MAE 552
Heuristic Optimization

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Lecture #33

4/22/02

Fully Stressed Design

NN's - Threshold

We said that the *threshold* has the effect of lowering the activation energy of the neuron (or raising it in the case of a bias).

So it is the only means we have to prevent a neuron from firing based on undesirable inputs.

Consider the following example.

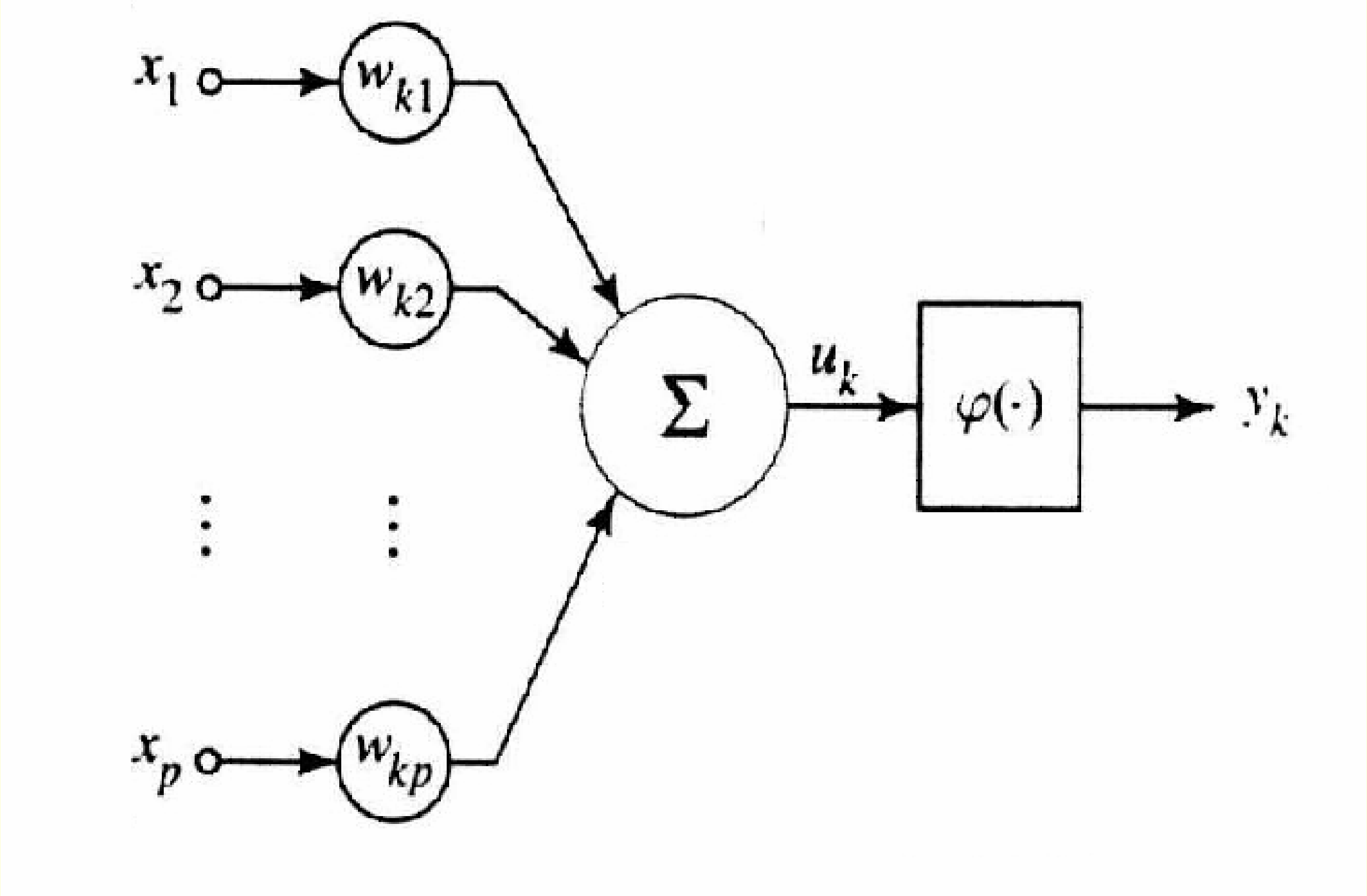
NN’s - Threshold

Suppose we want to teach a neuron to compute the logical “and” operation for a given set of binary inputs [0, 1].

Could we do this without a threshold value?

