Dielectric behavior

Topic 9

Reading assignment

• Shackelford, Materials Science for Engineering, Sec. 15.4.
• Chung, Composite Materials, Ch. 7.

Insulators and dielectric properties

• Materials used to insulate an electric field from its surroundings are required in a large number of electrical and electronic applications.
• Electrical insulators obviously must have a very low conductivity, or high resistivity, to prevent the flow of current.
• Porcelain, alumina, cordierite, mica, and some glasses and plastics are used as insulators.

Dielectric strength

• Maximum electric field that an insulator can withstand before it loses its insulating behavior
• Lower for ceramics than polymers
• Dielectric breakdown - avalanche breakdown or carrier multiplication
Polarization in dielectrics

- **Capacitor** – An electronic device, constructed from alternating layers of a dielectric and a conductor, that is capable of storing a charge. These can be single layer or multi-layer devices.
- **Permittivity** - The ability of a material to polarize and store a charge within it.
- **Linear dielectrics** - Materials in which the dielectric polarization is linearly related to the electric field; the dielectric constant is not dependent on the electric field.
- **Dielectric strength** - The maximum electric field that can be maintained between two conductor plates without causing a breakdown.

Polarization mechanisms in materials:

- (a) electronic,
- (b) atomic or ionic,
- (c) high-frequency dipolar or orientation (present in ferroelectrics),
- (d) low-frequency dipolar (present in linear dielectrics and glasses),
- (e) interfacial-space charge at electrodes, and
- (f) interfacial-space charge at heterogeneities such as grain boundaries.

A charge can be stored at the conductor plates in a vacuum (a). However, when a dielectric is placed between the plates (b), the dielectric polarizes and additional charge is stored.
\[ D_0 = \frac{Q}{A}, \]
\[ \Sigma = \frac{V}{d}, \]
\[ D_0 = \varepsilon_0 \Sigma \]
\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ C/(V.m)} \]

Slope = \[ C_0 = \frac{Q}{V} = \frac{\varepsilon_0 \Sigma A}{\Sigma d} = \frac{\varepsilon_0 A}{d}, \]
\[ D_m = \kappa D_o = \frac{kQ}{A}, \]
\[ D_m = \kappa \varepsilon_o \Sigma = \varepsilon \Sigma \]
\[ C_m = \frac{kQ}{V} = \frac{k\varepsilon_o \Sigma A}{\Sigma d} = \frac{k\varepsilon_o A}{d} = kC_o, \]
\[ P = D_m - D_o = \kappa \varepsilon_o \Sigma - \varepsilon_o \Sigma = (\kappa - 1) \varepsilon_o \Sigma \]

**Slope** = \((\kappa - 1) \varepsilon_o\)

**Free charges**

**Bound charges**

Center of negative charge

Center of positive charge
\[
\frac{\kappa Q - Q}{Q} = \kappa - 1, \\
\chi = \kappa - 1 = \frac{P}{\varepsilon_0 \Sigma}, \\
(\text{bound charge})d = (\kappa - 1) Qd
\]

Dipole moment = \frac{(\kappa - 1)Qd}{Ad} = \frac{(\kappa - 1)Q}{A} = P, \\

\[
V = \frac{\kappa Q}{C_m}, \\
\kappa Q = D_m A = \varepsilon \Sigma A \\
C_m = \frac{\kappa \varepsilon_0 A}{x} = \frac{\varepsilon A}{x} \\
V = \frac{\varepsilon \Sigma A}{x} = \Sigma x
\]

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric constant, ( \kappa )</th>
<th>Dielectric strength (kV/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Al}_2\text{O}_3 ) (99.9%)</td>
<td>10.1</td>
<td>9.1(^b)</td>
</tr>
<tr>
<td>( \text{Al}_2\text{O}_3 ) (99.5%)</td>
<td>9.8</td>
<td>9.5(^b)</td>
</tr>
<tr>
<td>BeO (99.5%)</td>
<td>6.7</td>
<td>10.2(^a)</td>
</tr>
<tr>
<td>Cordierite</td>
<td>4.1-5.5</td>
<td>2.4-7.9(^a)</td>
</tr>
<tr>
<td>Nylon 66-reinforced with 33%</td>
<td>3.7</td>
<td>20.5</td>
</tr>
<tr>
<td>glass fibers (dry-as-molded)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nylon 66-reinforced with 33%</td>
<td>7.8</td>
<td>17.3</td>
</tr>
<tr>
<td>glass fibers (50% relative humidity)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Acetal (50% relative humidity)</td>
<td>3.7</td>
<td>19.7</td>
</tr>
<tr>
<td>Polyester</td>
<td>3.6</td>
<td>21.7</td>
</tr>
</tbody>
</table>

Source: Data from Ceramic Source \#6, American Ceramic Society, Columbus, OH, 1985, and Design Handbook for Du Pont Engineering Plastics.

