CONDUCTION

$$q = -A \times k \times \frac{dT}{dx}$$

$$\frac{BTU}{hr} = \left(ft^2\right)\left(\frac{BTU}{hr\ ft\ {}^{O}R}\right) \qquad \left(\frac{{}^{O}R}{ft}\right)$$

$$\frac{W}{hr} = \left(m^2\right)\left(\frac{W}{hrm\ {}^{O}K}\right) \qquad \left(\frac{{}^{O}K}{m}\right)$$

for steady state, linear T(x)

C - Conductance

$$q = A \times C \times \varphi T$$

$$C = \frac{k}{\varphi x} \qquad \frac{BTU}{hr\ ft^2\ F}, \frac{W}{m^2\ C}$$

R – Resistance

$$q = A \times \frac{1}{R} \times \varphi T$$

$$R = \frac{1}{C} = \frac{\varphi x}{k} \qquad \frac{hr\ ft^2 F}{BTU}, \frac{m^2\ C}{w} \qquad (5-7)$$

Table 5-1*b* (*Continued*)

Description	Thickness, mm	Density ρ, kg/m^3	Conductivity k, W/(m-K)	Conductance C, $W/(m^2\text{-}K)$	Specific Heat, kJ/(kg-K)
Mineral fiber with resin binder	—	240	0.042	—	—
Core or roof insulation	—	260–270	0.049	—	—
Acoustical tile	12.7	—	—	4.5	—
Acoustical tile	19.0	—	—	3.0	—
Loose Fill					
Cellulosic insulation (milled paper or wood pulp)	—	37–51	0.039–0.046	—	1.398
Perlite, expanded	—	32–66	0.039–0.045	—	1.09
	—	66–120	0.045–0.052	—	—
	—	120–180	0.052–0.060	—	—
Mineral fiber (rock, slag, or glass)					
approx. 95–130 mm	—	9.6–3.2	—	0.52	0.71
approx. 170–220 mm	—	9.6–3.2	—	0.31	—
approx. 190–250 mm	—	9.6–3.2	—	0.26	—
approx. 260–350 mm	—	9.6–3.2	—	0.19	5.28
Mineral fiber (rock, slag or glass)					
approx. 90 trim (closed sidewall application)	—	32–56	2.1–2.5	—	—
Vermiculite, exfoliated	—	110–130	0.068	—	1.34
	—	64–96	0.063	15.7	—
Metals					
Aluminum (1100)	—	2660	221.5	—	0.9
Steel, mild	—	7600	45.3	—	0.5
Steel, stainless	—	7680	15.6	—	0.46
Roofing					
Asbestos–cement shingles	—	1900	—	27.0	1.00
Asphalt roll roofing	—	1100	—	36.9	1.51
Asphalt shingles	—	1100	—	12.9	1.26
Built-up roofing	10	1100	—	17.0	1.46
Slate	13	—	—	114	1.26
Wood shingles, plain and plastic film faced	—	—	—	6.0	1.30

MATERIAL PROPERTIES

	k	Ɉ	c_p	t	Ů
	$in \frac{BTU}{hrft^2\,^{O}F}$	$\frac{lbm}{ft^3}$	$\frac{BTU}{^{O}F}$	thicknss	emmisivity
MINIMUM					
glass wool + air	.026	.4	.18		
Air	.015	.076	.24		
Argon	.0088	.1034	.1245		
MAXIMUM					
Aluminum	1536	171	.214		
water	.34	62.3	1.00		
RANGE	.008 - 1500	.06 - 200	.18 - 1.0		

MATERIAL PROPERTIES

	k	Ɉ	c_p	t	Ů
	$m \frac{W}{hr\ m^{2}\ {}^{O}K}$	$\frac{kg}{m^{3}}$	$\frac{W}{{}^{O}K}$	thicknss	emmisivity
MINIMUM					
glass wool + air	.079	6.32	.7		
Air	.0455	.85	1.005		
Argon	.026	1.012	.5203		
MAXIMUM					
Aluminum	220.	2660.	.9		
water	1.031	988.	4.18		
RANGE	.026 - 220.	.85 - 2660.	.7 - 4.18		

