Electrical behavior

Topic 3

Supplementary reading

Shackelford, Materials Science for Engineers, 6th Ed., Ch. 15.

<table>
<thead>
<tr>
<th>Conducting range</th>
<th>Material</th>
<th>Conductivity, $\sigma$ (S$^{-1}$ m$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conductors</td>
<td>Aluminum (annealed)</td>
<td>$35.36 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>Copper (annealed standard)</td>
<td>$58.00 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>Iron (99.99% purity)</td>
<td>$10.30 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>Steel (wire)</td>
<td>$5.71 - 9.35 \times 10^6$</td>
</tr>
<tr>
<td>Semiconductors</td>
<td>Germanium (high purity)</td>
<td>$2.0$</td>
</tr>
<tr>
<td></td>
<td>Silicon (high purity)</td>
<td>$0.40 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>Lead sulfide (high purity)</td>
<td>$384$</td>
</tr>
<tr>
<td>Insulators</td>
<td>Aluminum oxide</td>
<td>$10^{-10} \text{ to } 10^{-12}$</td>
</tr>
<tr>
<td></td>
<td>Borosilicate glass</td>
<td>$10^{-12}$</td>
</tr>
<tr>
<td></td>
<td>Polyethylene</td>
<td>$10^{-12} \text{ to } 10^{-15}$</td>
</tr>
<tr>
<td></td>
<td>Nylon 66</td>
<td>$10^{-17} \text{ to } 10^{-19}$</td>
</tr>
</tbody>
</table>


Reading assignment

- Chung, Multifunctional cement-based Materials, Ch. 2.

![Graph of electrical conductivity](image-url)
Figure 18.2 (a) Charge carriers, such as electrons, are deflected by atoms or defects and take an irregular path through a conductor. The average rate at which the carriers move is the drift velocity \( v \). (b) Valence electrons in the metallic bond move easily. (c) Covalent bonds must be broken in semiconductors and insulators for an electron to be able to move. (d) Entire ions must diffuse to carry charge in many ioniically bonded materials.

Mean free path –
The average distance that electrons can move without being scattered by other atoms.
The current flowing through per unit cross-sectional area.

**Current**

\[ I = \frac{\text{charge}}{\text{time}} \]

1 ampere \( = \frac{1 \text{ coulomb}}{\text{sec}} \).

**Current density**

\[ \bar{J} = \frac{I}{A} \]
**Electrical resistance**

\[ R \propto \frac{\ell}{A} \]

\[ R = \frac{1}{\sigma} \frac{\ell}{A} \]

where \( \sigma \) is the electrical conductivity

\[ R = \frac{V}{I} \]

\[ \sigma = \frac{\ell}{A V} = \frac{(l/A)}{(V/\ell)} \]

\[ \sigma = \frac{\tilde{j}}{(V/\ell)} \]

**Electric field**

The voltage gradient or volts per unit length.

\[ \frac{dV}{dx} = -\frac{V}{\ell} \]

Electric field

\[ E = -\frac{dV}{dx} \]

\[ V = -\frac{dV}{\ell dx} = E \]

\[ \sigma = \frac{\tilde{j}}{E} \]

**Drift velocity**

\[ v \propto E \]

\[ v = \mu E \]

where \( \mu \) is the mobility
• Drift velocity - The average rate at which electrons or other charge carriers move through a material under the influence of an electric or magnetic field.
• Mobility - The ease with which a charge carrier moves through a material.

\[
I = qnvA
\]

\[
\tilde{J} = \frac{I}{A} = \frac{qnvA}{A} = qnv
\]

\[
\sigma = \frac{\tilde{J}}{E} = \frac{qnv}{E}
\]

\[
\frac{v}{E} = \mu
\]

\[
\sigma = q n \mu
\]

Flux
\[
J_n = -D_n \frac{dn}{dx}
\]

Current density
\[
\tilde{J}_n = (-q) J_n
\]

\[
\tilde{J}_n = (-q) \left( -D_n \frac{dn}{dx} \right)
\]

\[
= q D_n \frac{dn}{dx}
\]

Flux
\[
J_p = -D_p \frac{dp}{dx}
\]

Current density
\[
\tilde{J}_p = q J_p = q \left( -D_p \frac{dp}{dx} \right) = -q D_p \frac{dp}{dx}
\]
Einstein relationship

\[
\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{q}
\]

Energy levels of an isolated atom

\[
\begin{align*}
3s & \underline{\quad} \\
2p & \underline{\quad} \\
2s & \underline{\quad} \\
1s & \underline{\quad}
\end{align*}
\]

Isolated Na atom

Hypothetical Na₄ molecule

Electrons

\[
\begin{align*}
3s & \underline{2N \text{ electrons}} \\
2p & \underline{6N \text{ electrons}} \\
2s & \underline{2N \text{ electrons}} \\
1s & \underline{2N \text{ electrons}}
\end{align*}
\]

1 atom \quad 2 atoms \quad N atoms
Energy bands of solid sodium

• Valence band - The energy levels filled by electrons in their lowest energy states.
• Conduction band - The unfilled energy levels into which electrons can be excited to provide conductivity.
• Energy gap (Bandgap) - The energy between the top of the valence band and the bottom of the conduction band that a charge carrier must obtain before it can transfer a charge.
Figure 15.5
The Fermi function, \( f(E) \), describes the relative filling of energy levels. At 0 K, all energy levels are completely filled up to the Fermi level, \( E_F \), and are completely empty above \( E_F \).

