

Radiation

$$q = - A F_{12} (T_2^4 - T_1^4)$$

= emissivity, blackbody – 1., earth $\approx .9$, metals $\approx .4$

F_{12} = view factor

– Steffan Boltzman Constant

$$= .1713 \times 10^{-8} \frac{\text{BTU}}{\text{ft}^2 \text{ hr } {}^\circ\text{R}^4}$$

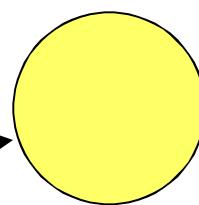
$$= 5.66964 \times 10^{-8} \frac{\text{W}}{\text{m}^2 {}^\circ\text{K}^4}$$

EARTH
Radius 3960 miles



15°C

93 million miles

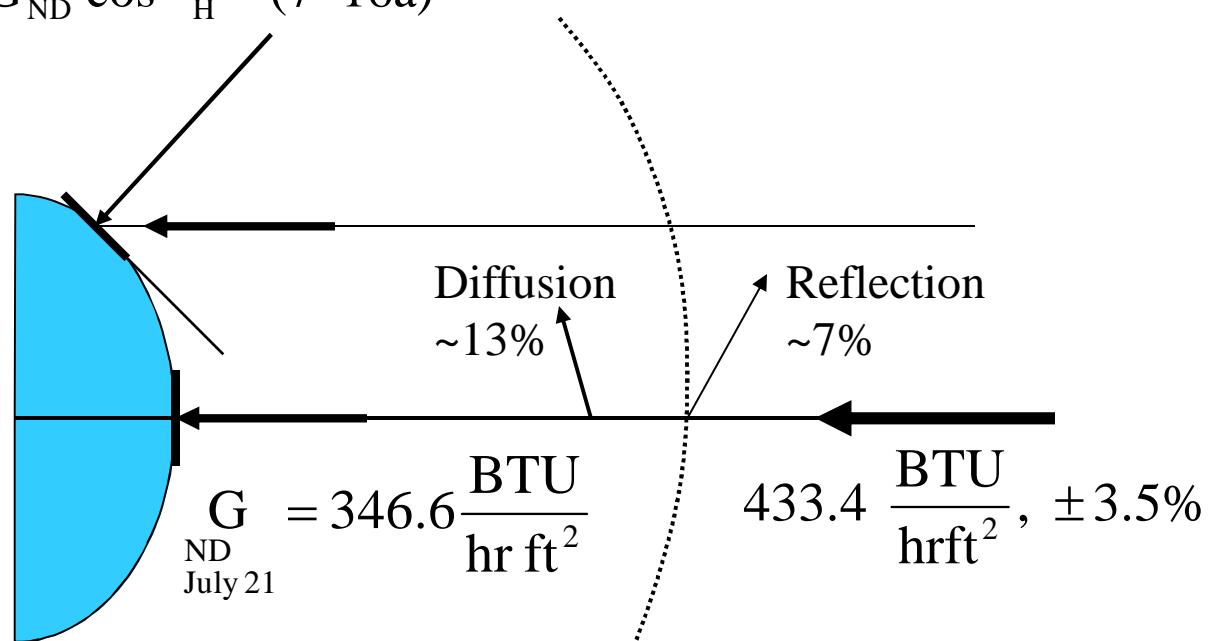


SUN

$10,800^\circ\text{C}$

SOLAR RADIATION

$$G_D = \frac{G}{\text{Direct}} = G_{ND} \cos H \quad (7-16a)$$



$$\frac{G_{NORMAL}}{DIRECT} = CN \times \frac{A}{\exp\left(\frac{B}{\sin}\right)} \quad (7-15), \text{ Table 7-2}$$

Table 7-2 Solar Data for Twenty-First Day of Each Month^a

	Equation of Time, min	Declination, degrees	A, Btu hr-ft ²	A, W m ²	B, Dimensionless	C, Dimensionless
Jan	-11.2	-20.2	381.0	1202	0.141	0.103
Feb	-13.9	-10.8	376.2	1187	0.142	0.104
Mar	-7.5	0.0	368.9	1164	0.149	0.109
Apr	1.1	11.6	358.2	1130	0.164	0.120
May	3.3	20.0	350.6	1106	0.177	0.130
June	-1.4	23.45	346.1	1092	0.185	0.137
July	-6.2	20.6	346.4	1093	0.186	0.138
Aug	-2.4	12.3	350.9	1107	0.182	0.134
Sep	7.5	0.0	360.1	1136	0.165	0.121
Oct	15.4	-10.5	369.6	1166	0.152	0.111
Nov	13.8	-19.8	377.2	1190	0.142	0.106
Dec	1.6	-23.45	381.6	1204	0.141	0.103

^aA, B, C, coefficients are based on research by Machler and Iqbal (6).

Source: Reprinted by permission from ASHRAE *Cooling and Heating Load Calculation Manual*, 2nd ed., 1992.

LOCAL CIVIL TIME

$$LCT = \text{Standard Time} + \frac{30 \text{ min}}{7.5^\circ} (\text{Time Zone Longitude} - \text{Local Longitude})$$

for Central Standard Time Zone

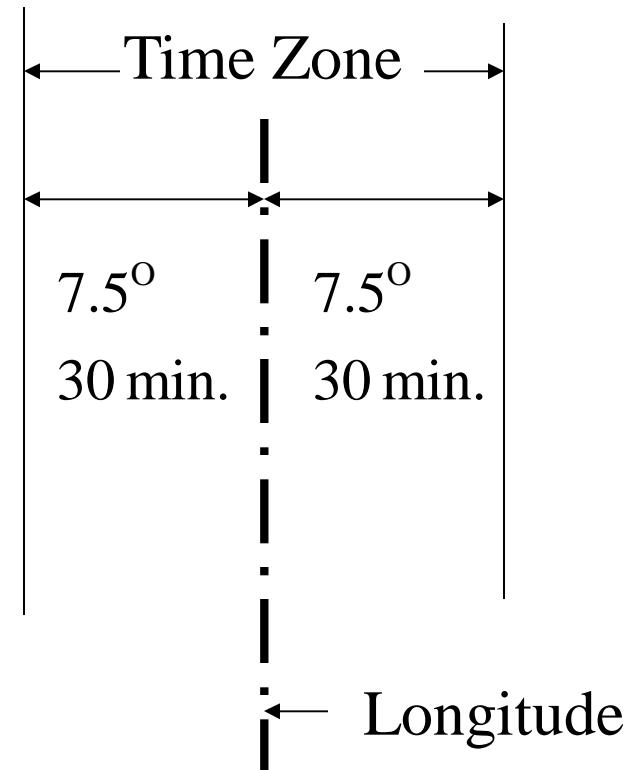
$$LCT = CST + \frac{30 \text{ min}}{7.5^\circ} (90^\circ - \text{Longitude})$$

for Chicago

$$LCT = CST + \frac{30 \text{ min}}{7.5^\circ} (90^\circ - 87.91^\circ)$$

$$LCT = CST + \frac{30 \text{ min}}{7.5^\circ} (2.09^\circ)$$

$$LCT = CST + 8.36 \text{ min}$$



75° Eastern Standard Time

90° Central Standard Time

105° Mountain Standard Time

120° Pacific Standard Time

LOCAL SOLAR TIME

The earth doesn't travel the same orbital distance each day of the year. The Equation of Time corrects for this from +13.9 to -13.9 degrees over the year.

$$\text{Local Solar Time} = \text{Local Civil Time} + \text{Equation of Time}$$
$$\text{LST} = \text{LCT} + \text{EOT(N)}$$

$$\text{EOT(N)} = \text{Equation of Time} \quad (7-4) \text{ page 186}$$
$$N = \frac{(n-1)}{365 \text{ days}} \times 360 \text{ degrees}$$