(P.S. don’t say yes if you are thinking of changing the summing junction into an “and-ing” junction. No cheating.)

NN's - Threshold



NN’s - Threshold

Maybe it could be done but not easily I would say.

On the other hand, is it an easy task if we incorporate a threshold?

Consider the following neuron model settings.

NN's - Threshold

- All true weights set to 1.

- Activation function: *Threshold function*.

$$\varphi(u) = \begin{cases} 1 & if\ u \geq 0 \\ 0 & otherwise \end{cases}$$

- Threshold value set to p (number of inputs).

OC Methods

- Intuitive optimality criteria (OC) methods:
  - An OC method consists of 2 parts.
    1. A statement of optimality criteria (two basic types)
       - Rigorous mathematical statements like:

         “The K-T conditions must be met”
       - Intuitive statements like

         “The strain energy density in the structure must be uniform”
    2. A resizing algorithm used to attempt to meet the optimality criteria.

Fully Stressed Design (FSD)

- References:

- Venkayya, V.B., “Design of Optimum structures”, Comput. Struc., 1, pp. 265-309, 1971

Fully Stressed Design

FSD is probably the most successful of the OC methods and is responsible for sparking the most interest in developing these sorts of methods.

This method is widely used in the design of structures.

It is applicable to problems with only stress and minimum gage constraints.

Fully Stressed Design

- The optimality criteria statement for FSD is as follows:

  "For the optimum design, each member of the structure that is not at its minimum gage must be fully stressed under at least one of the design load conditions."

Fully Stressed Design

The statement seems perfectly reasonable but there is an implication that the structure is member separable.

So adding or removing material from a member effects only the stress in that member and not in any others.

Fully Stressed Design

Some advantages of FSD:

There is usually a fully stressed structure that lies somewhere near the true optimum.

A great deal of design improvement is likely with a relatively low amount of analysis (very few iterations).

The savings for such improvements are likely to be great.

No derivatives are necessary.

Fully Stressed Design

Some disadvantages of FSD:

It may not find a truly optimal solution.

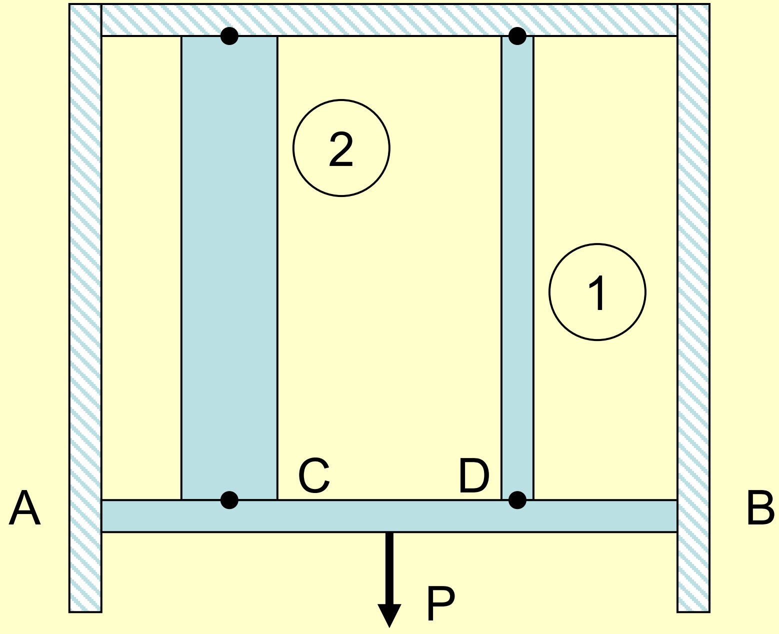
It does not perform well for structures made of more than 1 material.

It may not perform well for statically indeterminate or highly redundant structures because of multiple load paths.

Fully Stressed Design

We will learn this method using examples.

Example 1: Consider this structure shown below.



Perfectly rigid platform AB

Axially loaded members 1 and 2

Fully Stressed Design

Rigid member AB remains exactly horizontal by displacing the load P to the left or right.

Members 1 and 2 are made of different steel alloys with the same Young's Modulus but different densities (ρ_1, ρ_2) and different yield stresses (σ_{01}, σ_{02})

Our objective is to find the minimum mass design by altering the cross-sectional areas (A_1,A_2) without exceeding the yield strength for either member.

Fully Stressed Design

A minimum gage (A_0) is stipulated for both members.