CONVECTION

$$\frac{BTU}{hr} = (ft^2)\left(\frac{BTU}{hr\, ft\, {}^{O}R}\right) \left(\frac{{}^{O}R}{ft}\right)$$

$$\frac{W}{hr} = (m^2)\left(\frac{W}{hrm\, {}^{O}K}\right) \left(\frac{{}^{O}K}{m}\right)$$

$$q = -A \times h \times (T_2 - T_{wall}) \quad (5\text{-}8a)$$

$$C = h$$

$$R = \frac{1}{h} = \frac{1}{C} \quad (5-9b)$$

h from correlations of dimensionless parameters

Forced Convection

horizontal plate, turbulent $\frac{hl}{k} = .664\left(\frac{c_p \varepsilon}{k}\right)^{\frac{1}{3}}\left(\frac{\rho Vl}{\varepsilon}\right)^{\frac{1}{2}}$

$$N_u = .664 \times P_R^{\frac{1}{3}} \times N_{RE}^{.5}$$

N_u – Nusselt Number

P_R – Prandtl Number

N_{RE} = Reynolds Number

Free Convection

vertical, turbulent $h = .19\Delta^{.333}$

coefficients - Figure 5-2a

CONVECTION HEAT TRANSFER COEFFICIENTS

Table 5-2*a* Surface Unit Conductances and Unit Resistances for Air[a]

		Surface Emittances											
		$\epsilon = 0.9$				$\epsilon = 0.2$				$\epsilon = 0.05$			
		h		*R*		*h*		*R*		*h*		*R*	
Position of Surface	Direction of Heat Flow	$\frac{\text{Btu}}{\text{hr-ft}^2\text{-F}}$	$\frac{\text{W}}{\text{m}^2\text{-C}}$	$\frac{\text{hr-ft}^2\text{-F}}{\text{Btu}}$	$\frac{\text{m}^2\text{-C}}{\text{W}}$	$\frac{\text{Btu}}{\text{hr-ft}^2\text{-F}}$	$\frac{\text{W}}{\text{m}^2\text{-C}}$	$\frac{\text{hr-ft}^2\text{-F}}{\text{Btu}}$	$\frac{\text{m}^2\text{-C}}{\text{W}}$	$\frac{\text{Btu}}{\text{hr-ft}^2\text{-F}}$	$\frac{\text{W}}{\text{m}^2\text{-C}}$	$\frac{\text{hr-ft}^2\text{-F}}{\text{Btu}}$	$\frac{\text{m}^2\text{-C}}{\text{W}}$
Still Air													
Horizontal	Upward	1.63	9.26	0.61	0.11	0.91	5.2	1.10	0.194	0.76	4.3	1.32	0.232
Sloping—45 degrees	Upward	1.60	9.09	0.62	0.11	0.88	5.0	1.14	0.200	0.73	4.1	1.37	0.241
Vertical	Horizontal	1.46	8.29	0.68	0.12	0.74	4.2	1.35	0.238	0.59	3.4	1.70	0.298
Sloping—45 degrees	Downward	1.32	7.50	0.76	0.13	0.60	3.4	1.67	0.294	0.45	2.6	2.22	0.391
Horizontal	Downward	1.08	6.13	0.92	0.16	0.37	2.1	2.70	0.476	0.22	1.3	4.55	0.800
Moving Air													
(any position) Wind is 15 mph or 6.7 m/s (for winter)	Any	6.0	34.0	0.17	0.029								
Wind is 7½ mph or 3.4 m/s (for summer)	Any	4.0	22.7	0.25	0.044								

[a] Conductances are for surfaces of the stated emittance facing virtual blackbody surroundings at the same temperature as the ambient air. Values are based on a surface–air temperature difference of 10 F and for a surface temperature of 70 F.