Table 7-2 page 187

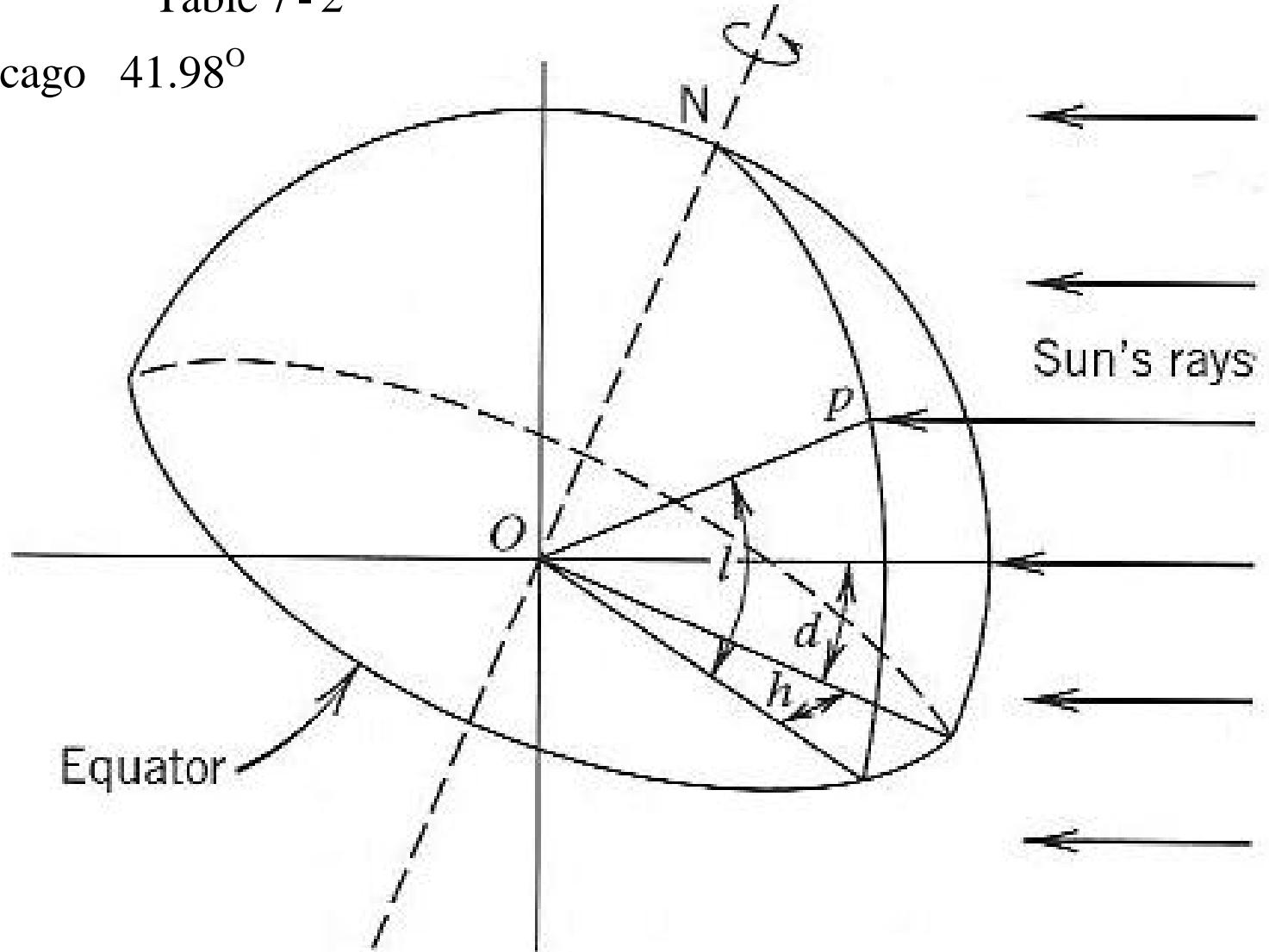
$$n = \text{day of the year}$$

h – hour angle, 15° /hour from noon LST - AM, + PM

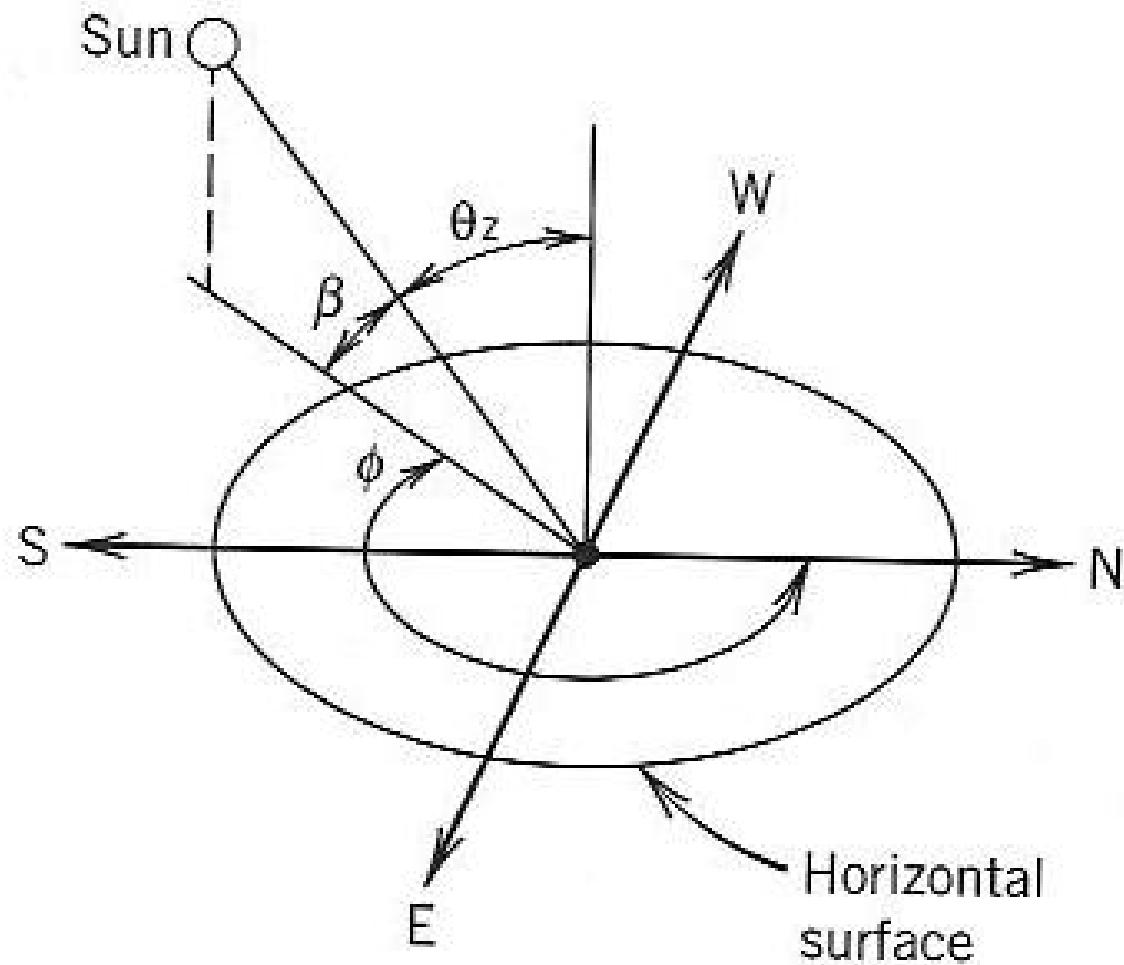
d – declination $\pm 23.45^\circ$ over year

Table 7 - 2

1 – latitude, Chicago 41.98°



ϕ – clockwise angle between projection of solar ray and north
– angle between solar ray and horizontal surface



SOLAR ANGLES

l – latitude of location, degrees

δ – declination, angle between sun ray and equitorial plane Figure 7 – 2

h – angle hour, $\pm 15^\circ$ /hour from South, noon, - AM, + PM

ϕ - angle between the projection of solar ray on a horizontal surface and north, solar azimuth, 180° at noon, less in AM and PM
- angle between solar ray and horizontal surface, solar altitude

$$\sin \text{Altitude} = \cos(l) \times \cos(h) \times \cos(d) + \sin(l) \times \sin(d) \quad (7-8)$$

$$\cos \phi = \frac{\sin \text{Altitude} - \cos(l) \times \sin(l) \times \cos(h)}{\cos(l)} \quad (7-11)$$

γ - angle between normal to surface and horizontal projection of sun ray.

- facing angle, clockwise from north to surface normal.

$$= |(-\phi)| \quad (7-12) \text{ require inverse } \phi \text{ angle in PM}$$

$$= ABS(180 \times (1 + (h/ABS(h)))) - (h/ABS(h))\psi - \phi, h \text{ negative AM, positive PM}$$

$$\text{the above gives } {}_{AM} = |(-\phi)|, \quad {}_{PM} = |(360 - -\phi)|$$

θ - angle between a normal to the surface and sun ray, angle of incidence

θ is the primary angle in radiation calculations

α - angle between normal to surface and normal to horizontal

Arbitrary surface $\cos \theta = \cos \beta \times \cos \gamma \times \sin \alpha + \sin \beta \times \cos \alpha \quad (7-13a)$

Vertical Surface $\alpha = 90, \sin \alpha = 1, \cos \alpha = 0$

$$\cos \theta = \cos \beta \times \cos \gamma \quad (7-13b)$$

Horizontal Surface $\alpha = 0, \sin \alpha = 0, \cos \alpha = 1$

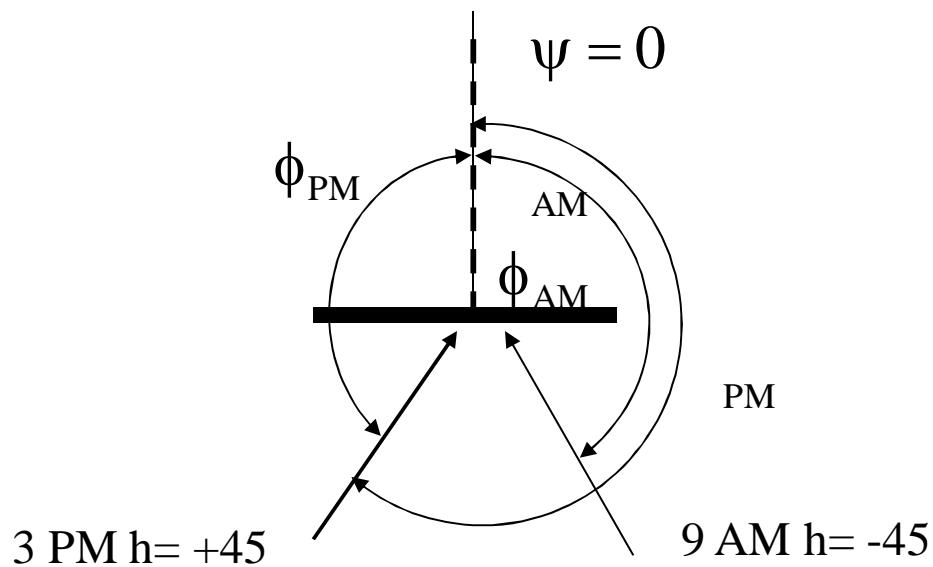
$$\cos \theta = \sin \beta$$

ϕ – angle between projection of solar ray and South

γ – angle between projection of solar ray and surface normal

$$= \text{ABS}(180 \times (1 + (h/\text{ABS}(h))) - (h/\text{ABS}(h))\psi - \phi), h \text{ negative AM, positive PM}$$

NORTH FACING

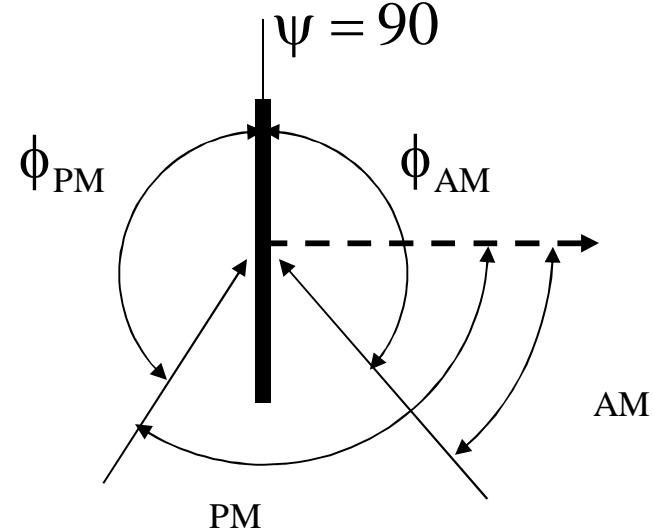


for $\phi = 135$

$$\gamma_{AM} = |0 - 135 - 45 \times (1 - 1)| = 135$$

$$\gamma_{PM} = |0 - 135 - 45 \times (1 + 1)| = 225$$

EAST FACING



for $\phi = 135$

$$\gamma_{AM} = |90 - 135 - 45 \times (1 - 1)| = 45$$

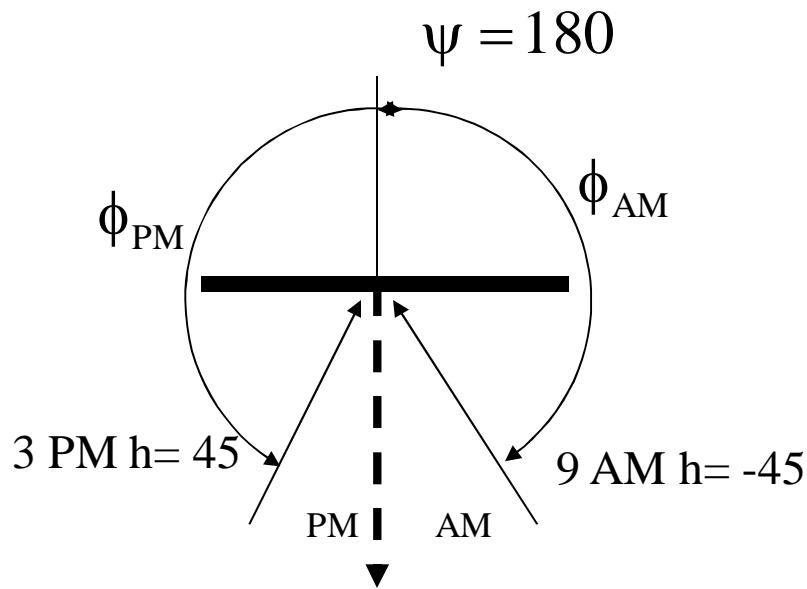
$$\gamma_{PM} = |90 - 135 - 45 \times (1 + 1)| = 135$$