\(^a\) At 10 \(^5\) Hz.
\(^b\) Average root-mean-square (RMS) values at 60 Hz.
Table 7.6  Values of the relative dielectric constant $\kappa$ of various dielectric materials at 1 kHz (Data from Ceramic Source ’86, American Ceramic Society, Columbus, Ohio, 1985, and Design Handbook for DuPont Engineering Plastics).

<table>
<thead>
<tr>
<th>Material</th>
<th>$\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al2O3 (99.5%)</td>
<td>9.8</td>
</tr>
<tr>
<td>BeO (99.5%)</td>
<td>6.7</td>
</tr>
<tr>
<td>Cordierite</td>
<td>4.1-5.3</td>
</tr>
<tr>
<td>Nylon-66 reinforced with glass fibers</td>
<td>3.7</td>
</tr>
<tr>
<td>Polyester</td>
<td>3.6</td>
</tr>
</tbody>
</table>

\[
\Sigma = \hat{\Sigma} e^{i\omega t} = \hat{\Sigma}(\cos \omega t + i \sin \omega t),
\]
\[
D_m = \hat{D}_m e^{i(\omega t - \delta)} = \hat{D}_m [\cos(\omega t - \delta) + i \sin(\omega t - \delta)],
\]
\[
\hat{D}_m e^{i(\omega t - \delta)} = \epsilon \hat{\Sigma} e^{i\omega t},
\]
\[
\epsilon = \frac{\hat{D}_m e^{-i\delta}}{\hat{\Sigma}} = \frac{\hat{D}_m}{\hat{\Sigma}} (\cos \delta - i \sin \delta),
\]
\[
\tan \delta = -\frac{\text{Imaginary part of } \kappa}{\text{Real part of } \kappa},
\]

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric Constant (at 1 kHz)</th>
<th>Dielectric Strength (10^9 W/m)</th>
<th>tan $\delta$ (at 10^6 Hz)</th>
<th>Resistivity (ohm-cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polyethylene</td>
<td>23</td>
<td>23</td>
<td>20</td>
<td>0.0072</td>
</tr>
<tr>
<td>Epoxy</td>
<td>21</td>
<td>21</td>
<td>20</td>
<td>0.00017</td>
</tr>
<tr>
<td>Epoxy</td>
<td>25</td>
<td>25</td>
<td>20</td>
<td>0.00020</td>
</tr>
<tr>
<td>PVC</td>
<td>35</td>
<td>32</td>
<td>40</td>
<td>0.05000</td>
</tr>
<tr>
<td>PTFE</td>
<td>40</td>
<td>36</td>
<td>20</td>
<td>0.04000</td>
</tr>
<tr>
<td>PTFE</td>
<td>40</td>
<td>32</td>
<td>24</td>
<td>0.05000</td>
</tr>
<tr>
<td>PTFE</td>
<td>40</td>
<td>36</td>
<td>12</td>
<td>0.05000</td>
</tr>
<tr>
<td>Glass</td>
<td>40</td>
<td>36</td>
<td>18</td>
<td>10^12</td>
</tr>
<tr>
<td>Glass</td>
<td>23</td>
<td>23</td>
<td>10</td>
<td>10^12</td>
</tr>
<tr>
<td>Glass</td>
<td>38</td>
<td>36</td>
<td>10</td>
<td>0.00004</td>
</tr>
<tr>
<td>Soda lime glass</td>
<td>70</td>
<td>70</td>
<td>10</td>
<td>0.00000</td>
</tr>
<tr>
<td>MgO</td>
<td>90</td>
<td>65</td>
<td>6</td>
<td>0.00100</td>
</tr>
<tr>
<td>MgO</td>
<td>10</td>
<td>120</td>
<td>8</td>
<td>0.00020</td>
</tr>
<tr>
<td>MgO</td>
<td>70</td>
<td>40</td>
<td>40</td>
<td>10^12</td>
</tr>
<tr>
<td>MgO</td>
<td>2000-4000</td>
<td>12</td>
<td>-0.0001</td>
<td>10^8</td>
</tr>
<tr>
<td>Glass</td>
<td>76</td>
<td>76</td>
<td>76</td>
<td>10^12</td>
</tr>
</tbody>
</table>
\[ i_c = \frac{dQ}{dt} = C \frac{dv}{dt}, \]
\[ v = V \sin \omega t \]
\[ \omega = 2\pi f = \frac{2\pi}{T}, \]
\[ i_c = C \frac{dv}{dt} = \alpha CV \cos \omega t, \]
\[ = \frac{V}{1/\alpha C} \cos \omega t \]