Source: Adapted by permission from *ASHRAE Handbook, Fundamentals Volume*, 1989.

COMBINED CONDUCTION CONVECTION

$$q = h\,A\,\varphi T_o = \frac{k\,A}{\varphi x_1}\varphi T_1 = \frac{k\,A}{\varphi x_2}\varphi T_2 = h\,A\,\varphi T_i$$

solving for the temperature differences with $R = \frac{\varphi x}{k}, R = \frac{1}{h}$

Conductance, $C = \frac{1}{R}$ listed in Table 5–1a

$$\varphi T_O = \frac{q}{hA} = \frac{q}{A_o} R_o$$

$$\varphi T_1 = \frac{q}{\frac{kA}{\varphi x_1}} = \frac{q}{A} R_1$$

$$\varphi T_2 = \frac{q}{\frac{kA}{\varphi x_2}} = \frac{q}{A} R_2$$

$$\varphi T_i = \frac{q}{hA} = \frac{q}{A} R_i$$

the overall temperature difference is the sum of the individual temperature differences

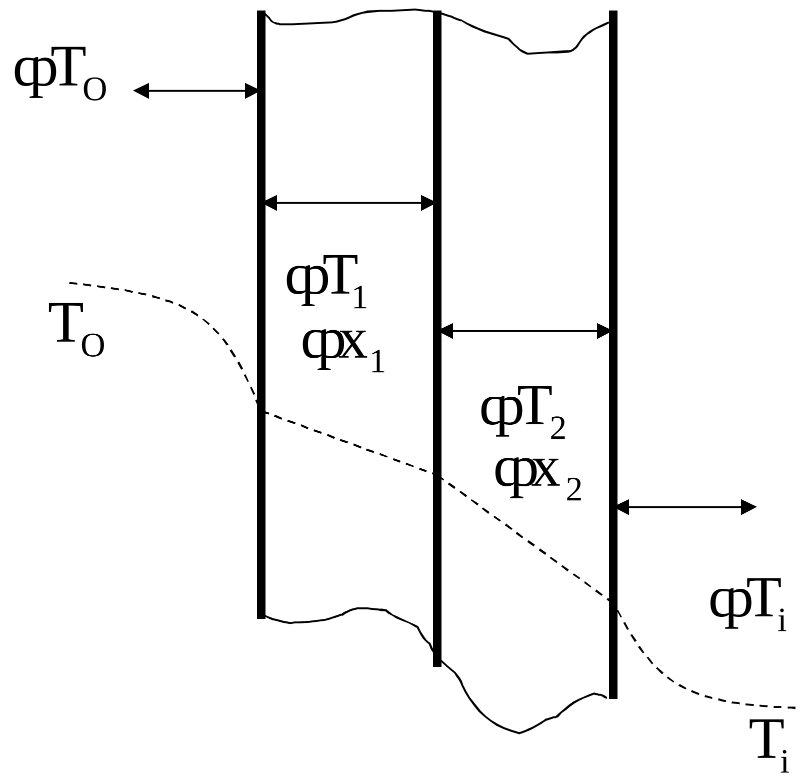
$$T_o - T_i = \sum \varphi T = \frac{q}{A} R_o + \frac{q}{A} R_1 + \frac{q}{A} R_2 + \frac{q}{A} R_i$$

$$T_o - T_i = \frac{q}{A} \sum R$$

$$q = \frac{1}{\sum R} A\left(T_o - T_i\right)$$

$$q = UA\left(T_o - T_i\right) = \frac{1}{\sum R} A\left(T_o - T\right)$$

$$U = \frac{1}{\sum R} = \frac{1}{\frac{1}{h_o} + \frac{\varphi x_1}{k_1} + \frac{\varphi x_2}{k_2} + \frac{1}{h_i}}$$