ϕ – angle between projection of solar ray and South

γ – angle between projection of solar ray and surface normal

$$= ABS(180 \times (1 + (h/ABS(h))) - (h/ABS(h))\psi - \phi), h \text{ negative AM, positive PM}$$

SOUTH FACING

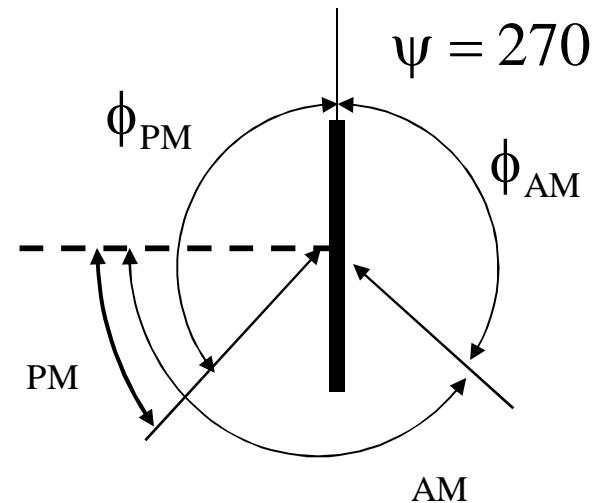


for $\phi = 135$

$$_{AM} = |180 - 135 - 45 \times (1 - 1)| = 45$$

$$_{PM} = |180 - 135 - 45 \times (1 + 1)| = 45$$

WEST FACING



for $\phi = 135$

$$\gamma_{AM} = |270 - 135 - 45 \times (1 - 1)| = 135$$

$$\gamma_{PM} = |180 - 135 - 45 \times (1 + 1)| = 45$$

$$AM \ h < 0, \quad _{AM} = |- \phi| = |270 - 135| = 135$$

$$PM \ h > 0, \quad _{PM} = |360 - - \phi| = |360 - 180 - 135| = 45$$

Wall facing 12° west of south
longitude = 90°
latitude, $l = 40^{\circ}$
October 21, 3:30 CDT

$$CST = CDT - 1 = 3:30 - 1:00 = 2:30$$

$$LCST = 2:30 + \frac{30 \text{ min}}{7.5^{\circ}} (90^{\circ} - 90^{\circ}) = 2:30$$

$$LST = 2:30 + EOT \quad \text{Table 7.2 at October 21}$$

$$LST = 2:30 + 15.4 \text{ min} = 2.757 \text{ hr}$$

$$h = 2.757 \text{ hr} \times 15^{\circ} / \text{hr} = 41.35^{\circ}$$

$$d = -10.5 \quad \text{Table 6.1 at October 21}$$

$$l = 40^{\circ} \text{ latitude}$$

$$\sin h = \cos l \times \cos d \times \cos \phi + \sin l \times \sin d \quad (7-8)$$

$$\sin h = .766 \times .7513 \times .6428 + .6428 \times (-.18223)$$

$$\sin h = .4487, \quad h = 26.66^{\circ}$$

$$\cos \phi = \frac{\sin l \times \cos d - \cos l \times \sin d \times \cos h}{\cos l}$$

$$\cos \phi = \frac{-.18224 \times .76604 - .98325 \times .64279 \times .75069}{.89368}$$

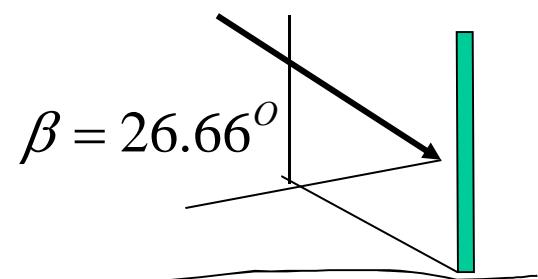
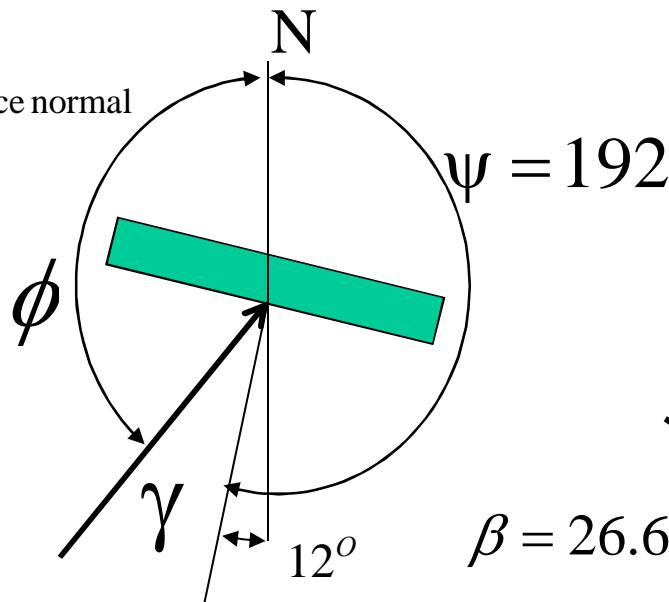
$$\cos \phi = -.6871$$

$$\phi = 133.4$$

ERROR

$$= \text{abs}(\phi - 133.4) = \text{abs}(192 - 133.4) = 58.6 \quad (7-12)$$

- ray to horizontal
- ϕ - ray projection to north
- ray projection to surface normal
- ray to surface normal



$$\begin{aligned} \text{If } h \geq 0 \quad & \gamma = 306 + \psi + \beta = 360 - 192 - 133.4 = 34.6 \\ & = ABS(180 \times (1 + (h/ABS(h))) - (h/ABS(h))\psi - \phi), \quad h \text{ positive PM} \\ & = |180(1 + (41.35/41.35))| - 192(41.35/41.35) - 133.4 = 34.6 \end{aligned}$$

for a vertical surface,

$$\cos v = \cos h \times \cos \phi = \cos 26.66 \times \cos 34.6 \quad (7-13b)$$

$$\cos v = .7356, \quad v = 42.63$$

the angle between the wall normal and the sunray

$$\cos H = \sin h = .4487, \quad H = 63.34$$

longitude							
I	40	Latitude					
d	-10.5	Declination					
PSI	192	Surface orientation CW from North, SW					
Local Solar Time, LST	hour angle h	BETA ray to horizontal (EQ 7-8)	PHI ray projection to North (EQ7-11)	GAMA ray projection to wall normal (EQ7-12)	THETA H sun ray to horizontal normal (EQ7-11)	THETA V sun ray to wall normal (EQ7-11)	
0.001	-180	-60.500	0.029951	191.97	150.50	118.80	
1.000	-165	-57.639	28.38842	163.61	147.64	120.90	
2.000	-150	-50.304	50.3282	141.67	140.30	120.07	
3.000	-135	-40.522	66.15455	125.85	130.52	116.43	
4.000	-120	-29.587	78.29552	113.70	119.59	110.46	
4.480	-113	-24.143	83.37246	108.63	114.14	106.95	
5.000	-105	-18.185	88.5537	103.45	108.18	102.76	
6.000	-90	-6.727	98.08073	93.92	96.73	93.89	
7.000	-75	4.463	107.7042	84.30	85.54	84.31	
8.000	-60	15.039	118.1493	73.85	74.96	74.42	
9.000	-45	24.549	130.1501	61.85	65.45	64.59	
10.000	-30	32.355	144.4099	47.59	57.64	55.27	
11.000	-15	37.619	161.2593	30.74	52.38	47.09	
12.001	0.015	39.500	179.9809	11.98	50.50	40.99	
13.000	15	37.619	161.2593	6.74	52.38	38.13	
14.000	30	32.355	144.4099	23.59	57.64	39.27	
14.757	41.36	26.631	133.3844	34.62	63.37	42.64	
15.000	45	24.549	130.1501	37.85	65.45	44.09	
16.000	60	15.039	118.1493	49.85	74.96	51.49	
17.000	75	4.463	107.7042	60.30	85.54	60.39	
18.000	90	-6.727	98.08073	69.92	96.73	70.06	
19.000	105	-18.185	88.5537	79.45	108.18	79.98	
19.520	112.8	-24.143	83.37246	84.63	114.14	85.10	
20.000	120	-29.587	78.29552	89.70	119.59	89.74	
21.000	135	-40.522	66.15455	101.85	130.52	98.98	
22.000	150	-50.304	50.3282	117.67	140.30	107.25	
23.000	165	-57.639	28.38842	139.61	147.64	114.06	

SOLAR ANGLE LIMITS

- these limits can be used to verify that spread sheet equations are correct

- 1) At sunrise, $\phi = 0^\circ, 0 \text{ rad}$, $\theta_H = 90^\circ, .785 \text{ rad}$
- 2) At sunset, $\phi = 0^\circ, 0 \text{ rad}$, $\theta_H = 90^\circ, .785 \text{ rad}$
- 3) At noon, $\phi = 180^\circ, 3.1416 \text{ rad}$
- 4) At $\phi = 0$, $\theta_H =$
sunrise - 6am LST with $\phi = 90^\circ, .785 \text{ rad}$
sunset - 6 pm LST with $\phi = 90^\circ, .785 \text{ rad}$
- 5) On March 21 and September 21
All changes of angle and heat flux with solar time are smooth and continuous.
- 6) $\theta_V > 90^\circ, .785 \text{ rad} \Rightarrow$ shade
diffuse and reflected radiation, G_{dH}, G_{dV}, G_R , reach the surface.
no direct radiation reaches the surface, G_{dH} and G_{dV} edited to 0.
- 7) $\theta_V < 90^\circ, .785 \text{ rad} \Rightarrow$ shade,
diffuse and reflected radiation, G_{dH}, G_{dV}, G_R , reach the surface.
no direct radiation reaches the surface, G_{dH} and G_{dV} edited to 0.
- 8) All heat fluxes must be less than G_{ND}
 $G_{ND} <$ constant A, Table 7-2.
- 9) At noon $\theta_H = 90^\circ$

