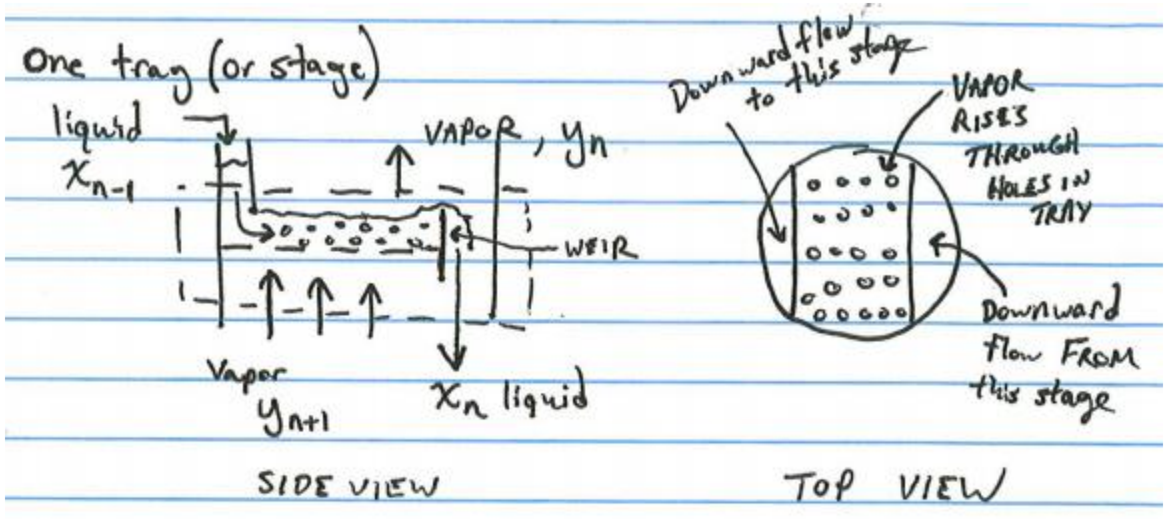
Absorption Tower Analysis

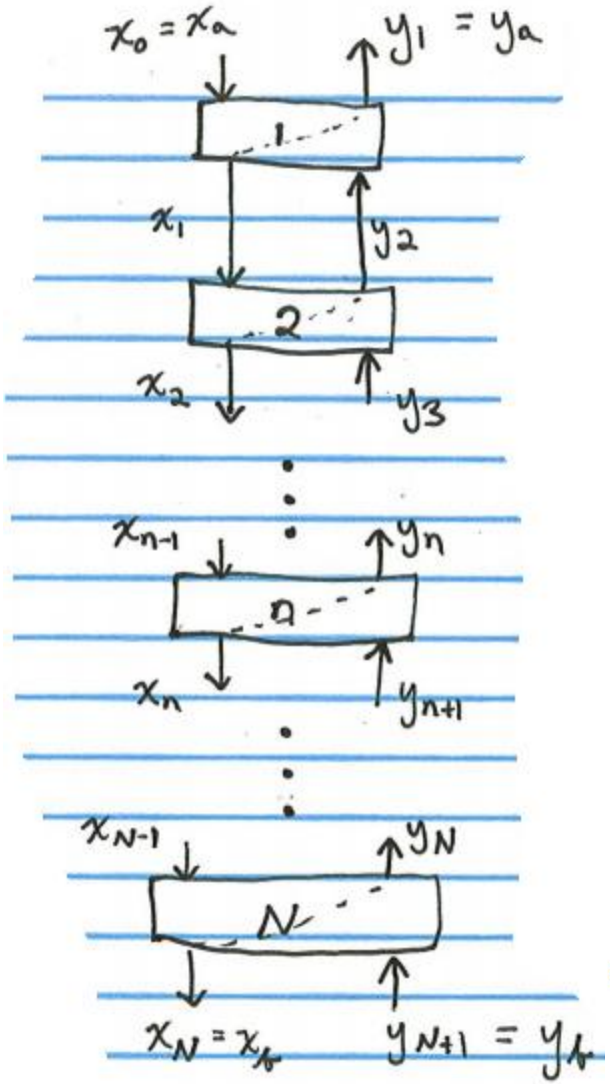
- A tower has many stages (or trays)
- Liquid flows down channel on left hand side of this drawing
- Liquid flows across the tray
  - Level is set by weir
- Vapor flows up through holes in tray
- Mass transfer occurs between the bubbles and the liquid
- Liquid flows down the channel on right hand side of this drawing



