MAE 552
Heuristic Optimization

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Lecture #20

3/10/02

Taguchi's Orthogonal Arrays

ANOVA

Now that we have a predicted optimum observation value for our experiment, we should compare that to our actual observed value. (We talked about performing the optimum experiment in a previous lecture).

If the predicted value is close to the actual, then we can say that the additive model is sufficient to describe our system and visa versa. The opposing case indicates that we probably have interactions amongst our variables.

ANOVA

So how close is close enough?

To determine this, we have to find our Variance of prediction error. The error value is given by the difference between the predicted and observed optimal.

It has 2 independent components:

– That caused by estimates of m, m_{A1}, m_{B1}.
– That caused by repetition error in an experiment.

ANOVA

•These two components are independent of one another and therefore, the variance of the prediction error is the sum of the variances of these to error components.

To compute the component of the variance due to the first term, we must find the equivalent sample size n_0. It is given as follows:

$$\frac{1}{n_0} = \frac{1}{n} + \left[\frac{1}{n_{A1}} - \frac{1}{n}\right] + \left[\frac{1}{n_{B1}} - \frac{1}{n}\right]$$

Where:

n_{A1} is replication # of A1

n_{B1} is replication # of B1

n is # of experiments.

ANOVA

- The first components contribution will then be

$$\left[\frac{1}{n_0}\right]\sigma_e^2$$

- The second component is a function of the number of times that we repeat the verification experiment, n_r (we called it the replication error). It is given by:

$$\left[\frac{1}{n_r}\right]\sigma_e^2$$

ANOVA

- So our final calculation (for our experiment) is:

$$\sigma^2_{pred} = \left[\frac{1}{n_0}\right]\sigma^2_e + \left[\frac{1}{n_r}\right]\sigma^2_e$$

- You should verify that in our example, this comes our to a value of 80.6(dB)^2.

ANOVA

- So our corresponding 2σ confidence limits are ±17.96 dB. A prediction error outside these limits is a strong indication that the additive model was not appropriate and that there likely exists some factor interactions.

- Final note on prediction error. The prediction error does not only apply to our optimum experiment. It applies to any experiment we wish to try that is not part of our matrix experiment.

Interactions

- Control Factor Interactions.
  - This entire time, we have been operating under the assumption that an additive model will suit us. Thus we used the variables separable approach.
  - So what happens if our ANOVA tells us that the additive model is not a good approximation of our process?
  - Let's start by defining a factor interaction.

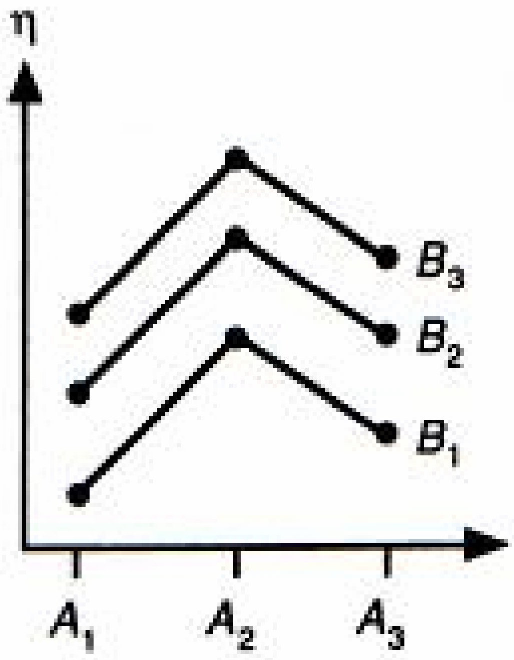
Interactions

Interaction:

When the effect of changing 2 or more factors simultaneously is not the simple sum of the effects of changing them one at a time, there exists an interaction between some or all of these variables.

Consider the following set of figures.