Direct, Diffuse and Reflected Solar Radiation

$$G_d \quad (7-22)$$

$$G_{dV}$$

Diffuse on Vertical

$$G_{dH} \quad (7-17)$$

Diffuse on Horizontal

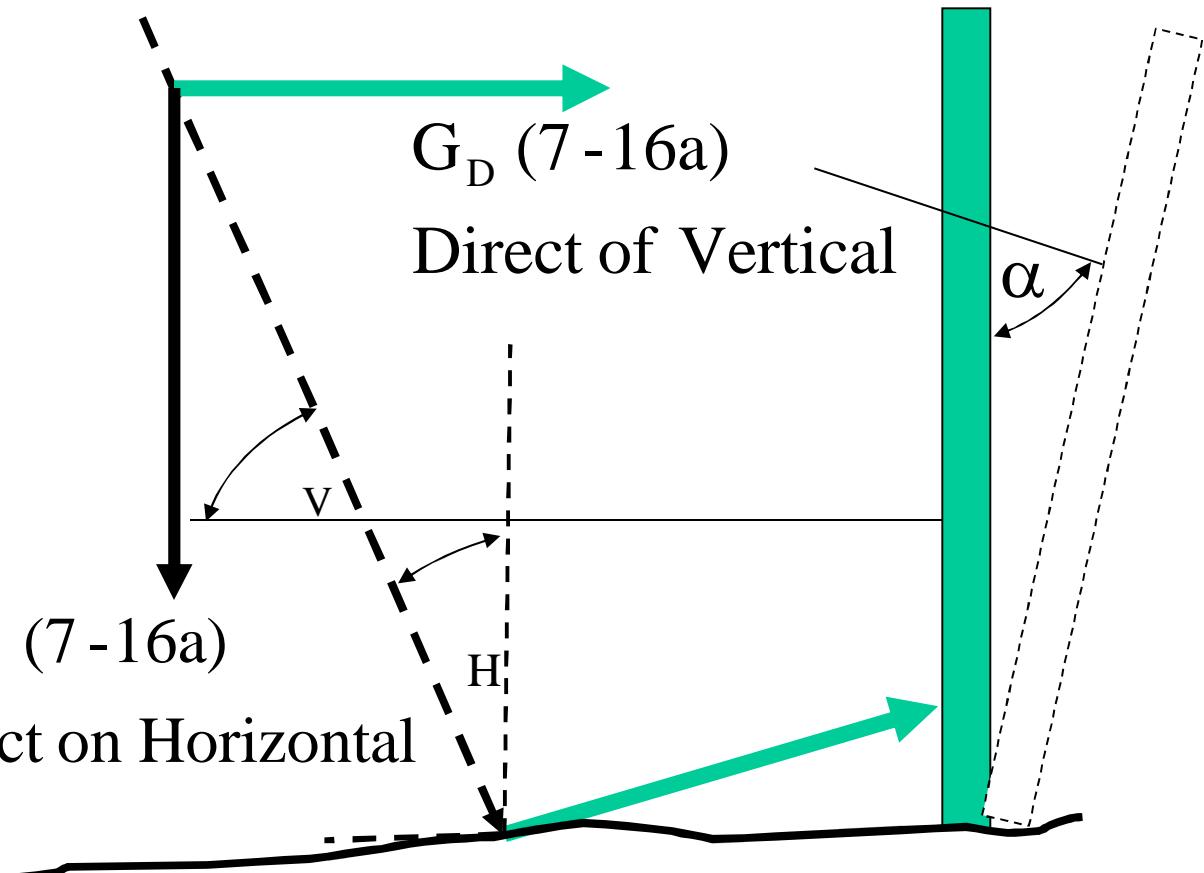
$$G_{DH} \quad (7-16a)$$

Direct on Horizontal

$$G_R \quad (7-23)$$

Reflected on Vertical

$$G_{ND} = C_N \times \frac{A}{\exp\left(\frac{B}{\sin\beta}\right)} \quad (7-15)$$



Direct, Diffuse and Reflected Solar Radiation

$$G_d = C G_{ND} F_{ws} \quad (7-18)$$

diffuse on non-horiz

$$F_{ws} = (1 + \cos \alpha)/2 = .5 \text{ for vertical} \quad (7-20)$$

$$G_{dH} = G_{DH} (.55 + .437 \cos \alpha + .313 \cos^2 \alpha) \quad (7-21)$$

diffuse on vert

$$G_{dH} = C G_{ND} \quad (7-17)$$

diffuse on horiz

$$G_{total \text{ on vert}} = G_D + G_d + G_R$$

$$G_{total \text{ on horiz}} = G_{DH} + G_{dH}$$

$$G_{ND} = C_N \times \frac{A}{\exp\left(\frac{B}{\sin \beta}\right)} \quad (7-15)$$

$$(7-21)$$

$$G_D = G_{ND} \cos \alpha \quad (7-16a)$$

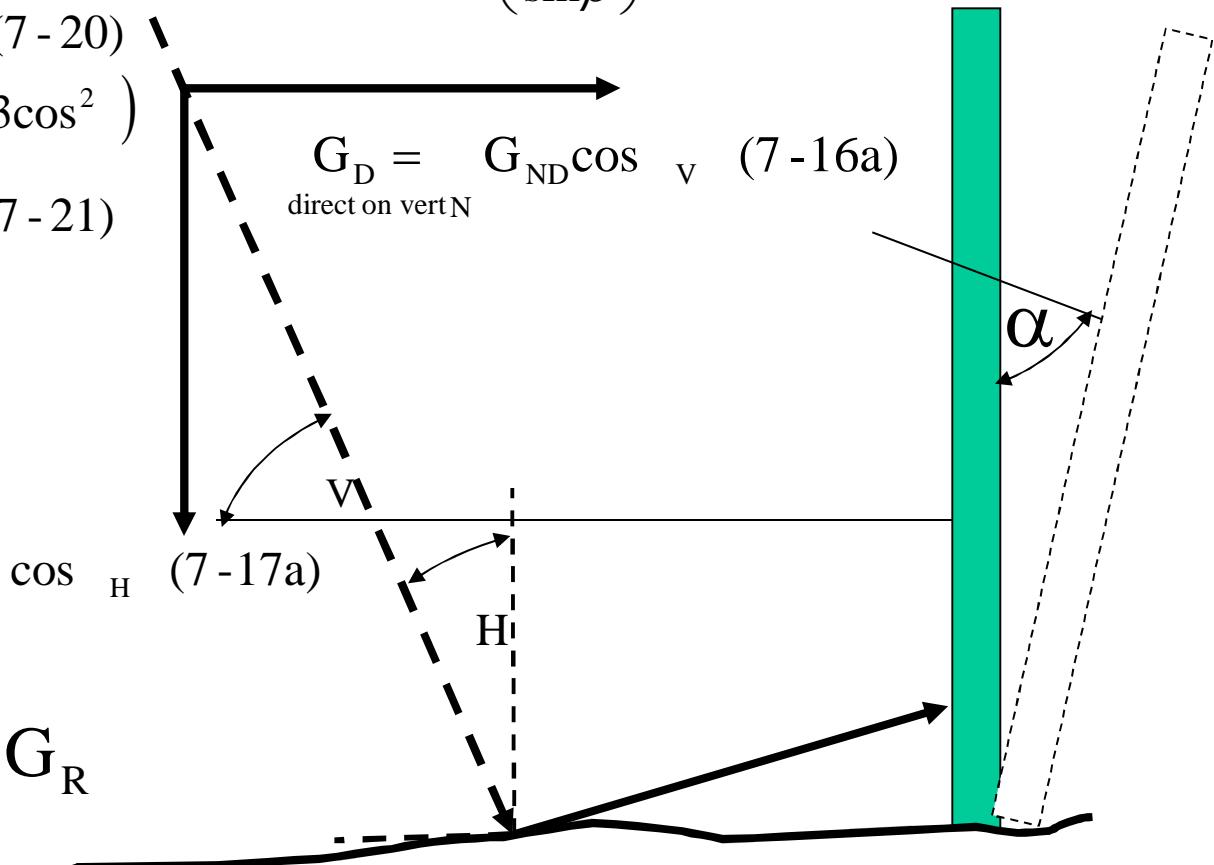
direct on vertN

$$G_{DH} = G_{ND} \cos \alpha \quad (7-17a)$$

direct on horz

$$G_R = (G_{DH} + G_{dH}) F_{wg} \quad (7-23)$$

$$F_{wg} = (1 - \cos \alpha)/2 = .5 \text{ for vertical} \quad (7-24)$$



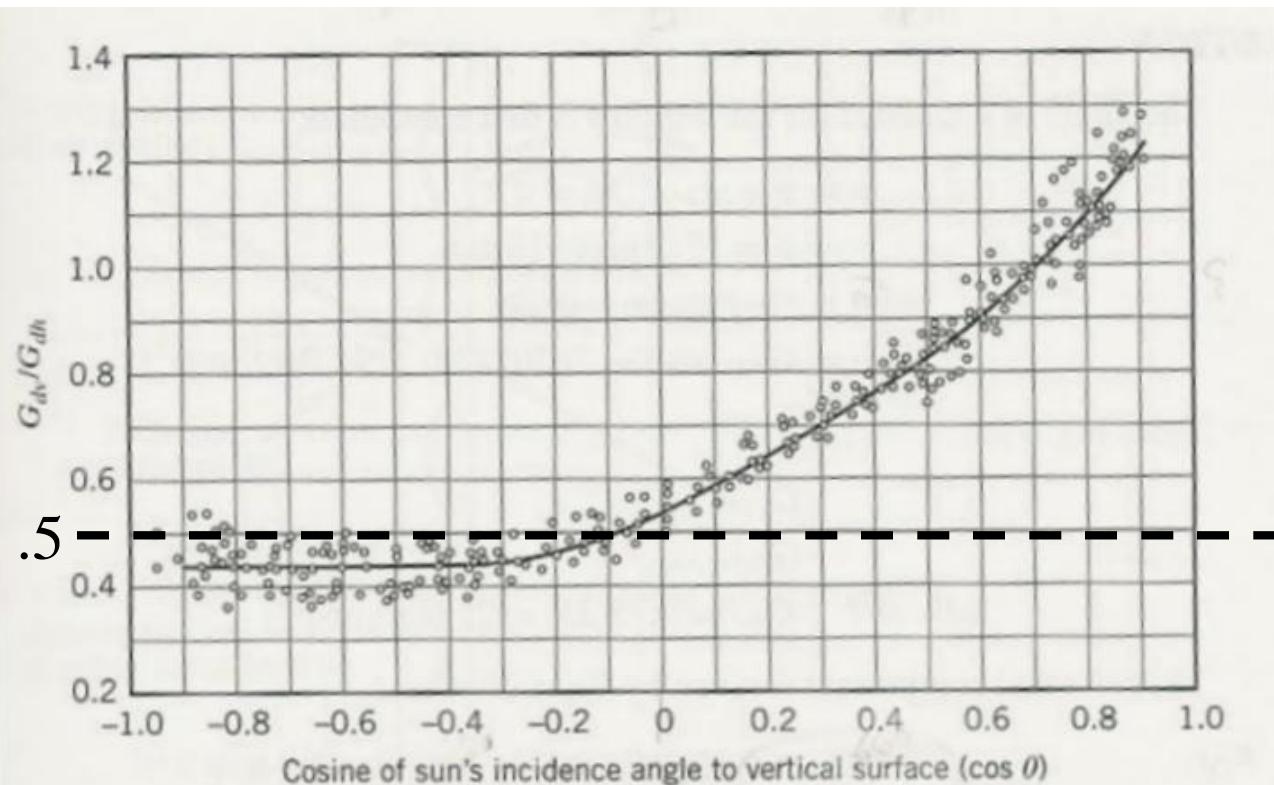


Figure 6-9 Ratio of diffuse sky radiation incident on a vertical surface to that incident on a horizontal surface during clear days. (Reprinted by permission from *ASHRAE Transactions*, Vol. 69, p. 29)

$$\frac{\text{Diffusion on a Vertical Surface}}{\text{Diffusion on a Horizontal Surface}} = \frac{G_{dv}}{G_{dH}} = .55 + .437\cos v + .313\cos^2 v \quad (7-22)$$

$$\text{for } v = 90^\circ, \quad \frac{G_d}{G_{dH}} = \frac{C \times G_{ND} \times F_{ws}}{C \times G_{ND}} = .5 \text{ for a vertical surface}$$