RADIATION

$$q = -\text{Ůů}\ A\left(T_1^{\ 4} - T_2^{\ 4}\right)$$

Ů= emissivity, blackbody $= 1.$, earth $\approx .9$, metals $\approx .4$

ů – Steffan Boltzman Constant

$$\text{ů} = .1713\times10^{-8}\ \frac{BTU}{ft^2\ hr\ {}^oR^4}$$

$$\text{ů} = 5.66964\times10^{-8}\ \frac{W}{m^{2\ o}K^4}$$

Radiation heat transfer from a 10 ft^2 radiator with an emissivity of .9 at 180^O F in a 40^O room

$$q = .1713\times10^{-8}\times.9\times10\ ft^2\times\left((180+460)^4-(40+460)^4\right)$$

$$q = 1623.9\ \frac{BTU}{hr}$$

CYLINDIRCAL COORDINATES

for a cylindirical wall

$$\text{with } q = -A \times k \times \frac{dt}{dx}$$

writen in cylindrical coordinates

$$R = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\,\text{´}\,L\,k} \quad (5-6) \quad (5-16,\ 5-17)$$

What is the heat transfer through 1 square foot of a wall made up of .625 in gypsum board and .625 in Douglas Fir plywood with a wall temperature difference of 10 degrees F?

.5 in gypsum board Table 5-1a

$$R_{.5} = \frac{1}{C} = \frac{1}{2.22} = \frac{\Delta x}{k}$$

$$k = C \times \Delta x$$

$$R = \frac{\Delta x}{k} = \frac{.625}{.50 \times 2.22} = .563 \frac{\text{hr ft}^2\ \text{F}}{\text{BTU}}$$

Douglas Fir plywood Table 5-1a

$$R = \frac{\varphi x}{k} = \frac{.625\ \text{in}}{.8 \frac{\text{BTU in}}{\text{hr ft}^2\ \text{F}}} = .781 \frac{\text{hr ft}^2\ \text{F}}{\text{BTU}}$$

$$U = \frac{1}{\sum R} = \frac{1}{.563 + .781} = .744 \frac{\text{BTU}}{\text{hr ft}^2\ \text{F}}$$

$$q = UA\left(T_{\substack{\text{wall}\\ \text{outside}}} - T_{\substack{\text{wall}\\ \text{inside}}}\right) = .744 \times 1 \times 10 = 7.44 \frac{\text{BTU}}{\text{hr}}$$

What is the heat transfer for a still air temperature difference of 10 degrees F.

$$R_o = R_i = .68 \ \text{Table } 5-2a$$

$$U = \frac{1}{R_o + R_{\text{gypsum}} + R_{\text{plywood}} + R_i}$$

$$U = \frac{1}{.68 + 1.344 + .68} = .37 \frac{\text{BTU}}{\text{hrft}^2\text{F}}$$

$$q = UA\left(T_{\substack{\text{airl}\\ \text{outside}}} - T_{\substack{\text{air}\\ \text{inside}}}\right)$$