\[ \sin \left( \frac{\omega t + \pi}{2} \right) \]
\[ = \sin \omega t \cos \frac{\pi}{2} + \cos \omega t \sin \frac{\pi}{2} \]
\[ = \cos \omega t, \]
\[ i_c = \frac{V}{1/\alpha C} \sin \left( \omega t + \frac{\pi}{2} \right), \]
\[ i_R = \frac{V}{R} \sin \omega t, \]

\[ i_c = \frac{V}{1/\alpha C} \sin \left( \omega t + \frac{\pi}{2} \right), \]
\[ i_R = \frac{V}{R} \sin \omega t, \]
\[ \tan \delta = \frac{V/R}{\omega C} = \frac{1}{\omega CR}, \]
Energy stored = \int_0^\tau v_C \, dt ,
= \int_0^\tau V^2 \omega C \sin \omega t \cos \omega t \, dt ,
= \int_0^\tau \frac{V^2 \omega C}{2} \sin 2 \omega t \, dt ,
= -\frac{V^2 \omega C}{4 \omega} [\cos 2 \omega t]_0^\tau ,
= -\frac{1}{4} CV^2 (\cos 2 \omega \tau - 1) ,

Maximum energy stored = \frac{1}{2} CV^2
This occurs when \cos 2\omega t = -1

Energy loss per cycle due to conduction through the resistor R

Energy loss = \frac{V^2}{R} \int_0^{2\pi/\omega} \sin \omega t \sin \omega t \, dt
= \frac{V^2}{\omega R} \left[ \frac{1}{2} \int_0^{2\pi/\omega} (1 - \cos 2 \omega t) \, d(\omega t) \right]
= \frac{V^2}{\omega R} \left[ \frac{1}{2} \left( \frac{\omega t}{2} - \frac{1}{2} \sin 2 \omega t \right) \right]_0^{2\pi/\omega}
= \frac{V^2}{\omega R} \left[ \frac{1}{2} (2\pi - 0 - 0 + 0) \right]
= \frac{V^2 \pi}{\omega R} .
The smaller is R, the greater is the energy loss.

\[ \text{Energy lost per cycle} = \frac{V^2 \pi / \omega R}{2 \pi \times \text{maximum energy stored}} = \frac{V^2 \pi / \omega R}{2 \pi CV^2 / 2} = \frac{1}{\omega CR} = \tan \delta \]
Frequency dependence of polarization mechanisms. On top is the change in the dielectric constant with increasing frequency, and the bottom curve represents the dielectric loss.

Quartz – polarization only under stress.
(a) The oxygen ions are at face centers, Ba\(^{+2}\) ions are at cube corners and Ti\(^{+4}\) is at cube center in cubic BaTiO\(_3\).

(b) In tetragonal BaTiO\(_3\), the Ti\(^{+4}\) is off-center and the unit cell has a net polarization.
Different polymorphs of BaTiO$_3$ and accompanying changes in lattice constants and dielectric constants.

Table 7.3 Contribution to dipole moment of a BaTiO$_3$ unit cell by each type of ion.