REFLECTION

$$G_R = (G_{DH} + G_{dH}) F_{wg} \quad (7-23)$$

reflection on vert

$$F_{wg} = (1 - \cos \alpha)/2 = .5 \text{ for vertical} \quad (7-24)$$

Ground Reflectance, $R_g = f(\text{incidence})$

new snow .87

dirty snow .5

new concrete .35

old concrete .25

green grass .25

brown grass .2

asphalt .1

bare soil .1

water .06

Chicago, July 21, LST=10 AM, SE facing, $\psi = 135$

$l = 41.98^\circ \text{N}$ latitude

$C_N = .95$ clearness number Fig 7.7
 $= .3$ reflectivity

10 AM Solar Time

$d = f(N) = 20.6$ Table 7-2

$\psi = 135$

$$h = \left(\frac{12:-10:}{24} \right) \times 360 = 30^\circ$$

$$\sin \beta = \cos l \cos h \cos d + \sin l \sin d \quad (7-8)$$

$$\sin \beta = \cos 41.98 \times \cos 30 \times \cos 20.6 + \sin 41.98 \times \sin 20.6$$

$$\sin \beta = .8379, \quad = 56.923^\circ$$

$$\cos \phi = \frac{\sin l \cos h - \sin l \cos d}{\cos \beta} \quad (7-11)$$

$$\cos \phi = -.51424, \quad \phi = 120.94$$

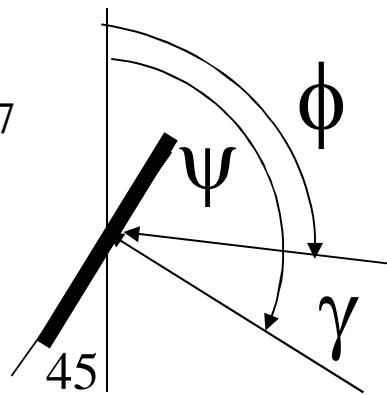
$$\gamma = \text{abs}(\psi - \phi) = \text{abs}(135 - 120.93) = 14.06$$

$$\cos \theta_v = \cos \beta \times \cos \gamma$$

$$\theta_v = 58.03$$

$$\cos \theta_h = \sin \beta$$

$$\theta_h = 33.08$$



$$G_{ND} = C_N \frac{A}{\exp\left(\frac{B}{\sin \beta}\right)} \quad (7-15), \text{ Table 7-2}$$

$$G_{ND} = .95 \times 346.6 / \exp(.186/.8379)$$

$$G_{ND} = 263.72 \text{ BTU/hr ft}^2$$

HORIZONTAL SURFACE

$$G_{dH} = C \times G_{ND} \quad (7-16a)$$

$$G_{dH} = .138 \times 263.72$$

$$G_{dH} = 36.39 \text{ BTU/hr ft}^2$$

$$G_{DH} = G_{ND} \cos \theta_h \quad (7-16a)$$

$$G_{DH} = 263.72 \times \cos(33.08)$$

$$G_{DH} = 220.97 \text{ Btu/hr ft}^2$$

Chicago Office Building, SE wall

VERTICAL SURFACE

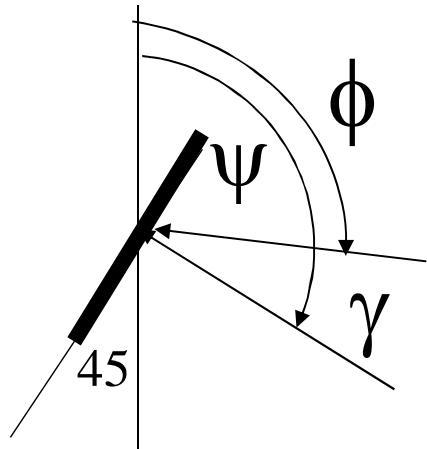
$$G_D = G_{ND} \cos_v \quad \text{direct on vert N} \quad (7-16a)$$

$$G_D = 263.72 \times \cos(58.03) = 139.63 \text{ Btu/hr ft}^2 \quad \text{direct on vert}$$

$$\frac{G_{dV}}{G_{dH}} = .55 + .437 \cos + .313 \cos^2 = .8691 \quad (7-21)$$

$$G_d = \frac{G_{dV}}{G_{dH}} \times G_{dH} \quad \text{diffuse on vert} \quad (7-22)$$

$$G_d = .8691 \times 36.39 = 31.63 \text{ Btu/hr ft}^2 \quad \text{diffuse on vert}$$



$$G_R = (G_{DH} + G_{dH})_g F_{ws} \quad \text{reflection on vert} \quad (7-23)$$

$$G_R = (220.97 + 36.39) \times .3 \times .5 = 38.6 \text{ Btu/hr ft}^2 \quad \text{reflection on vert}$$

$$G_{\text{total on vert}} = G_D + G_d + G_R$$

$$G_{\text{total on vert}} = \text{direct} + \text{diffuse} + \text{reflected}$$

$$G_{\text{total on vert}} = 139.63 + 31.36 + 38.60 = 209.53 \text{ Btu/hr ft}^2$$

Chicago, SE Facing $\psi = 135$															
longitude				A		346.6									
I	41.98	Latitude		B	0.186	C	0.138	CN	0.95	Reflectivity	0.3				
PSI	135	Surface orientation CW from North													
DELTA	45	Tilt angle, between arbitrary surface normal and horizontal normal													
Local Solar Time, LST	h	BETA ray to horizontal	PHI ray projection to North	GAMA ray projection to wall normal	THETA H sun ray to horizontal normal	THETA V sun ray to wall normal	THETA A suns ray to arbitrary surface normal	Normal Direct Sun Ray G ND	Direct on horizontal surface G DH	Diffues on horizontal surface G dH	TOTAL horizontal direct + diffuse	Direct on vertical surface G DV	Diffuse on Vertical Surface GdV	Reflecte d on arbitrary surface GRV	TOTAL direct + diffuse + reflecte d
0.001	-180	-27.420	0.015818	134.98	117.42	128.87	140.29	493.1	-227.1	68.1	-159.0	0.0	27.2	-23.9	3.3
1.000	-165	-25.900	15.62386	119.38	115.90	116.18	128.38	504.1	-220.2	69.6	-150.6	0.0	29.1	-22.6	6.5
2.000	-150	-21.548	30.21186	104.79	111.55	103.73	115.31	546.4	-200.7	75.4	-125.3	0.0	35.0	-18.8	16.2
3.000	-135	-14.874	43.22322	91.78	104.87	91.72	101.70	679.6	-174.4	93.8	-80.7	0.0	50.4	-12.1	38.3
4.000	-120	-6.464	54.67035	80.33	96.46	80.39	87.80	1718.0	-193.4	237.1	43.7	286.8	149.8	6.5	443.1
4.480	-113	-1.966	59.7036	75.30	91.97	75.31	81.08	74407.5	-2553.3	10268.2	7715.0	18874.9	6992.6	1157.2	27024.8
5.070	-104	3.875	65.57922	69.42	86.12	69.47	72.80	21.0	1.4	2.9	4.3	7.4	2.2	0.6	10.2
6.000	-90	13.611	74.3887	60.61	76.39	61.51	59.76	149.4	35.2	20.6	55.8	71.2	17.1	8.4	96.7
7.000	-75	24.547	83.71982	51.28	65.45	55.32	45.89	210.4	87.4	29.0	116.5	119.7	26.1	17.5	163.3
8.000	-60	35.680	93.63508	41.36	54.32	52.44	32.49	239.4	139.6	33.0	172.6	145.9	30.8	25.9	202.6
9.000	-45	46.667	105.3078	29.69	43.33	53.41	20.63	255.0	185.5	35.2	220.6	152.0	32.4	33.1	217.5
10.000	-30	56.925	120.9504	14.05	33.07	58.03	14.79	263.7	221.0	36.4	257.4	139.6	31.6	38.6	209.9
11.000	-15	65.158	144.7826	9.78	24.84	65.54	20.86	268.2	243.4	37.0	280.4	111.1	29.0	42.1	182.2
12.001	0.015	68.620	179.9615	45.04	21.38	75.07	32.80	269.7	251.1	37.2	288.3	69.5	25.4	43.2	138.1
13.000	15	65.158	144.7826	80.22	24.84	85.91	46.20	268.2	243.4	37.0	280.4	19.1	21.6	42.1	82.8
14.000	30	56.925	120.9504	104.05	33.07	97.61	60.08	263.7	221.0	36.4	257.4	0.0	18.1	38.6	56.7
15.000	45	46.667	105.3078	119.69	43.33	109.87	74.10	255.0	185.5	35.2	220.6	0.0	15.4	33.1	48.5
16.000	60	35.680	93.63508	131.36	54.32	122.47	88.12	239.4	139.6	33.0	172.6	0.0	13.4	25.9	39.3
17.000	75	24.547	83.71982	141.28	65.45	135.21	102.01	210.4	87.4	29.0	116.5	0.0	11.5	17.5	29.0
18.000	90	13.611	74.3887	150.61	76.39	147.87	115.62	149.4	35.2	20.6	55.8	0.0	8.3	8.4	16.7
18.800	102	5.202	66.83717	158.16	84.80	157.58	126.13	42.3	3.8	5.8	9.7	0.0	2.4	1.5	3.9
20.000	120	-6.464	54.67035	170.33	96.46	168.39	140.56	1718.0	-193.4	237.1	43.7	0.0	100.1	6.5	106.7
21.000	135	-14.874	43.22322	181.78	104.87	165.02	149.84	679.6	-174.4	93.8	-80.7	0.0	39.4	-12.1	27.3
22.000	150	-21.548	30.21186	194.79	111.55	154.07	153.59	546.4	-200.7	75.4	-125.3	0.0	30.9	-18.8	12.1
23.000	165	-25.900	15.62386	209.38	115.90	141.62	149.67	504.1	-220.2	69.6	-150.6	0.0	27.8	-22.6	5.2

VERTICAL SURFACE

DIRECT + DIFFUSE + REFLECTED

$$G_D = G_{ND} \cos_v$$

direct on vert

$$G_{dV} = G_{dH} (.55 + .437 \cos_v + .313 \cos^2_v)$$

diffuse on vert

$$G_R = (G_{DH} + G_{dH}) F_{ws}$$

reflection on vert

$$\cos_v = \cos \cos$$

HORIZONTAL SURFACE

DIRECT + DIFFUSE

$$G_{DH} = G_{ND} \cos$$

direct on horz N

$$G_{dH} = C G_{ND}$$

diffuse on horiz

$$\cos_h = \sin$$

CALCULATION

CDT—CivilDaylightSavingsTime

CST—CivilStandardTime

LCST—LocalCivilStandardTime

LST—LocalSolarTime

h—hour angle

d—declination

l—latitude

$$\downarrow$$

$$\cos \phi$$

$$\cos$$

surface orientation

$$\downarrow$$

$$\theta_H, \theta_V, \theta_{inclined}$$

SOLAR RADIATION EQUATION SUMMARY

$$G_{ND} = C_N \times A / \exp(B / \sin)$$

HORIZONTAL

$$\cos \gamma_H = \sin$$

$$G_{DH} = G_{ND} \cos \gamma_H$$

direct on horzN

$$G_{dH} = C G_{ND}$$

diffuse on horiz

NON-HORIZONTAL

VERTICAL, $\gamma = 90^\circ$

$$\cos \gamma_V = \cos \gamma \cos$$

$$\frac{G_{dv}}{G_{dh}} = (0.55 + 0.437 \cos \gamma_V + 0.313 \cos^2 \gamma_V)$$

$$G_{dV} = G_{dH} \left(\frac{G_{dv}}{G_{dh}} \right)$$

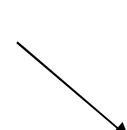
diffuse on vert

ANGLED, $\gamma = 0$ to 90°

$$\cos \gamma = \sin \gamma \times \cos \gamma \times \sin \gamma + \sin \gamma \times \cos \alpha$$

