STATE UNIVERSITY OF NEW YORK AT BUFFALO

Department of Mechanical and Aerospace Engineering

MAE 589 Diffraction, Microscopy and Spectroscopy Techniques

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Homework No. 8
Solution

1. (a)

(i) ^{40}Ar $\quad m = 40$

minimum $M \longrightarrow k_M = \frac{E}{E_0} = 0 \Rightarrow M = 40$

(ii) ^{4}He $\quad m = 4$

minimum $M \longrightarrow k_M = \frac{E}{E_0} = 0 \Rightarrow M = 4$.

(b) $\theta = 138°$, $E_0 = 600$ eV

(i) ^{40}Ar $\quad m = 40$

Let $M_1 = 100$, $M_2 = 101$.

$$k_{M_1} = \left[\frac{40 \cos 138° + (100^2 - 40^2 \sin^2 138°)^{\frac{1}{2}}}{40 + 100}\right]^2 = 0.226$$

$$E_1 = k_{M_1} E_0 = 135.6 \text{ eV}$$

$$k_{M_2} = \left[\frac{40 \cos 138° + (101^2 - 40^2 \sin^2 138°)^{\frac{1}{2}}}{40 + 101}\right]^2 = 0.230$$

$$E_2 = k_{M_2} E_0 = 138.15 \text{ eV}$$

$$\Delta E = E_2 - E_1 = \underline{\underline{2.55 \text{ eV}}}$$

(ii) ^{4}He $\quad m = 4$

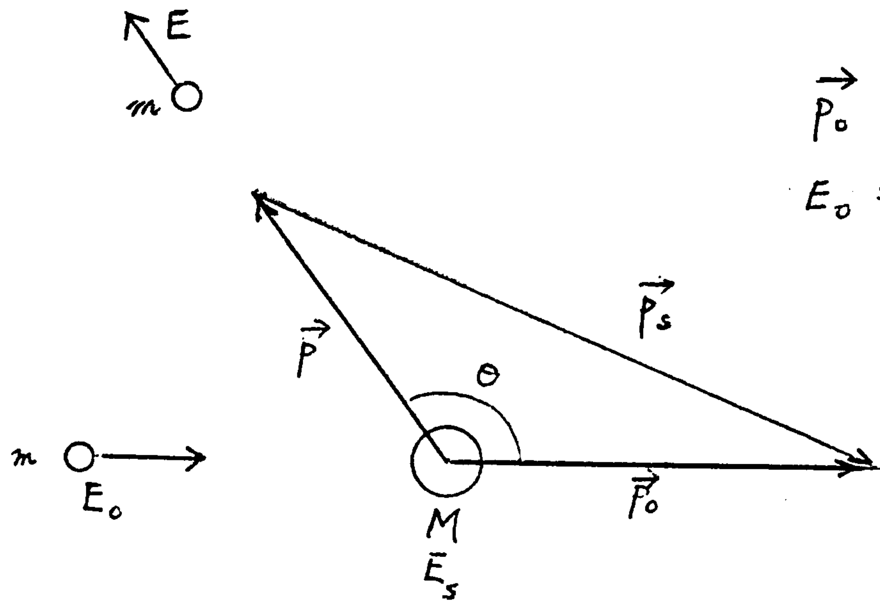
$k_{M_1} = 0.870$, $\quad E_1 = 521.86$ eV

$k_{M_2} = 0.871$, $\quad E_2 = 522.53$ eV

$\Delta E = \underline{\underline{0.67 \text{ eV}}}$

(c) Advantage of ^{40}Ar: resolve heavy elements better.
Disadvantage of ^{40}Ar: can't detect elements lighter than 40.

2.



$\vec{p}_o = \vec{p} + \vec{p}_s$

$E_o = E + E_s$

m = mass of projectile ion

M = mass of surface atom

$\vec{p}_o$ = momentum of incident ion

$\vec{p}$ = momentum of deflected ion

$\vec{p}_s$ = momentum of surface atom

E_o = energy of incident ion

E = energy of deflected ion

E_s = energy of surface atom

$$E_o = E + E_s \quad \Rightarrow \quad \frac{p_o^2}{2m} = \frac{p^2}{2m} + \frac{p_s^2}{2M}$$

$$\text{or} \quad \frac{m}{M} p_s^2 = p_o^2 - p^2 \qquad (1)$$

Applying the law of cosines to the above triangle,

$$p_s^2 = p_0^2 + p^2 - 2\,p_0 p \cos\theta \qquad (2)$$

Dividing Eq. (2) by Eq. (1),

$$\frac{M}{m} = \frac{p_0^2 + p^2 - 2 p_0 p \cos\theta}{p_0^2 - p^2} \qquad (3)$$