- **ASSUME THAT VAPOR LEAVING A GIVEN STAGE IS IN EQUILIBRIUM WITH THE LIQUID LEAVING THAT SAME STAGE**

Absorption Tower Analysis

- Stages are numbered from top to bottom
- **n** is an arbitrary stage location
- **N** is the bottom stage
- Subscripts for **x** and **y** refer to the stage from which that flow originates (i.e. which stage did it come from)
- For stage **n**:
  - Vapor enters from stage **n+1** and has mole fraction $\mathbf{y_{n+1}}$
  - Liquid enters from stage **n-1** and has mole fraction $\mathbf{x_{n-1}}$
  - Vapor leaving stage **n** has mole fraction $\mathbf{y_n}$
  - Liquid leaving stage **n** has mole fraction $\mathbf{x_n}$
- $\mathbf{x_n}$ and $\mathbf{y_n}$ are related via Equilibrium Relations
- $\mathbf{x_n}$ and $\mathbf{y_{n+1}}$ are related via the Operating Line
  - Note that $\mathbf{x_n}$ and $\mathbf{y_{n+1}}$ cross the control volume we used to analyze the operating line



Equilibrium Relationships

- Raoult's Law $y_i P = x_i P_i^{sat}(T)$

$$y_n = \frac{P_i^{sat}(T)}{P} * x_n$$

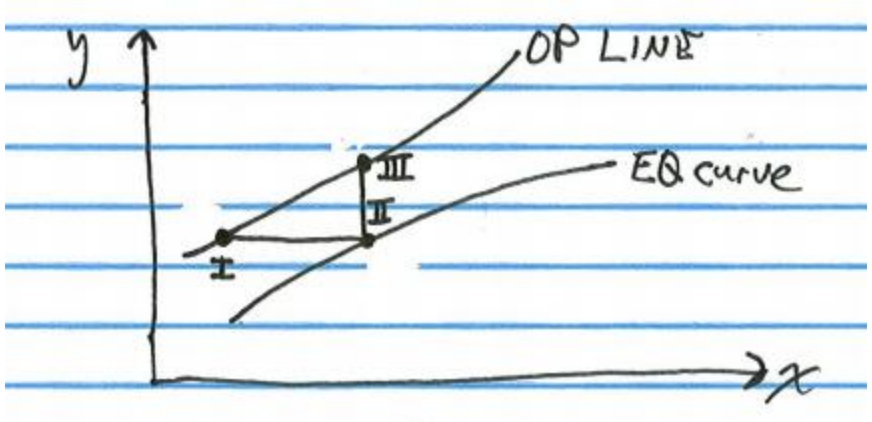
- Henry's Law $y_i P = x_i \gamma_i^{\infty} P_i^{sat}(T)$

$$y_n = \frac{k_i(T)}{P} x_n$$

  - Note that subscript **n** refers to stage number – it is understood that it refers to the solute
- Vapor Liquid Equilibria (VLE) data supplied
  - Be cautious of the units!
    - They will often be mass ratios, mass fractions, partial pressures, etc
    - You **MUST** convert to mole fractions