$$F_{wall\ sky} = (1 + \cos \gamma) / 2$$

$$G_d = C G_{ND} F_{ws}$$



$$G_{DV} = G_{ND} \cos \gamma_V$$

direct on vert

$$F_{wall\ ground} = (1 - \cos \gamma) / 2$$



$$G_R = (G_{DH} + G_{dH}) F_{wg}$$

reflection on vert

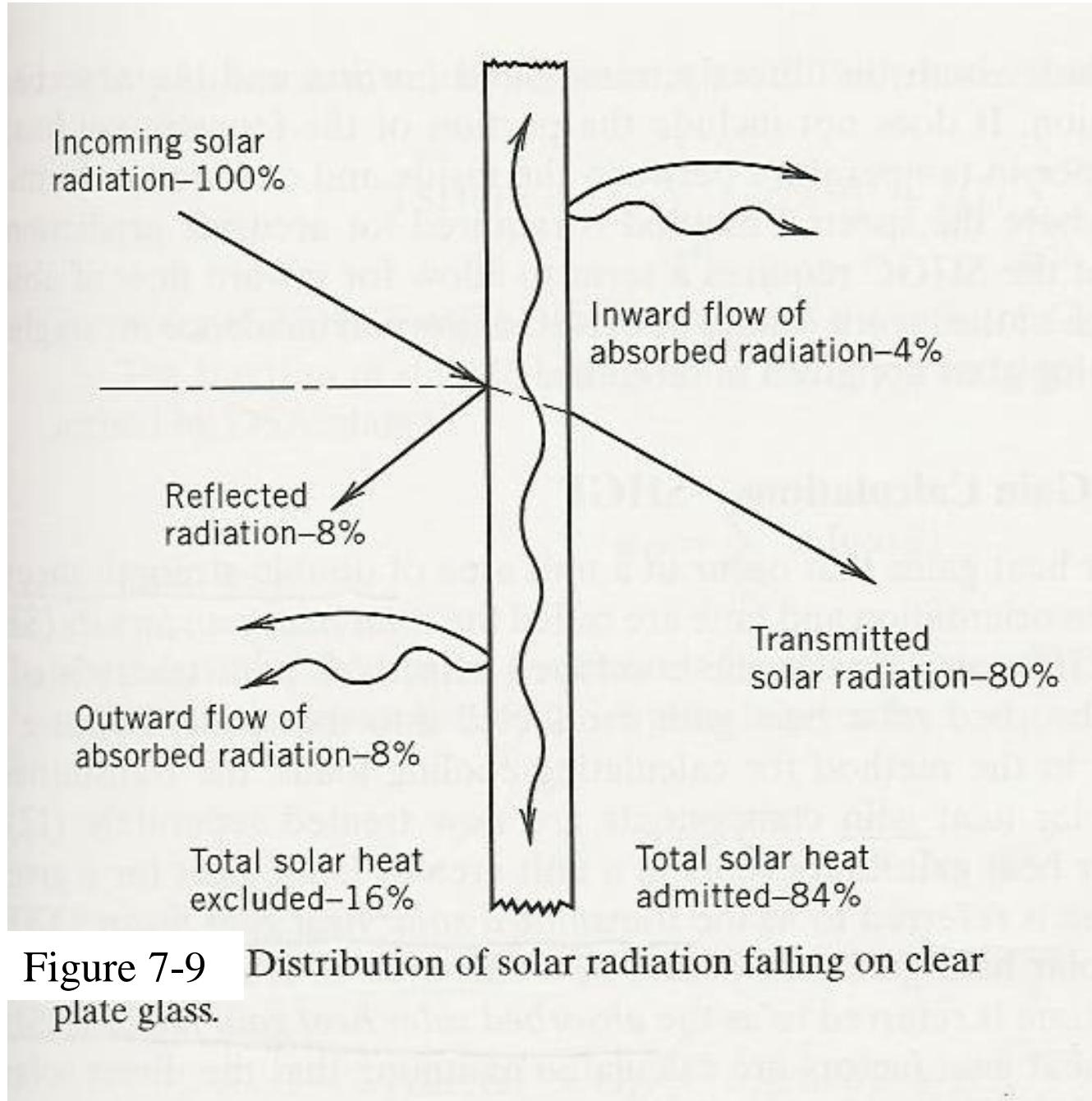
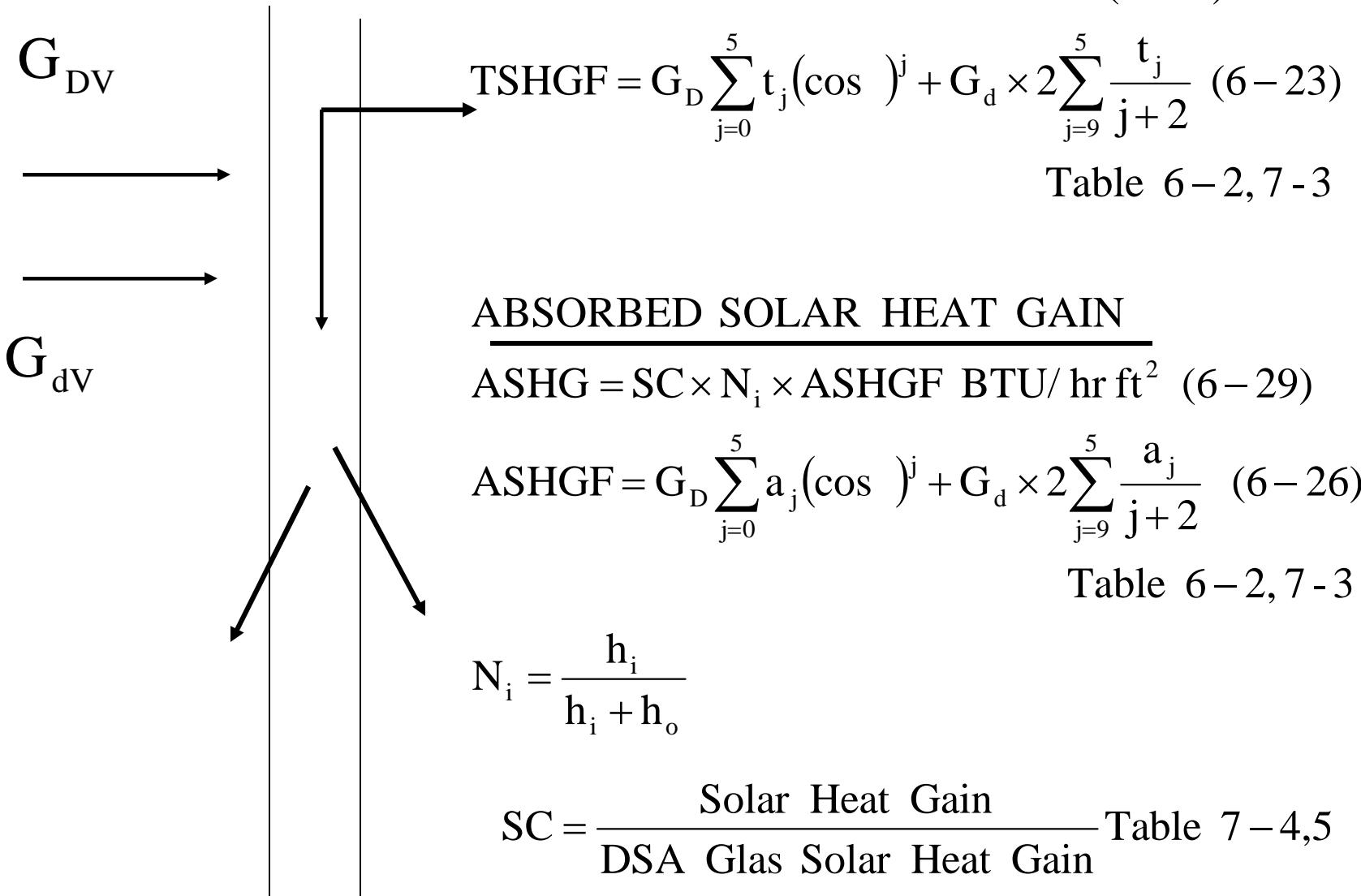


Figure 7-9 Distribution of solar radiation falling on clear plate glass.

TRANSMITTED SOLAR HEAT GAIN

$$TSHG = SC \times TSHGF \text{ BTU/hr ft}^2 \quad (6-28)$$



Simplified Solar Heat Gain Calculations—SHGF

The hourly solar heat gains that occur in a unit area of double-strength sheet glass (DSA) for a given orientation and time are called the *solar heat gain factors* (SHGF). The term takes into account for the combined effects of both transmitted solar heat gain and absorbed solar heat gain conducted into the space. Because of refinements made in the method for calculating cooling loads, the transmitted and the absorbed solar heat gain components are now treated separately (12). The transmitted solar heat gain that occurs in a unit area of DSA glass for a given orientation and time is referred to as the *transmitted solar heat gain factor* (TSHGF). The absorbed solar heat gain that occurs in a unit area of DSA glass for a given orientation and time is referred to as the *absorbed solar heat gain factor* (ASHGF).

Both solar heat gain factors are calculated assuming that the direct solar irradiation G_D and the diffuse solar irradiation G_d have already been determined. The procedures for doing this will now be given.

The transmittance τ_D of DSA glass to direct (beam) radiation incident at an angle θ is

$$\tau_D = \sum_{j=0}^5 t_j [\cos \theta]^j \quad (6-22a)$$

where t_j is the transmission coefficients for glass (Table 6-2) (5). The transmittance τ_d of DSA glass to diffuse radiation is given by

$$\tau_d = 2 \sum_{j=0}^5 \frac{t_j}{j+2} \quad (6-22b)$$

Note that both calculations use the transmission coefficients for glass found in Table 6-2. These coefficients give a normal transmittance for DSA glass of 0.88, which is

Table 6-2 Coefficients for DSA
Glass for Calculation of
Transmittance and Absorptance

j	a_j	t_j
0	0.01154	-0.00885
1	0.77674	2.71235
2	-3.94657	-0.62062
3	8.57811	-7.07329
4	-8.38135	9.75995
5	3.01188	-3.89922

Source: Reprinted by permission
from *ASHRAE Handbook, Funda-
mentals Volume*, 1989.

slightly higher than values sometimes used. The transmitted solar heat gain factor is

$$\text{TSHGF} = G_D \sum_{j=0}^5 t_j [\cos \theta]^j - 2G_d \sum_{j=0}^5 \frac{t_j}{j+2} \quad (6-23)$$

The units of TSHGF will be consistent with the units of G_D and G_d .