Dividing by p_0^2 and noting that $p^2/p_0^2 = E/E_0$,

$$\frac{M}{m} = \frac{1 + E/E_0 - 2\sqrt{E/E_0}\,\cos\theta}{1 - E/E_0}$$

$$\frac{M}{m} - \frac{M}{m}\frac{E}{E_0} = 1 + \frac{E}{E_0} - 2\sqrt{\frac{E}{E_0}}\,\cos\theta$$

$$0 = \frac{E}{E_0}\left(1 + \frac{M}{m}\right) + \left(1 - \frac{M}{m}\right) - 2\sqrt{\frac{E}{E_0}}\,\cos\theta$$

Let $\sqrt{\frac{E}{E_0}} = x$

$$\left(1 + \frac{M}{m}\right)x^2 - (2\cos\theta)\,x + \left(1 - \frac{M}{m}\right) = 0$$

$$x^2 - \frac{2\cos\theta}{1 + \frac{M}{m}}\,x + \frac{1 - \frac{M}{m}}{1 + \frac{M}{m}} = 0$$

$$x = \frac{\dfrac{2\cos\theta}{1 + \frac{M}{m}} \pm \sqrt{\left(\dfrac{2\cos\theta}{1 + \frac{M}{m}}\right)^2 - 4\left(\dfrac{1 - \frac{M}{m}}{1 + \frac{M}{m}}\right)}}{2}$$

$$= \frac{\cos\theta}{1 + \frac{M}{m}} \pm \sqrt{\left(\frac{\cos\theta}{1 + \frac{M}{m}}\right)^2 - \left(\frac{1 - \frac{M}{m}}{1 + \frac{M}{m}}\right)}$$

$$\chi^2 = \left(\frac{\cos\theta}{1+\frac{M}{m}}\right)^2 + \left(\frac{\cos\theta}{1+\frac{M}{m}}\right)^2 - \left(\frac{1-\frac{M}{m}}{1+\frac{M}{m}}\right)$$

$$\pm\ 2\ \frac{\cos\theta}{1+\frac{M}{m}}\sqrt{\left(\frac{\cos\theta}{1+\frac{M}{m}}\right)^2 - \left(\frac{1-\frac{M}{m}}{1+\frac{M}{m}}\right)}$$

$$\chi^2 = \left(1+\frac{M}{m}\right)^{-2}\left[2\cos^2\theta - 1 + \left(\frac{M}{m}\right)^2 \pm (2\cos\theta)\sqrt{\cos^2\theta - 1 + \left(\frac{M}{m}\right)^2}\right]$$

$$= \left(1+\frac{M}{m}\right)^{-2}\left\{\cos^2\theta + \left(\frac{M}{m}\right)^2 - \sin^2\theta \pm 2\cos\theta\left[\left(\frac{M}{m}\right)^2 - \sin^2\theta\right]^{\frac{1}{2}}\right\}$$

$$= \left(1+\frac{M}{m}\right)^{-2}\left\{\cos\theta \pm \left[\left(\frac{M}{m}\right)^2 - \sin^2\theta\right]^{\frac{1}{2}}\right\}^2$$

Since $\frac{M}{m} \geq 1$,

$$\chi^2 = \left(1+\frac{M}{m}\right)^{-2}\left\{\cos\theta + \left[\left(\frac{M}{m}\right)^2 - \sin^2\theta\right]^{\frac{1}{2}}\right\}^2$$

$$= \frac{m^2}{(m+M)^2}\left\{\cos\theta + \left[\left(\frac{M}{m}\right)^2 - \sin^2\theta\right]^{\frac{1}{2}}\right\}^2$$

$$= \frac{\left[m\cos\theta + (M^2 - m^2\sin^2\theta)^{\frac{1}{2}}\right]^2}{(m+M)^2}$$

(a) $m = M$

$$k_M = \frac{1}{4}(\cos\theta + \sin\theta)^2$$

(b) $m \ll M$

$$k_M = \left[\frac{\cancel{m\cos\theta} + M}{M}\right]^2 = 1$$

(c) $\theta = 90^\circ$

$$k_M = \left[\frac{(M^2 - m^2)^{1/2}}{m + M}\right]^2$$