<table>
<thead>
<tr>
<th>Ion</th>
<th>Charge (C)</th>
<th>Displacement (m)</th>
<th>Dipole moment (C.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ba$^{2+}$</td>
<td>(+2)(1.6 x 10$^{-19}$)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ti$^{4+}$</td>
<td>(+4)(1.6 x 10$^{-19}$)</td>
<td>+0.10(10$^{-10}$)</td>
<td>6.4 x 10$^{-30}$</td>
</tr>
<tr>
<td>O$_2^-$ (side of cell)</td>
<td>2(-2)(1.6 x 10$^{-19}$)</td>
<td>-0.10(10$^{-10}$)</td>
<td>6.4 x 10$^{-30}$</td>
</tr>
<tr>
<td>O$_2^-$ (top and bottom of cell)</td>
<td>(-2)(1.6 x 10$^{-19}$)</td>
<td>-0.13(10$^{-10}$)</td>
<td>4.2 x 10$^{-30}$</td>
</tr>
</tbody>
</table>
| Total = 17 x 10$^{-30}$

$$P = \frac{17 \times 10^{-30} \text{ C.m}}{4.03 \times 3.98^2 \times 10^{-30} \text{ m}^3} = 0.27 \text{ C.m}^2$$
The effect of temperature and grain size on the dielectric constant of barium titanate. Above the Curie temperature, the spontaneous polarization is lost due to a change in crystal structure and barium titanate is in the paraelectric state. The grain size dependence shows that similar to yield-strength dielectric constant is a microstructure sensitive property.
Ferroelectric - A material that shows spontaneous and reversible dielectric polarization.

Piezoelectric – A material that develops voltage upon the application of a stress and develops strain when an electric field is applied.

Depoling
Piezoelectric aging rate $r$

$$\frac{u_2 - u_1}{u_1} = r \log \frac{t_2}{t_1},$$

$u$: parameter such as capacitance
$t$: number of days after polarization
The (a) direct and (b) converse piezoelectric effect.

In the direct piezoelectric effect, applied stress causes a voltage to appear. In the converse effect (b), an applied voltage leads to development of strain.
Direct piezoelectric effect

\[ P = d \sigma \]

\[ \partial P = d \sigma = d \frac{\partial}{\partial \sigma} \]

\[ d = \varepsilon_o \frac{\partial \kappa}{\partial \sigma} \]

\( d \): Piezoelectric coupling coefficient (piezoelectric charge coefficient)

### Table 7.1 The piezoelectric constant \( d \) (longitudinal) for selected materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Piezoelectric constant ( d ) (C/N = m/V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>2.3 x 10^{-12}</td>
</tr>
<tr>
<td>BaTiO(_3)</td>
<td>100 x 10^{-12}</td>
</tr>
<tr>
<td>PbZrTiO(_6)</td>
<td>250 x 10^{-12}</td>
</tr>
<tr>
<td>PbNb(_2)O(_6)</td>
<td>80 x 10^{-12}</td>
</tr>
</tbody>
</table>

\[ P = D_m - D_o = \kappa \varepsilon_o \Sigma - \varepsilon_o \Sigma = (\kappa - 1) \varepsilon_o \Sigma \]

\[ \partial \Sigma = \frac{\partial P}{\varepsilon_o (\kappa - 1)} = d \sigma \]

\[ \partial V = \ell \partial \Sigma \]

\[ \partial V = \frac{\ell d \partial \sigma}{\varepsilon_o (\kappa - 1)} \]

\( g \): piezoelectric voltage coefficient

\[ g = \frac{d}{(\kappa - 1) \varepsilon_o} \]

\[ \partial V = \ell g \partial \sigma \]
Reverse piezoelectric effect

\[ S = d\Sigma \]

\[ \dfrac{\Sigma}{\sigma} = \dfrac{S}{(\kappa - 1)\varepsilon_o \Sigma} \]

\[ \dfrac{\partial \Sigma}{\partial \sigma} = \dfrac{\partial S}{(\kappa - 1)\varepsilon_o \partial \Sigma} \]

\[ \dfrac{S}{\sigma} = \dfrac{d}{(\kappa - 1)\varepsilon_o \Sigma} \]

\[ \dfrac{\partial \Sigma}{\partial \sigma} = \dfrac{d}{(\kappa - 1)\varepsilon_o} \]
\[
\frac{\Sigma}{\sigma} = \frac{d}{(\kappa - 1)\varepsilon_0}
\]

\[g = \frac{d}{(\kappa - 1)\varepsilon_0}\]

\[\Sigma = g\sigma\]

\[\partial\Sigma = g\partial\sigma\]

**Hooke’s law**

\[\sigma = ES\]

\[\Sigma = g\sigma\]

\[\Sigma = gES\]