We are given the following relations.

$$\rho_1 = 0.9\rho_2 \qquad \sigma_{01} = 2\sigma_{02}$$

Fully Stressed Design

We can simply evaluate the mass of a design using the following equation.

$$m = l(\rho_1 A_1 + \rho_2 A_2)$$

And the stresses by another equation

$$\sigma_1 = \sigma_2 = \frac{P}{A_1 + A_2}$$

Fully Stressed Design

So which of our two stress constraints will be the limiting or driving constraint?

$$\sigma_1 \le \sigma_{01} \quad \text{OR} \quad \sigma_2 \le \sigma_{02}$$

Clearly, since σ_1 is twice σ_2, σ_2 will become critical first and we can use this information to devise the following expression:

$$A_1 + A_2 = P / \sigma_{02}$$

Fully Stressed Design

The minimum mass design will make max use of the superior alloy (#1) by driving the area of the inferior member toward minimum gage (#2).

So plugging in A_0 for A_2 in our previous relation and rearranging gives:

$$A_1 = \frac{P}{\sigma_{02}} - A_0$$

Where it must be true that:

$$\left\{ \frac{P}{\sigma_{02}} \geq 2A_0 \right\}$$

Fully Stressed Design

The previous equation provides a solution to our problem. But is it a fully stressed solution?

No. We do have that σ_2 is at its bound and that A_2 is at min gage which is good. But we also have that σ_1 is ½ the allowable and A_1 is not at min gage.

So our optimality criteria is not yet met.

Fully Stressed Design

The actual fully stressed design is (take my word for it for now):

$A_1 = A_0$

$A_2 = P/\sigma_{02}-A_1$

Where member 2 is fully stressed and member 1 is at its gage value.

Does anyone see a reason why this isn't good?

Fully Stressed Design

Recall that our density relation told us that the alloy of member 2 was heavier.

So we have a larger volume of the heavier material in our FSD solution which will increase the value of our objective function.

Let’s compare our two solutions by plugging in some numbers.

Fully Stressed Design

Say that:

$P/\sigma_{02} = 20A_0$

Then according to solution 1:

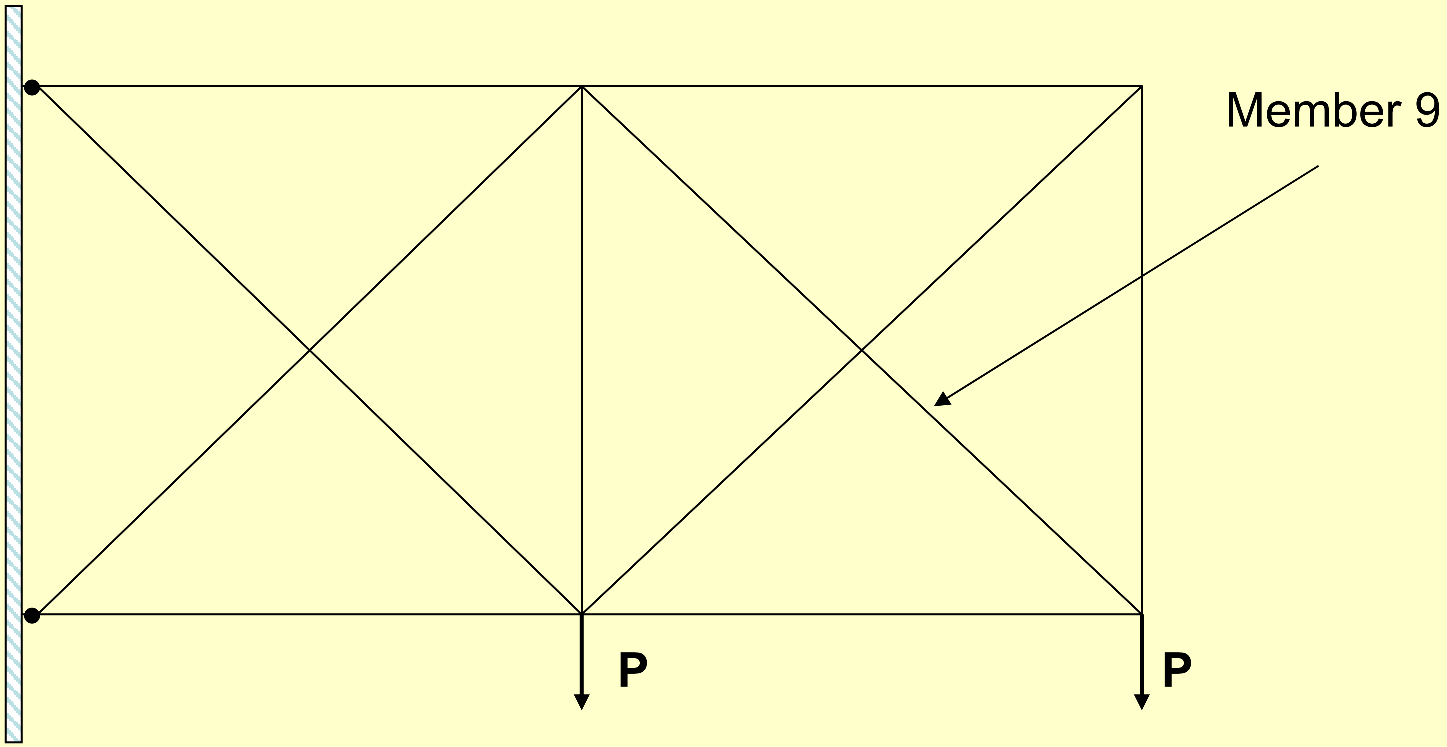
$A_1 = 19A_0$, $A_2 = A_0$, and $m = 18.1\rho_2 A_0 l$