Interactions

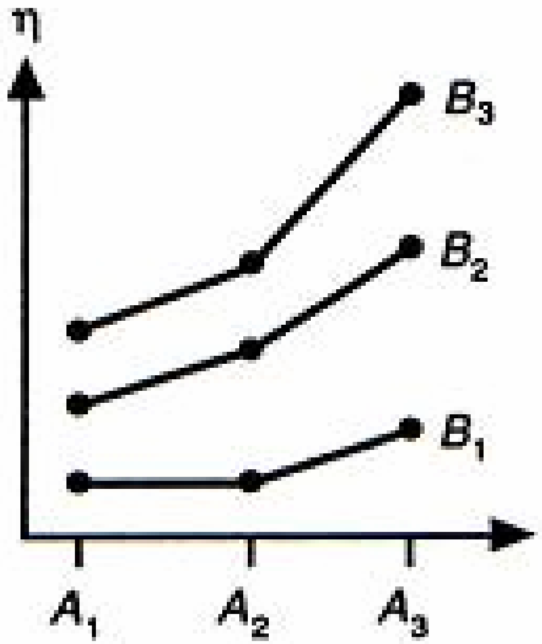


(a) No Interaction

Notice that the change in performance when varying factor A through each of its 3 levels is the same regardless of the setting of factor B.

In this case, there is said to be no interaction between factors A and B.

Interactions

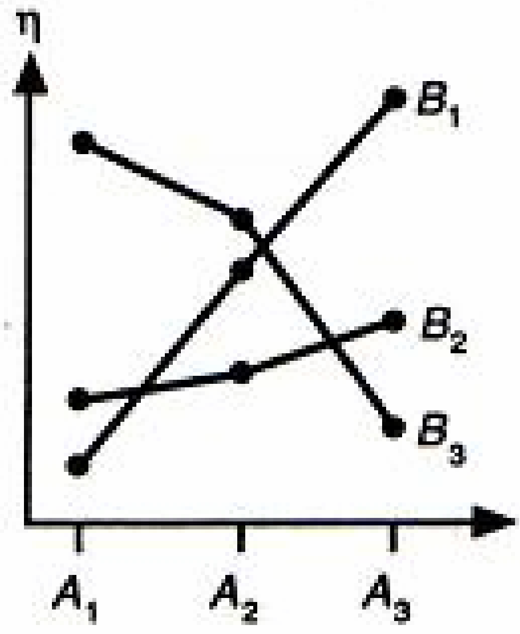


(b) Synergistic Interaction

Here, the lines are not parallel but the direction of improvement is consistent.

In this case, our additive model will give useful results although our predicted outcomes will be inaccurate.

Interactions



(c) Antisynergistic Interaction

Here, the lines are not parallel and the direction of improvement is inconsistent.

In this case, our additive model will give unpredictable results.

This is least desirable.

Interactions

- The presence of an interaction necessitates the use of a cross product term to describe the variation in eta.

- These cross products are treated like factors in a matrix experiment which increases the needed dimensionality and likewise the required number of experiments. Therefore, it is desirable to eliminate interactions whenever possible.

Interactions

•Keep in mind that the mere presence of an interaction does not mean that we must study it. Usually, an engineer is able to judge which factor interactions will be important and which will not based on his/her knowledge of the problem.

•Analysis note, the degrees of freedom for an interaction is equal to the product of the degrees of freedom of the factors involved.

Experiment Planning

The question now is:

"Do we have all the tools we need to properly fit an OA to our design task?"

Let's consider this question with our new knowledge about handling factor interactions.

Experiment Planning

- What information do we need?

  - The number of factors.
  - The number of levels for each factor.
  - The number of interactions to be studied.

Let's consider an example.

Experiment Planning

- We have a problem that consists of the following:
  - A single 2-level factor (A).
  - 5 three level factors (B-F).
  - A two level interaction between A and B.

What matrix should we use?

Experiment Planning

- Let's look at the L_{18} (2^1 x 3^7) array. This experiment is set up to handle 1 factor with 2 levels and 7 factors with 3 levels each.

- Clearly, our experiment will fit in here but what do we do with the extra column? Can we ignore it and still maintain orthogonality?

- Is it wise to ignore it? Could it be put to better use?

Experiment Planning

- Let’s consider another example such as the following:

  - 3 factors with 3 levels each (A-C)
  - 1 factor with 2 levels (D)
  - 0 interactions.

Experiment Planning

- This is very similar to our CVD example right? So could we use the L_9 array?

- If we do, what do we do about the fact that factor D has only 2 levels?

- The answer is that we can still use the array and there are techniques that will allow us to fill in the missing levels sensibly.

Experiment Planning

- **Dummy Level Technique**
  - User can fit a factor which has m levels into a column that accommodates n levels (where n>m).

  For example:

  2-level factor A assigned to a 3-level column.

  Set $A_3 = A_1$ or A_2

Deciding which is up to the designer. The effect of doing this is that you unbalance your experiment. You will in fact estimate the effect of A_3 to a greater precision.