The fraction of direct (beam) solar radiation incident at an angle θ that is absorbed by DSA glass is

$$\alpha_D = \sum_{j=0}^5 a_j [\cos \theta]^j \quad (6-24)$$

where a_j is the absorption coefficients for glass (Table 6-2) (5). The fraction of diffuse solar radiation absorbed by DSA glass is given by

$$\alpha_d = 2 \sum_{j=0}^5 \frac{a_j}{j+2} \quad (6-25)$$

The absorbed solar heat gain factor is then given by

$$\text{ASHGF} = G_D \sum_{j=0}^5 a_j [\cos \theta]^j + 2G_d \sum_{j=0}^5 \frac{a_j}{j+2} \quad (6-26)$$

For $\theta = 0$, a ray perpendicular to DSA glass, $\cos \theta = 1$

$$TSHGF = G_D \sum_{j=0}^5 t_j (\cos \theta)^j + G_d \times 2 \sum_{j=9}^5 \frac{t_j}{j+2} \quad (6-23) \quad \text{Table } 6-2$$

$$\text{Transmisivity } T_D = \sum_{j=0}^5 t_j (1)^j = .87 \quad T_d = 2 \sum_{j=9}^5 \frac{t_j}{j+2} = .799 \text{ (independent of } \theta\text{)}$$

$$ASHGF = G_D \sum_{j=0}^5 a_j (\cos \theta)^j + G_d \times 2 \sum_{j=9}^5 \frac{a_j}{j+2} \quad (6-26) \quad \text{Table } 6-2$$

$$\text{Absorbitvty } A_D = \sum_{j=0}^5 a_j (1)^j = .05 \quad A_d = 2 \sum_{j=9}^5 \frac{a_j}{j+2} = .054 \text{ (independent of } \theta\text{)}$$

Table 7-3 Solar Heat Gain Coefficient (SHGC), Solar Transmittance (T), Front Reflectance (R_f), Back Reflectance (R_b), and Layer Absorptances (a_{hi}) for Glazing Window Systems

Glazing Systems				Center-of-Glazing Properties								Total Window SHGC at Normal Incidence			
				Incidence Angles								Aluminum		Other Frames	
ID	Glass Thick., in. (mm)			Normal	0.0	40.0	50.0	60.0	70.0	80.0	Diffuse	Operable	Fixed	Operable	Fixed
1a	1/8 (3.2)	<i>Uncoated Single Glazing, CLR</i>	SHGC	0.86	0.84	0.82	0.78	0.67	0.42	0.78	0.75	0.78	0.64	0.75	
			T	0.83	0.82	0.80	0.75	0.64	0.39	0.75					
			R_f	0.08	0.08	0.10	0.14	0.25	0.51	0.14					
			R_b	0.08	0.08	0.10	0.14	0.25	0.51	0.14					
			a_{hi}	0.09	0.10	0.10	0.11	0.11	0.11	0.10					
5a	1/8 (3.2)	<i>Uncoated Double Glazing, CLR CLR</i>	SHGC	0.56	0.7										
			T	0.70	0.6										
			R_f	0.13	0.1										
			R_b	0.13	0.1										

Solar Heat Gain Coefficient
Solar Transmittance
Front Refelctance
Rear Reflectance
Single Pane Absorptance

SIMPLIFIED METHOD

$$q = G_{\text{incident}} \times \text{SHGC}$$

$$q = (G_{\text{DV}} + G_{\text{dV}} + G_R) \times \text{SHGC}$$

Louisville, KY, July 21																			
longitude				A	346.4	SHGC single clear		0.78											
I	42.8	Latitude		B	0.186	SHGC case		0.58											
d	20.6	Declination		C	0.138														
				CN	0.95														
				Reflectivity	0.25														
PSI	270	Surface orientation CW from North																	
DELTA	45	Tilt angle, between arbitrary surface normal and horizontal normal				HORIZONTAL SURFACE			VERTICAL SURFACE			ARBITRARY SURFACE							
Local Solar Time, LST	h	BETA ray to horizontal	PHI ray projection to North	GAMA ray projection to wall normal	THETA H sun ray to horizontal normal	THETA V sun ray to wall normal	THETA A suns ray to arbitrary surface normal	Normal Direct Sun Ray G ND	Direct on horizontal surface G DH	Diffuse on horizontal surface G dH	TOTAL horizontal direct + diffuse	Direct on vertical surface G DV	Diffuse on Vertical Surface GdV	Reflecte d on arbitrary surface GRV	TOTAL direct + diffuse + reflected	Direct on arbitrary surface G DA	Diffuse on arbitrary Surface GdA	Reflected on arbitrary surface GRA	TOTAL direct + diffuse + reflected
0.001	-180	-26.630	0.015707	269.98	116.63	90.01	108.49	498.3	-223.4	68.8	-154.6	0.0	37.8	-19.3	18.5	0.0	58.7	-5.7	53.0
1.000	-165	-25.139	15.52261	254.48	115.14	104.02	118.14	509.9	-216.6	70.4	-146.2	0.0	32.5	-18.3	14.3	0.0	60.1	-5.4	54.7
2.000	-150	-20.865	30.05797	239.94	110.86	117.91	125.65	554.8	-197.6	76.6	-121.0	0.0	31.7	-15.1	16.6	0.0	65.3	-4.4	60.9
3.000	-135	-14.298	43.08262	226.92	104.30	131.44	129.99	698.9	-172.6	96.4	-76.2	0.0	38.4	-9.5	28.9	0.0	82.3	-2.8	79.5
4.000	-120	-6.007	54.59996	215.40	96.01	144.16	130.33	1946.1	-203.7	268.6	64.9	0.0	107.8	8.1	115.9	0.0	229.2	2.4	231.6
4.480	-113	-1.568	59.68256	210.32	91.57	149.65	129.01	294951.7	-8069.9	40703.3	32633.5	0.0	16524.5	4079.2	20603.7	0.0	34742.5	1194.8	35937.2
5.070	-104	4.202	65.63002	204.37	85.80	155.29	126.20	26.0	1.9	3.6	5.5	0.0	1.5	0.7	2.2	0.0	3.1	0.2	3.3
6.000	-90	13.823	74.57451	195.43	76.18	159.40	119.53	151.1	36.1	20.8	56.9	0.0	8.7	7.1	15.8	0.0	17.8	2.1	19.9
7.000	-75	24.631	84.07928	185.92	65.37	154.71	110.16	210.6	87.8	29.1	116.8	0.0	11.9	14.6	26.5	0.0	24.8	4.3	29.1
8.000	-60	35.626	94.20046	175.80	54.37	144.16	99.28	239.1	139.3	33.0	172.3	0.0	13.2	21.5	34.8	0.0	28.2	6.3	34.5
9.000	-45	46.453	106.1109	163.89	43.55	131.44	87.45	254.6	184.5	35.1	219.7	0.0	14.0	27.5	41.4	11.3	30.0	8.0	49.4
10.000	-30	56.513	121.977	148.02	33.49	117.91	75.00	263.3	219.6	36.3	255.9	0.0	15.0	32.0	47.0	68.1	31.0	9.4	108.5
11.000	-15	64.509	145.7407	124.26	25.49	104.02	62.16	267.8	241.7	37.0	278.7	0.0	17.1	34.8	51.9	125.1	31.5	10.2	166.8
12.001	0.01	67.830	179.9628	89.96	22.17	89.99	49.08	269.2	249.3	37.1	286.4	0.1	20.4	35.8	56.3	176.3	31.7	10.5	218.5
13.000	15	64.509	145.7407	55.74	25.49	75.98	35.94	267.8	241.7	37.0	278.7	64.9	24.9	34.8	124.6	216.8	31.5	10.2	258.6
14.000	30	56.513	121.977	31.98	33.49	62.09	22.97	263.3	219.6	36.3	255.9	123.2	29.9	32.0	185.1	242.4	31.0	9.4	282.8
15.000	45	46.453	106.1109	16.11	43.55	48.56	11.32	254.6	184.5	35.1	219.7	168.5	34.3	27.5	230.3	249.6	30.0	8.0	287.7
16.000	60	35.626	94.20046	4.20	54.37	35.84	9.90	239.1	139.3	33.0	172.3	193.8	36.6	21.5	252.0	235.6	28.2	6.3	270.0
17.000	75	24.631	84.07928	5.92	65.37	25.29	20.93	210.6	87.8	29.1	116.8	190.4	34.9	14.6	239.9	196.7	24.8	4.3	225.8
18.000	90	13.823	74.57451	15.43	76.18	20.60	33.82	151.1	36.1	20.8	56.9	141.4	25.7	7.1	174.2	125.5	17.8	2.1	145.4
18.800	102	5.512	66.90529	23.09	84.49	23.71	44.33	47.5	4.6	6.5	11.1	43.5	7.9	1.4	52.8	34.0	5.6	0.4	39.9
20.000	120	-6.007	54.59996	35.40	96.01	35.84	60.05	1946.1	-203.7	268.6	64.9	1577.6	298.1	8.1	1883.8	971.5	229.2	2.4	1203.1
21.000	135	-14.298	43.08262	46.92	104.30	48.56	72.94	698.9	-172.6	96.4	-76.2	462.6	94.2	-9.5	547.2	205.0	82.3	-2.8	284.6
22.000	150	-20.865	30.05797	59.94	110.86	62.09	85.46	554.8	-197.6	76.6	-121.0	259.6	63.0	-15.1	307.5	43.9	65.3	-4.4	104.8
23.000	165	-25.139	15.52261	74.48	115.14	75.98	97.42	509.9	-216.6	63.5	-153.1	123.5	42.8	-19.1	147.2	0.0	60.1	-5.6	54.5

6-17

Determine the amount of diffuse, direct and total radiation that would strike a South-facing surface tilted at 45 degrees on a clear day on December 21 in St Louis, MO at,

- a) noon solar time, b) 3 pm solar time c) all 24 hours

6-1

For a one square foot of DCS glass in the surface of problem 6-17 compute the total and absorbed radiation.