**Electromechanical coupling factor** (electromechanical coupling coefficient) \(k\)

\[k^2 = \frac{\text{output mechanical energy}}{\text{input electrical energy}}\]

\[k^2 = \frac{\text{output electrical energy}}{\text{input mechanical energy}}\]

\[S = d\Sigma\]

\[S = \frac{\Sigma}{gE}\]

\[d = \frac{1}{gE}\]

\[E = \frac{1}{gd}\]
Table 7.4 Properties of commercial PZT ceramics

<table>
<thead>
<tr>
<th>Property</th>
<th>PZT-5H (soft)</th>
<th>PZT4 (hard)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permittivity (κ at 1 kHz)</td>
<td>3400</td>
<td>1300</td>
</tr>
<tr>
<td>Dielectric loss (tan δ at 1 kHz)</td>
<td>0.02</td>
<td>0.004</td>
</tr>
<tr>
<td>Curie temperature (Tc, °C)</td>
<td>193</td>
<td>328</td>
</tr>
<tr>
<td>Piezoelectric coefficients (10⁻¹² m/V)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d₁₃</td>
<td>593</td>
<td>289</td>
</tr>
<tr>
<td>d₃₁</td>
<td>-274</td>
<td>-123</td>
</tr>
<tr>
<td>d₁₅</td>
<td>741</td>
<td>496</td>
</tr>
<tr>
<td>Piezoelectric coupling factors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>k₃₃</td>
<td>0.752</td>
<td>0.70</td>
</tr>
<tr>
<td>k₃₁</td>
<td>-0.388</td>
<td>-0.334</td>
</tr>
<tr>
<td>k₁₄</td>
<td>0.675</td>
<td>0.71</td>
</tr>
</tbody>
</table>
Table 7.2 Measured longitudinal piezoelectric coupling coefficient $d$, measured relative dielectric constant $\kappa$, calculated piezoelectric voltage coefficient $g$ and calculated voltage change resulting from a stress change of 1 kPa for a specimen thickness of 1 cm in the direction of polarization.

<table>
<thead>
<tr>
<th>Material</th>
<th>$d$ ($10^{-13}$ m/V)$^*$</th>
<th>$\kappa$</th>
<th>$g$ ($10^4$ m$^2$/C)$^+$</th>
<th>Voltage change (mV)$^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement paste (plain)</td>
<td>0.659 ± 0.031</td>
<td>35</td>
<td>2.2</td>
<td>2.2</td>
</tr>
<tr>
<td>Cement paste with steel fibers</td>
<td>208 ± 16</td>
<td>2700</td>
<td>8.7</td>
<td>8.7</td>
</tr>
<tr>
<td>and PVA</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cement paste with carbon fibers</td>
<td>3.62 ± 0.40</td>
<td>49</td>
<td>8.5</td>
<td>8.5</td>
</tr>
<tr>
<td>PZT</td>
<td>136</td>
<td>1024</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

---

**Bimorph (bi-strip)**

- Deflection
- Extension

Cantilever beam configuration for actuation

**Piezopolymer**

**Moonie**

**Cymbal**

Composites with piezoelectric/ferroelectric material sandwiched by metal faceplates for enhancing the piezoelectric coupling coefficient
**Pyroelectric** - The ability of a material to spontaneously polarize and produce a voltage due to changes in temperature.

\[
p = \frac{dP}{dT} = \varepsilon_0 \sum \frac{d\kappa}{dT},
\]

\( p = \text{pyroelectric coefficient} \)

\( P = \text{polarization} \)

---

Table 7.5 Pyroelectric coefficient (10^{-6} \text{ C/m}^2\text{K})

<table>
<thead>
<tr>
<th>Material</th>
<th>Coefficient (10^{-6} \text{ C/m}^2\text{K})</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaTiO_3</td>
<td>20</td>
</tr>
<tr>
<td>PZT</td>
<td>380</td>
</tr>
<tr>
<td>PVDF</td>
<td>27</td>
</tr>
<tr>
<td>Cement paste</td>
<td>0.002</td>
</tr>
</tbody>
</table>

\[
V = \frac{P x}{(\kappa-1) \varepsilon_0}
\]

**Voltage sensitivity**

\[
\frac{dV}{d\sigma} = \frac{P}{(\kappa-1) \varepsilon_0} \frac{dx}{d\sigma} + \frac{x}{(\kappa-1) \varepsilon_0} \frac{dP}{d\sigma}
\]

**Compliance**  
**Piezoelectric coupling coefficient d**
When any material undergoes polarization (due to an applied electric field), its ions and electronic clouds are displaced, causing the development of a mechanical strain in the material. This phenomenon is known as the electrostriction.