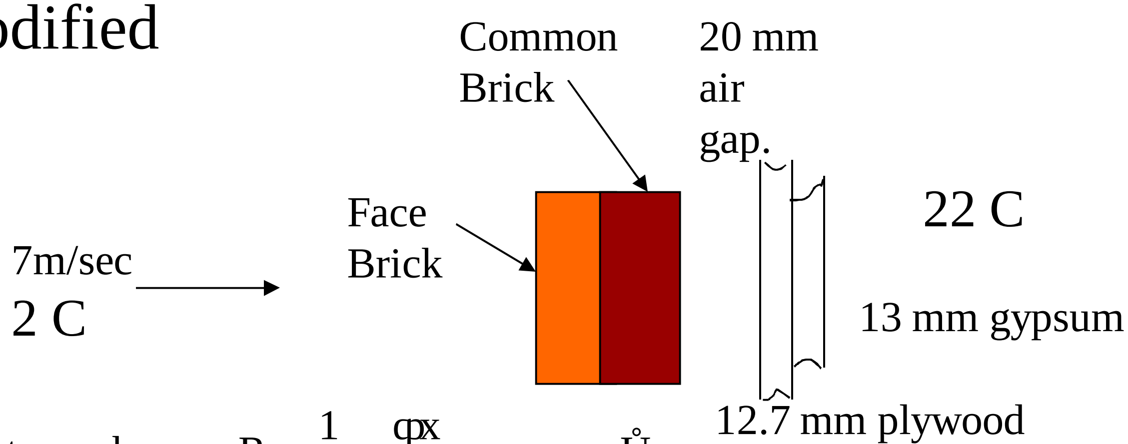
$$q = .37 \times 1 \times 10 = 3.7 \frac{\text{BTU}}{\text{hr}}$$

5-6. Compute the overall thermal resistance of a wall made up of 100 mm face brick (1920 kg/m^3) and 100-mm common brick (2080 kg/m^3) with a 20 mm air gap between. There is 13 mm of gypsum plaster on the inside. Assume a 7 m/s wind velocity on the outside and still air inside.

5-11. In an effort to save energy it is proposed to change the standard frame wall construction from 2×4 studs on 16 in. centers to 2×6 studs on 24 in. centers. Make a table similar to Table 5-4*a* showing both constructions. Use $5\frac{1}{2}$ in. and $3\frac{1}{2}$ in. fibrous glass insulation. Compare the two different constructions.

5-17. A wall is 12 ft wide and 8 ft high and has an overall heat transfer coefficient of 0.1 Btu/(hr-ft^2-F). It contains a solid wood door, $80 \times 32 \times 1\frac{3}{8}$ in., and a single glass window, 60×30 in. Assuming parallel heat flow paths for the wall, door, and window, find the overall thermal resistance and overall heat transfer coefficient for the combination. Assume winter conditions. The window is metal sash, no thermal break.

5.6 modified



	t	k	$R = \frac{1}{C} = \frac{\varphi x}{k}$	ɟ	Ů
	m	$m\frac{W}{m^2K}$	$\frac{{}^oKm^2}{W}$	$\frac{kg}{m^3}$	
Outside	-		.029 Fig 5 - 2a		
Face	.1		.0984	1920	
Common	.1		.112	2080	
Air gap	.02		.14 Fig 5 - 3a		.82
Gypsum	.013		.0778		
Plywood	.0127	.12	.106		
Inside			.120 Fig5 - 2a		

$$R = .6832\frac{{}^OK\,m^2}{W}$$

$$Q = U\,A\,\varphi T = \frac{1}{R}A\,\varphi T = \frac{1\,m\times(22-2)^OK}{.6832\frac{{}^OK\,m^2}{W}} = 29.3\frac{W}{m^2}$$