McCabe-Thiele Method

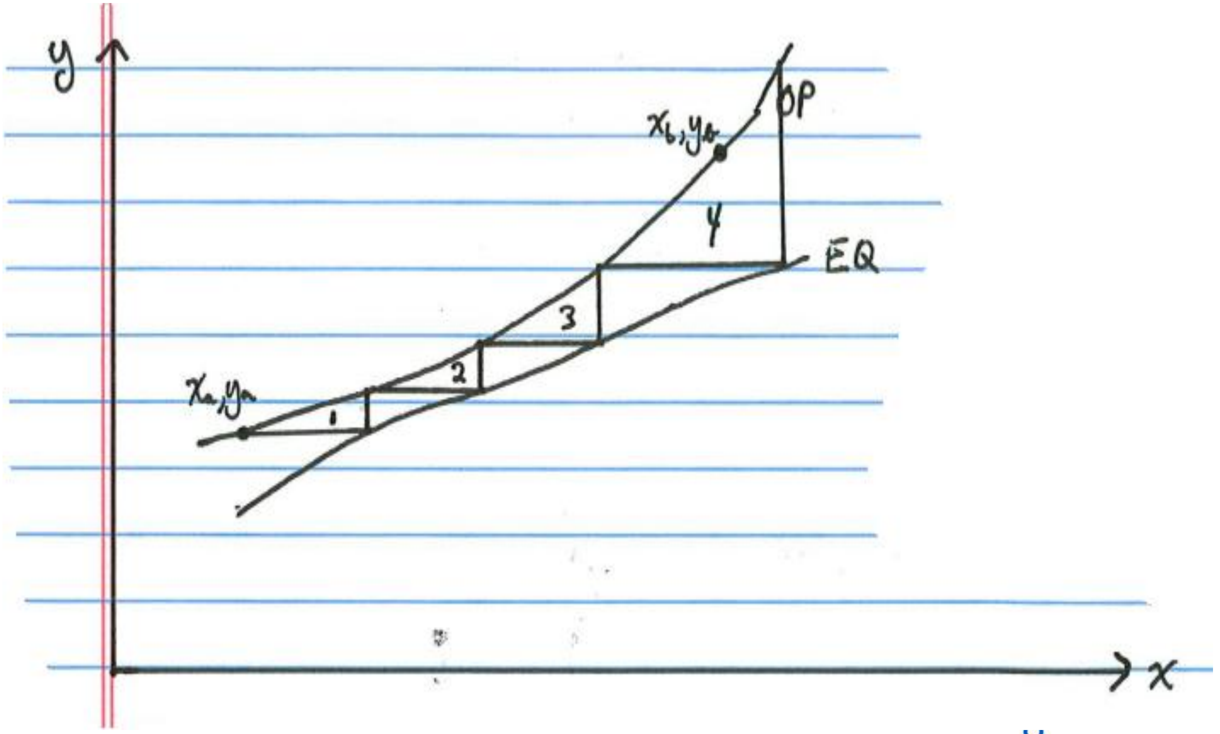
- From notes titled “Counter Current Absorption”, scheme 3 we can see that even a two-stage calculation is very complex. We want to solve for **N** stages…
- McCabe-Thiele is a graphical solution
- Plot both the Operating Line (mass balance) and the Equilibrium Curve on the same graph.
- Point I is ($\mathbf{x_a}$, $\mathbf{y_a}$) which is ($\mathbf{x_0}$, $\mathbf{y_1}$)
- Point II has the same value of y as Point I and is therefore $\mathbf{y} = \mathbf{y_1}$. As Point II lies on the equilibrium curve $\mathbf{x} = \mathbf{x_1}$. (Remember $\mathbf{x_n}$ and $\mathbf{y_n}$ are in equilibrium!)
- Point III has the same value of x as Point II and is therefore $\mathbf{x} = \mathbf{x_1}$. Because Point III is on the Operating Line we know that $\mathbf{y} = \mathbf{y_2}$. (Remember that ($\mathbf{x_n}$, $\mathbf{y_{n+1}}$) are on the Operating Line!)



- We have just determined the relationship from $y = y_1$ to $y = y_2$: we have moved one stage!

McCabe-Thiele

- Continue until we reach (**x_b**, **y_b**)
- Note that it is unlikely that any stage will land exactly on (**x_b**, **y_b**)
- This looks like about 3.6 stages
  - Obviously, there is no physical meaning of a partial stage
  - Report this on homework or in exam as "3.6 stages, to be rounded up to 4 stages"
- Stepping **ALWAYS** begins and finishes on the Operating Line!
- Later in the course we will learn how to determine the physical height of each stage
  - Rough estimate: 6" for small flows and 2'-3' for large flows

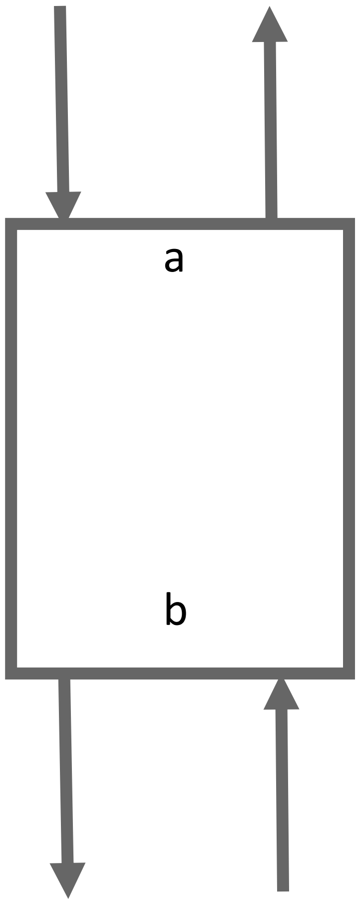


McCabe-Thiele Example

- By means of a plate column, acetone is absorbed from its mixture with air in a recycled nonvolatile absorption oil. The entering gas has a flow rate of 100 moles/minute and contains 10 mole percent acetone. The entering oil contains 0.1 mole percent acetone and has a liquid flow rate, L_a, of 200 moles/minute. Of the acetone in the air, 90 percent is to be absorbed. The equilibrium relationship is $y_e = 2\ x_e$. **Plot the operating line and determine the number of ideal stages.**
- Make the “usual assumption” that the air will not transfer into the oil and that the oil will not evaporate into the air.