Problem 6-17 INCLINED SURFACE

90.37 Longitude
 I = 38.8 Latitude
 F ss 0.8536
 F sg 0.1464
 d = -23.45 Declination

South Facing, 45 degree inclination

December

Problem 6-18

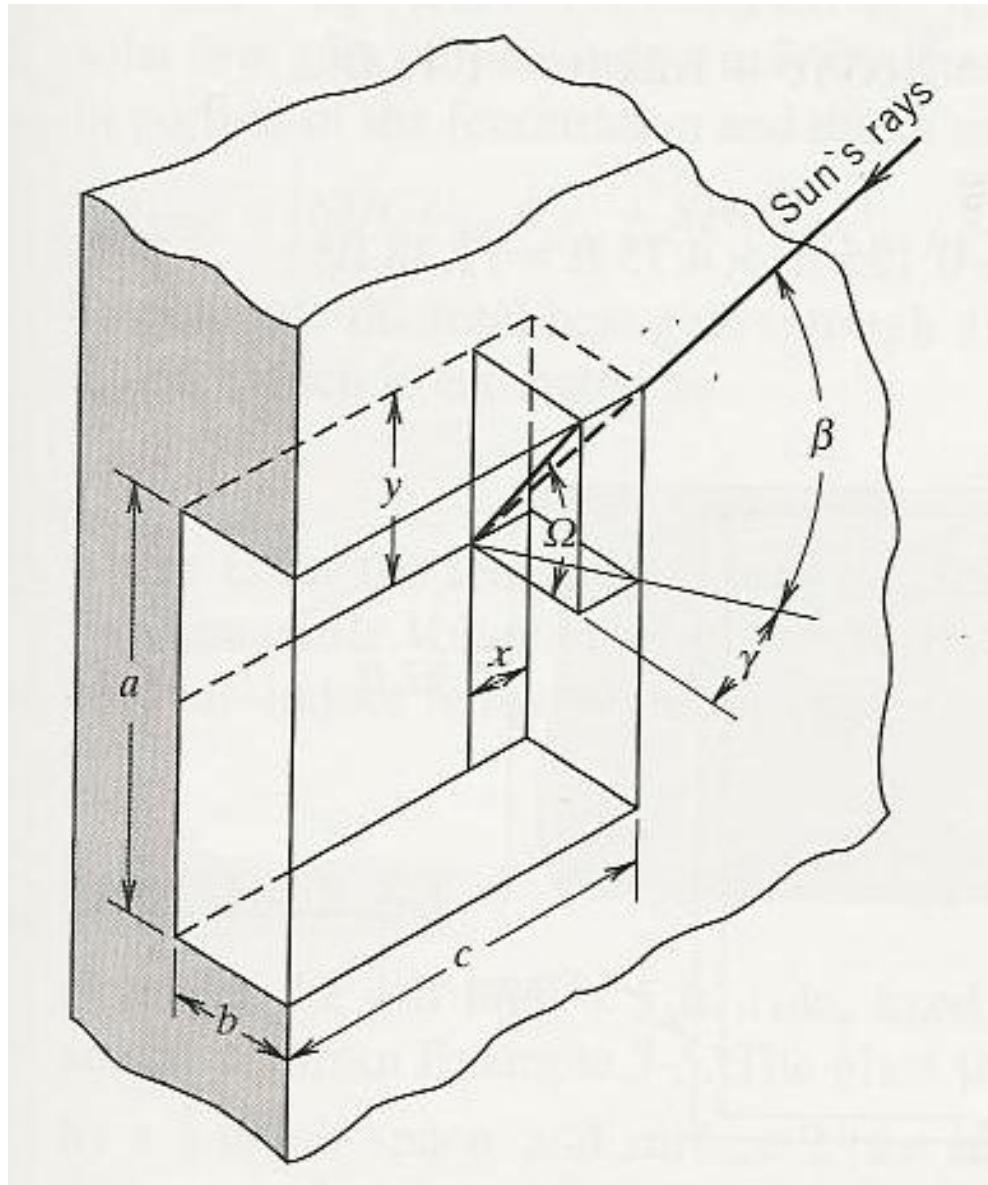
A= 381.80
 B= 0.1410
 C= 0.1030
 CN= 1.0000

Local Solar Time LST	h Hour Angle	BETA, Sun Ray to horiz.	PHI Sun Ray proj. to North	GAMA Sun Ray proj. to normal	THETA Sun Ray to Vertical Normal	V Normal (Sun Ray) G ND	HORIZONTAL		VERTICAL						TSHGF	ASHGF					
							DIRECT on Horizontal Surface G DH	DIFFUSE on Horizontal Surface G DV	DIRECT on vertical surface G DV	G dv /Gdh	DIFFUSE on vertical surface G dV	REFLECTE D direct +diffuse	TOTAL Surface G R +reflected	Local Solar Time LST	Total Direct DV	Total Diffused dV + R	cos theta V	Direct Transmitt D	Diffuse Transmitt tau		
0.0001	179.9985	-74.650	0.0051985	179.99	150.35		0.00	0.00	0.00	0.41	0.00	0.00	0.00	0	0.00	0.00	-0.87	9.31	-15.55	0.00	0.00
1	165	-70.046	44.087689	135.91	146.93		0.00	0.00	0.00	0.40	0.00	0.00	0.00	1	0.00	0.00	-0.84	7.87	-13.84	0.00	0.00
2	150	-60.289	67.745877	112.25	138.32		0.00	0.00	0.00	0.40	0.00	0.00	0.00	2	0.00	0.00	-0.75	4.51	-9.65	0.00	0.00
3	135	-49.018	81.552272	98.45	127.01		0.00	0.00	0.00	0.40	0.00	0.00	0.00	3	0.00	0.00	-0.60	1.27	-5.10	0.00	0.00
4	120	-37.361	91.637366	88.36	114.40		0.00	0.00	0.00	0.42	0.00	0.00	0.00	4	0.00	0.00	-0.41	-0.41	-1.87	0.00	0.00
5	105	-25.747	100.31963	79.68	101.13		0.00	0.00	0.00	0.48	0.00	0.00	0.00	5	0.00	0.00	-0.19	-0.49	-0.36	0.00	0.00
6	90	-14.439	108.67821	71.32	87.54		0.00	0.00	0.00	0.57	0.00	0.00	0.00	6	0.00	0.00	0.04	0.11	0.04	0.00	0.00
7	75	-3.687	117.37877	62.62	73.80		0.00	0.00	0.00	0.70	0.00	0.00	0.00	7	0.00	0.00	0.28	0.60	0.06	0.00	0.00
8	60	6.207	126.94774	53.05	60.07	103.64	11.21	10.67	51.72	0.85	9.11	0.64	61.47	8	51.72	9.75	0.50	0.80	0.06	48.94	3.41
9	45	14.845	137.8476	42.15	46.54	220.20	56.42	22.68	151.48	1.00	19.36	2.32	173.15	9	151.48	21.68	0.69	0.85	0.06	145.42	9.93
10	30	21.705	150.41546	29.58	33.61	260.77	96.44	26.86	217.18	1.13	22.93	3.61	243.71	10	217.18	26.54	0.83	0.87	0.05	209.45	12.47
11	15	26.184	164.65714	15.34	22.49	277.37	122.39	28.57	256.27	1.22	24.39	4.42	285.08	11	256.27	28.81	0.92	0.88	0.05	247.40	13.37
12.0001	-0.0015	27.750	179.99845	0.00	17.25	282.05	131.32	29.05	269.36	1.25	24.80	4.70	298.85	12	269.36	29.49	0.96	0.88	0.05	259.40	14.07
13	15	26.184	164.65714	15.34	22.49	277.37	122.39	28.57	256.27	1.22	24.39	4.42	285.08	13	256.27	28.81	0.92	0.88	0.05	247.40	13.37
14	30	21.705	150.41546	29.58	33.61	260.77	96.44	26.86	217.18	1.13	22.93	3.61	243.71	14	217.18	26.54	0.83	0.87	0.05	209.45	12.47
15	45	14.845	137.8476	42.15	46.54	220.20	56.42	22.68	151.48	1.00	19.36	2.32	173.15	15	151.48	21.68	0.69	0.85	0.06	145.42	9.93
16	60	6.207	126.94774	53.05	60.07	103.64	11.21	10.67	51.72	0.85	9.11	0.64	61.47	16	51.72	9.75	0.50	0.80	0.06	48.94	3.41
17	75	-3.687	117.37877	62.62	73.80		0.00	0.00	0.00	0.70	0.00	0.00	0.00	17	0.00	0.00	0.28	0.60	0.06	0.00	0.00
18	90	-14.439	108.67821	71.32	87.54		0.00	0.00	0.00	0.57	0.00	0.00	0.00	18	0.00	0.00	0.04	0.11	0.04	0.00	0.00
19	105	-25.747	100.31963	79.68	101.13		0.00	0.00	0.00	0.48	0.00	0.00	0.00	19	0.00	0.00	-0.19	-0.49	-0.36	0.00	0.00
20	120	-37.361	91.637366	88.36	114.40		0.00	0.00	0.00	0.42	0.00	0.00	0.00	20	0.00	0.00	-0.41	-0.41	-1.87	0.00	0.00
21	135	-49.018	81.552272	98.45	127.01		0.00	0.00	0.00	0.40	0.00	0.00	0.00	21	0.00	0.00	-0.60	1.27	-5.10	0.00	0.00
22	150	-60.289	67.745877	112.25	138.32		0.00	0.00	0.00	0.40	0.00	0.00	0.00	22	0.00	0.00	-0.75	4.51	-9.65	0.00	0.00
23	165	-70.046	44.087689	135.91	146.93		0.00	0.00	0.00	0.40	0.00	0.00	0.00	23	0.00	0.00	-0.84	7.87	-13.84	0.00	0.00
23.9999	179.9985	-74.650	0.0051985	179.99	150.35		0.00	0.00	0.00	0.41	0.00	0.00	0.00	24	0.00	0.00	-0.87	9.31	-15.55	0.00	0.00

j	a _j	t _j	t _j / j+2	a _j / j + 2
0.00	0.01154	-0.00885	0.00	0.01
1.00	0.77674	2.71235	0.90	0.26
2.00	-3.94657	-0.62062	-0.16	-0.99
3.00	8.57811	-7.07329	-1.41	1.72
4.00	-8.38138	9.75995	1.63	-1.40
5.00	3.01188	-3.89922	-0.56	0.43
SUM	0.05032	0.87032	0.39951	0.02703

$$TSHGF = G_D \sum_{j=0}^5 t_j (\cos \theta)^j + 2G_d \sum_{j=9}^5 \frac{t_j}{j+2} \quad (6-26)$$

$$ASHGF = G_D \sum_{j=0}^5 a_j (\cos \theta)^j + 2G_d \sum_{j=9}^5 \frac{a_j}{j+2} \quad (6-26)$$

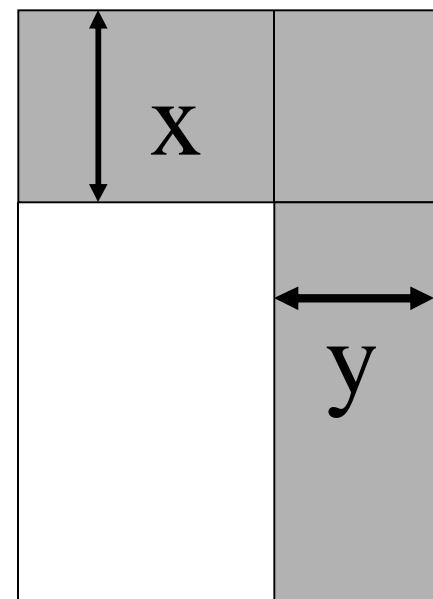


$$x = b \tan \gamma$$

$$y = b \tan \Omega$$

$$y = b \frac{\tan \beta}{\cos \gamma}$$

$$\Omega = \frac{\tan \beta}{\cos \gamma}$$



HOURLY ANALYSIS PROGRAM INPUT DATA

ZONE
SYSTEMS
Equipment Type
Air system
Number of zones

1 2 3

SPACE
Name
Floor area

Ceiling Height
Building weight
Ventilation 1
Ventilation 2

LIGHTS

Wattage
Schedule

EQUIPMENT

Type
Wattage
Schedule

PEOPLE

Number(Design)
Number (energy)

Schedule

WALLS

Exposure
Gross area

Construction
Exposure
Gross area

ZONE 1 2 3

Construction
Window type
Number
Window shade
Window type
Number
Window shade

Door Type/number
Door Type
Total Gross Wall Area

ROOF

Consturction
Exposure

INFILTRATION

FLOORS

Area
U
Perimeter
Edge Iresistance

CEILING PARTITION

Area
U
Max Unconditioned
Max ambient
Min Unconditioned
Min ambient

FUEL

\$/therm(100 kBtu/therm)

ELECTRICAL RATE

\$/kw hr

SCHEDULES

Occupancy
Lighting
Equipment

SPECIFICATION

Walls
Window
Roof
Doors