And according to our FSD solution:

$A_1 = A_0$, $A_2 = 19A_0$, and $m = 19.9\rho_2 A_0 l$

Fully Stressed Design

Example 2: 10-member truss – Highly Redundant



Fully Stressed Design

All members are made of the same material with the following properties:

$E = 10^7$ psi

$\rho = 0.1$ lb/in^3

Y = 25 ksi with the exception of number 9

We will consider member 9 with two different values of Y to demonstrate another problem with FSD.

Fully Stressed Design

If the yield stress of #9 is ≤ 37,500 psi, then the optimal and FSD solutions are identical.

If the yield stress of #9 is ≥ 37,500 psi, then the optimum design weighs 1497.6 lbs and member 9 is neither fully stressed or at minimum gage.

The FSD solution weighs 1725.2 lbs (15% heavier) and #9 is at min gage.

Fully Stressed Design

We will use this example to demonstrate another part of FSD.

Recall the 2 components for our OC methods. In our last example, we didn't talk about the resizing algorithm.

So what can we do?

Fully Stressed Design

We will assume that the load carried by a member is constant. That is, it does not change after resizing.

For axially loaded truss members with the areas as design variables, we know that:

$$F_i = \sigma_i A_i$$

Fully Stressed Design

Since F_i is constant (according to our assumption), we can say that the product of the stress and area before and after the resizing will be equal.

So this provides us with a stress ratio resizing update relation as follows:

$$\sigma_{new_i} A_{new_i} = \sigma_{old_i} A_{old_i} \rightarrow A_{new_i} = A_{old_i} \frac{\sigma_{old_i}}{\sigma_{0i}}$$

Fully Stressed Design

For a statically determinate structure, the assumption that the member forces are constant is exactly correct and thus the update relation is highly prudent.

For statically indeterminate structure, the relation is not exact and thus we need to apply the resizing algorithm iteratively until convergence to within some specified tolerance.

Fully Stressed Design

Let's look at this approach to see how we achieved the Fully stressed design in the 1st example.

Recall that we said the FSD was:

$$A_1 = A_0$$

$$A_2 = P/\sigma_{02} - A_1$$

Fully Stressed Design

We'll start with an initial design where both members are at minimum gage and the applied load is $20A_0\sigma_{02}$.

Recall that we had:

$$\sigma_1 = \sigma_2 = \frac{P}{A_1 + A_2} = \frac{20A_0\sigma_{02}}{A_1 + A_2}$$

and:

$$\sigma_{01} = 2\sigma_{02}$$

Fully Stressed Design

Iter	A_1/A_0	A_2/A_0	σ_1/σ_{01}	σ_2/σ_{02}
1	1.0	1.0	5.0	10
2	5.0	10.0	0.67	1.33
3	3.33	13.33	0.6	1.2
4	2.0	16.0	0.56	1.11
5	1.11	17.78	0.56	1.059
6	1.0	18.82	0.504	1.009
7	1.0	18.99	0.5	1.005

Fully Stressed Design

Recall that the FSD solution was:

$A_1 = A_0$

$A_2 = 19A_0$

$\sigma_1 = \sigma_{01}/2$

$\sigma_2 = \sigma_{02}$

This would be optimal if materials 1 and 2 were the same weight.