McCabe-Thiele Example

L_a = 200
x_a = 0.001
L_c = (1 – 0.001)(200) = 199.8 mol oil
$L_{i,\,a}$ = (0.001)(200) = 0.2 mol acetone



V_c = 90 mol air
$V_{i,\,a}$ = (1 – 0.9) * $V_{i,\,b}$ = 0.10 * 10
= 1.0 mol acetone

L_c = same = 199.8 mol oil
$L_{i,\,b}$ = ?
x_b = ?

V = 100 mol
y_b = 0.10
V_c = (1 - 0.1) * 100 = 90 mol air
$V_{i,\,b}$ = 0.1 * 100 = 10 mol acetone

McCabe-Thiele Example

Acetone Balance

In = Out

$$V_{i,b} + L_{i,a} = V_{i,a} + L_{i,b}$$
$$10 + 0.20 = 1.0 + L_{i,b}$$
$$L_{i,b} = 9.2\ mol\ acetone$$

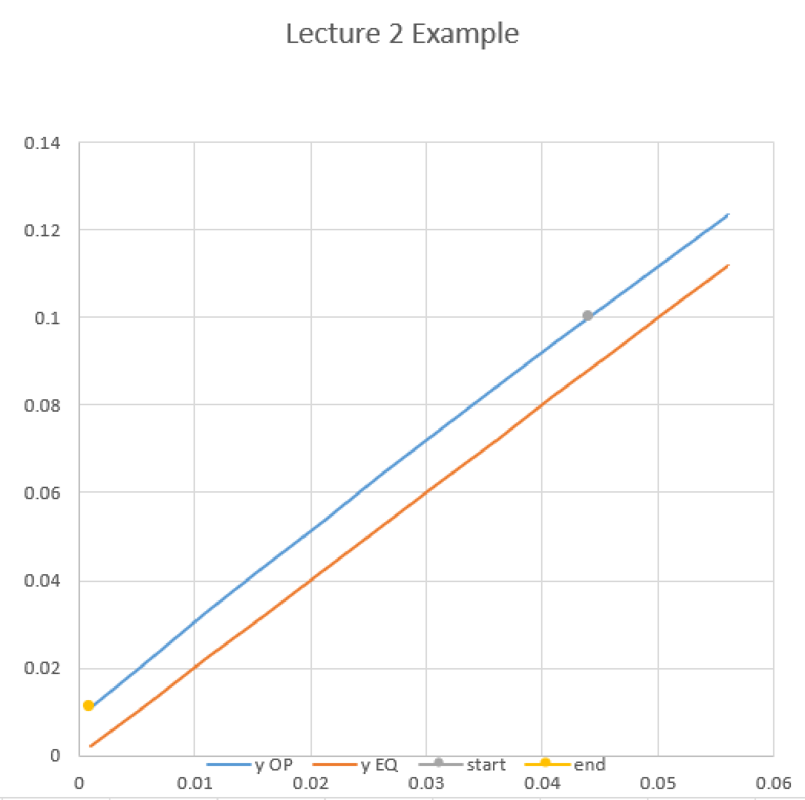
$$y_a = \frac{V_{i,a}}{V_{i,a} + V_C} = \frac{1.0}{1.0 + 90} = 0.011$$
$$x_b = \frac{L_{i,b}}{L_{i,b} + L_C} = \frac{9.2}{9.2 + 199.8} = 0.044$$

McCabe-Thiele Example

xa	ya	xb	yb	Lc	Vc		m
0.001	0.011	0.044	0.1	199.8	90		2

x	y OP	y EQ
0.001	0.011	0.002
0.006	0.021814	0.012
0.011	0.0325	0.022
0.016	0.04306	0.032
0.021	0.053497	0.042
0.026	0.063811	0.052
0.031	0.074007	0.062
0.036	0.084085	0.072
0.041	0.094048	0.082
0.046	0.103898	0.092
0.051	0.113636	0.102
0.056	0.123265	0.112

$$y = 1 - \left[\frac{1}{1 - y_a} + \frac{L_c}{V_c}\left(\frac{1}{1 - x} - \frac{1}{1 - x_a}\right)\right]^{-1}$$



McCabe-Thiele Example