<u>Gypsum</u> $-9.5mm \quad C = 17.6 \quad Fig5-1b$

$$R_{1.5} = \frac{1}{C} = \frac{1}{17.6} = \frac{\varphi x}{k}$$

$$k = 17.5\times 9.5 = 167.2\frac{W}{m^2K}$$

$$R_{13} = \frac{\varphi x}{k} = \frac{13}{167.2} = .0778\frac{{}^oKm^2}{W}$$

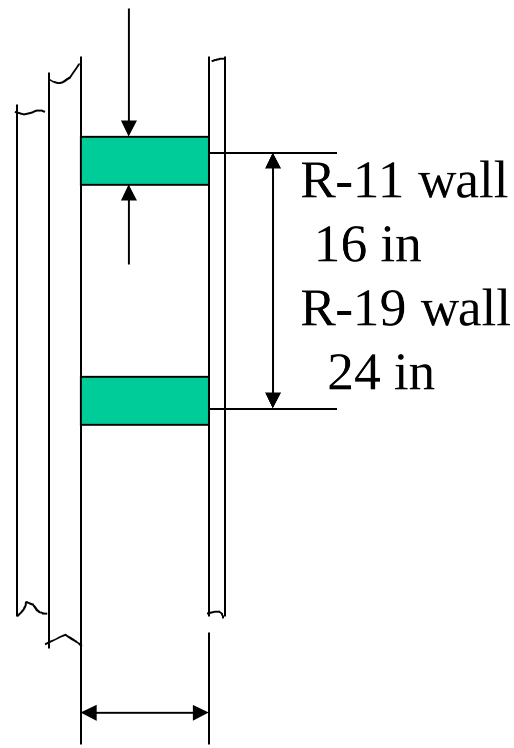
<u>Brick</u>

$$R = \frac{\varphi x}{k} = \frac{.1m}{\left(\frac{.92+1.112}{2}\right)\frac{W}{m^2K}}$$

$$R = .112\frac{{}^oKm^2}{W}$$

5-11

	t	R – 11 wall at stud R	R – 11 wall at insulation R	R – 19 wall at stud R	R – 19 wall at insulation R	
Outside		.17	.17	.17	.17	page 135
Siding		.79	.79	.79	.79	wood drop, 1x8
Plywood	.625	.75	.75	.75	.75	p 127
Insulation		-	11.	-	19.	
Studs		4.26	-	6.707	-	page 129
Gypsum	.50	.45	.45	.45	.45	page 127
Inside		.68	.68	.68	.68	page 135
		8.07	14.83	10.54	22.83	
AREA ft^2/ ft height		.125	1.2083	.125	1.875	



R-11 wall
ñ2x4ò- 3.5 in
R-19 wall
ñ2x6ò- 5.5 in

$$U = \frac{UA_{stud} + UA_{ins}}{A_{total}}$$

$$U_{11} = \frac{\frac{.125}{8.07} + \frac{1.2083}{14.83}}{1.332} = .0727 \qquad U_{19} = \frac{\frac{.125}{10.54} + \frac{1.875}{22.83}}{2} = .0471 \quad 1.54 \text{times}$$

Insulation and Stud Alone

$$U_{11} = \frac{\frac{.125}{4.26} + \frac{1.2083}{11}}{1.332} = .1044 \qquad U_{19} = \frac{\frac{.125}{6.707} + \frac{1.875}{19}}{2} = .0587 \quad 1.78 \text{times}$$

AIR GAP RESISTANCE

Table5 – 3a Combined conduction - convection

Vertical wall, horizontal heat flow, $\varepsilon = .82$

50 F mean temperature, 30 F temperature difference

Air gap thickness, in	Table 3a $R\ \dfrac{F\ ft^2\ hr}{BTU}$	Still Air Calculation $R = \dfrac{\Delta x}{k} = \dfrac{\Delta x}{.015}$
.5	.9	2.77
.75	.94	4.17
1.5	.90	8.33
3.5	.91	19.44

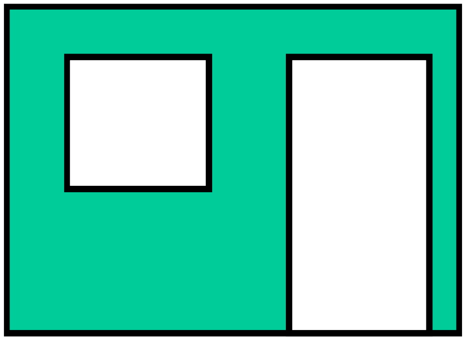
3 - 4 in mineral fiber batt, $R = 11.$ Table5 – 1a

5-17

Wall $U = .1\ \text{BTU/ft}^{2\,O}\text{F}$

Door $U = .39 \quad 1\ 3/8$ solid page 147

Window $U = 1.27$ page 148 assume Al sash



Window Area $= 60 \times 30/144 = 12.4\ \text{ft}^2$

Door Area $= 80 \times 32/144 = 17.8\ \text{ft}^2$

Wall Area $= 12 \times 8 - 12.4 - 17.8 = 65.7\ \text{ft}^2$

Total Area $= 96\ \ \text{ft}^2$

$$q_{overall} = q_{wall} + q_{window} + q_{door}$$

$$U_{overal} A\Delta T = U_{wall} A\Delta T + U_{window} A\Delta T + U_{door} A\Delta T$$

$$U = \frac{\sum UA}{A} = \frac{.1 \times 65.7 + .39 \times 17.8 + 1.27 \times 12.5}{96\ \text{ft}^2}$$

$$U_{overall} = .306\ \text{BTU/ ft}^2\text{hr}\ {}^O\text{F}$$

$U_{overall}$ is per unit area of composite wall

ROOF STRUCTURE

$$q_{w+r} = q_r + q_w$$

$$q = C \times A \times \varphi T$$

$$C_{w+r}A_{w+r} = C_r A_r + C_w A_w$$

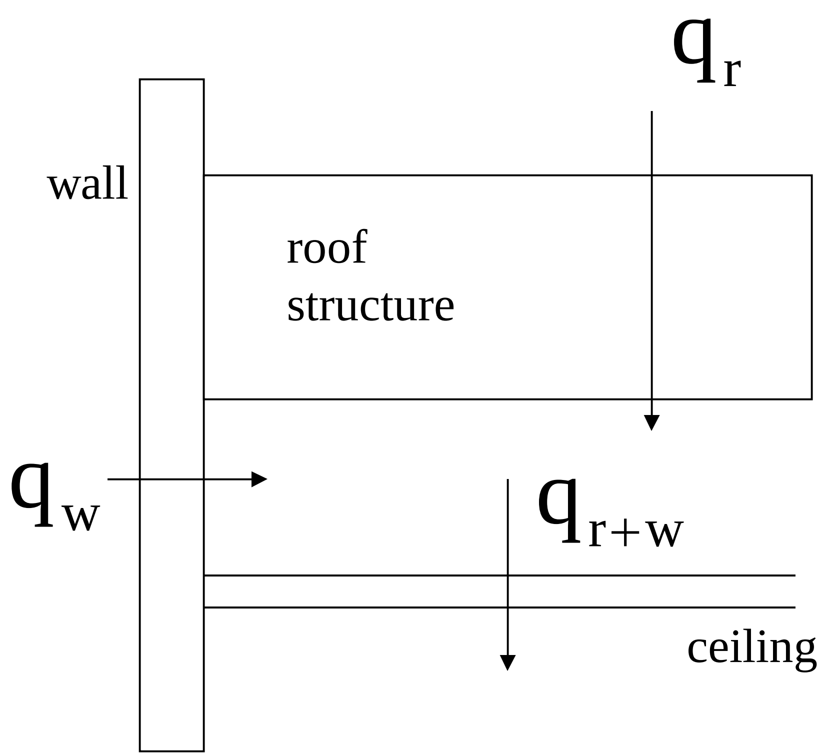
$$R'_{r+w} = \frac{1}{C_{w+r}A_{w+r}} = \frac{1}{C_r A_r + C_w A_w}$$

$$R'_{overall} = R'_c + R'_{w+r}$$

$$R'_{overall} = \frac{R}{A_c} = \frac{1}{C_c A_c} + \frac{1}{C_r A_r + C_w A_w}$$

$$R = \frac{1}{C_c} + \frac{1}{C_r + \left(\frac{A_w}{A_c}\right) C_w} \quad \text{page 147}$$

this resistance is per unit floor area.



Radiation effect Table 5-3a, 5-3b horizontal, up and down.

Emissivity Table 5-2b

Design Conditions Section 8.4 page 221

Typically outside temperatures are assumed to vary sinusoidally

$$T_{outside} = T_{design} - Range \times (\text{Table } 8-1) \qquad (8-2)$$

T_{design} (DB) and Range - Appendix B−1a

Chicago

% year	hrs	DB	Range
.4%		91	
1%	123	88	19.6
2%		86	

$$T_{outside} = 91. - 19.6 \times (\text{Table } 8-1 \text{ values, } 0\% \text{ to } 100\%)$$

Table B-1a Heating and Cooling Design Conditions—United States, Canada, and the World—English Units

Station	Lat., deg	Long., deg	Elev., ft.	Heating DB,F 99.6%	Heating DB,F 99%	MWS/MWD to DB 99.6% MWS, mph	MWS/MWD to DB 99.6% MWD, deg	MWS/MWD to DB 0.4% MWS, mph	MWS/MWD to DB 0.4% MWD, deg	Cooling DB/MWB 0.4% DB, F	Cooling DB/MWB 0.4% MWB, F	Cooling DB/MWB 1% DB, F	Cooling DB/MWB 1% MWB, F	Cooling DB/MWB 2% DB, F	Cooling DB/MWB 2% MWB, F	WB/MDB 1% WB, F	WB/MDB 1% MDB, F	HR 1% HR, gr	Range of DB, F
United States																			
Alabama, Birmingham	33.57	86.75	630	18	23	7	340	9	320	94	75	92	75	90	74	77	88	131	18.7
Alaska, Anchorage	61.17	150.02	131	−14	−9	4	10	8	290	71	59	68	57	65	56	58	66	64	12.6
Arizona, Tucson	32.12	110.93	2556	31	34	7	140	12	300	104	65	102	65	100	65	71	87	111	29.4
Arkansas, Little Rock	34.92	92.15	312	16	21	9	360	9	200	97	77	95	77	92	76	79	91	137	19.5
California, San Francisco	37.62	122.38	16	37	39	5	160	13	300	83	63	78	62	74	61	63	75	73	16.7
Colorado, Denver	39.75	104.87	5331	−3	3	6	180	9	160	93	60	90	59	87	59	63	80	90	26.9
Connecticut, Bridgeport	41.17	73.13	16	8	12	14	320	14	230	86	73	84	72	82	71	74	81	120	14.1
Delaware, Wilmington	39.68	75.60	79	10	14	11	290	11	240	91	75	89	74	86	73	76	85	125	17.0
Florida, Orlando	28.43	81.32	105	37	42	8	330	9	290	94	76	93	76	92	76	79	88	139	16.6
Georgia, Atlanta	33.65	84.42	1033	18	23	12	320	9	300	93	75	91	74	88	73	76	87	128	17.3
Hawaii, Honolulu	21.35	157.93	16	61	63	5	320	15	60	89	73	88	73	87	73	75	84	120	12.2
Idaho, Boise	43.57	116.22	2867	2	9	6	130	11	320	96	63	94	63	91	62	64	89	72	30.3
Illinois, Chicago	41.98	87.90	673	−6	−1	10	270	12	230	91	74	88	73	86	71	75	85	123	19.6
Indiana, Indianapolis	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

hourly outdoor temperature is given by

$$t_o = t_d - \mathrm{DR}(X) \tag{8-2}$$

where:

t_d = design dry bulb temperature, F or C
DR = daily range, F or C
X = percentage of daily range, from Table 8-1, divided by 100

Page 221

Table 8-1 Percentage of the Daily Range

Time, hr	Percent	Time, hr	Percent	Time, hr	Percent	Time, hr	Percent
1	87	7	93	13	11	19	34
2	92	8	84	14	3	20	47
3	96	9	71	15	0	21	58
4	99	10	56	16	3	22	68
5	100	11	39	17	10	23	76
6	98	12	23	18	21	24	82

Source: Reprinted by permission from *ASHRAE Cooling and Heating Load Calculation Manual*, 2nd ed., 1992.