Figure 15.6
At \( T > 0 \) K, the Fermi function, \( f(E) \), indicates promotion of some electrons above \( E_F \).

Figure 15.7
Variation of the Fermi function, \( f(E) \), with the temperature for a typical metal with \( E_F \approx 5 \) eV. Note that the energy range over which \( f(E) \) drops from 1 to 0 is equal to a few times \( kT \).

Figure 15.8
Comparison of the Fermi function, \( f(E) \), with the energy band structure for an insulator: Virtually no electrons are promoted to the conduction band if \( f(E) < 1 \) because of the magnitude of the band gap (\( > 2 \) eV).

Figure 15.9
Comparison of the Fermi function, \( f(E) \), with the energy band structure for a semiconductor: A significant number of electrons is promoted to the conduction band because of a relatively small band gap (\( < 2 \) eV). Each electron promotion creates a pair of charge carriers (i.e., an electron-hole pair).
• Holes are in the valence band.
• Conduction electrons are in the conduction band.

Holes - Unfilled energy levels in the valence band. Because electrons move to fill these holes, the holes move and produce a current.

Radiative recombination - Recombination of holes and electrons that leads to emission of light; this occurs in direct bandgap materials.

Electrical conduction through a composite material consisting of three components (1, 2 and 3) that are in a parallel configuration.
Resistance due to component $i$

$$R_i = \frac{\rho_i \ell}{A_i}$$

Current through component $i$

$$I_i = \frac{VA_i}{\rho_i \ell}$$

Total current through the composite

$$I = I_1 + I_2 + I_3 = \frac{V}{\ell} \left( \frac{A_1}{\rho_1} + \frac{A_2}{\rho_2} + \frac{A_3}{\rho_3} \right)$$

Total resistance

$$R = \frac{\ell}{\rho || (A_1 + A_2 + A_3)}$$

Total current

$$I = \frac{V}{R} = \frac{V (A_1 + A_2 + A_3)}{\rho || \ell}$$

$$\frac{V}{\ell} \left( \frac{A_1}{\rho_1} + \frac{A_2}{\rho_2} + \frac{A_3}{\rho_3} \right) = \frac{V}{\rho || \ell} (A_1 + A_2 + A_3)$$

Rule of Mixtures

$$\frac{1}{\rho ||} = \frac{1}{\rho_1 (A_1 + A_2 + A_3)} + \frac{1}{\rho_2 (A_1 + A_2 + A_3)} + \frac{1}{\rho_3 (A_1 + A_2 + A_3)}$$

$$= \frac{1}{\rho_1} f_1 + \frac{1}{\rho_2} f_2 + \frac{1}{\rho_3} f_3,$$

Electrical conduction through a composite material consisting of three components (1, 2 and 3) that are in a series configuration

$$V_i = IR_i = \frac{L_i}{A} I \rho_1$$

Total voltage drop

$$V = \frac{I}{A} \left( \rho_1 L_1 + \rho_2 L_2 + \rho_3 L_3 \right)$$

Total resistance

$$R = \rho_\perp \left( \frac{L_1 + L_2 + L_3}{A} \right)$$

Total voltage drop

$$V = IR = \frac{I \rho_\perp}{A} \left( \frac{L_1 + L_2 + L_3}{A} \right)$$
Total voltage drop

\[ V = \frac{I}{A} \left( \rho_1 L_1 + \rho_2 L_2 + \rho_3 L_3 \right) \]

\[ V = IR \]

\[ = I \rho \perp \frac{L_1 + L_2 + L_3}{A} \]

Conduction through a composite material with an insulating matrix and short conductive fibers

\[ \rho \perp = \frac{\left( \rho_1 L_1 + \rho_2 L_2 + \rho_3 L_3 \right)}{\left( L_1 + L_2 + L_3 \right)} \]

\[ = \perp_1 f_1 + \perp_2 f_2 + \perp_3 f_3 \]

Rule of Mixtures

Percolation threshold

Minimum volume fraction of conductive fibers (or particles) for adjacent fibers (or particles) to touch each other and form a continuous conductive path.
Conduction through an interface

Contact resistance

\[ \frac{1}{A} \]

\[ \frac{\rho_c}{A} \]

where \( \rho_c \) is the contact resistivity

Energy bands of an intrinsic semiconductor

Electrical conductivity of a semiconductor

\[ \sigma = q n \mu_n + q p \mu_p \]

where
- \( q \) = magnitude of the charge of an electron,
- \( n \) = number of conduction electrons per unit volume,
- \( p \) = number of holes per unit volume,
- \( \mu_n \) = mobility of conduction electrons,
- \( \mu_p \) = mobility of conduction holes.

Intrinsic silicon

For an intrinsic semiconductor (\( n = p \)),

\[ \sigma = q n (\mu_n + \mu_p) \]

Without thermal excitation

With thermal excitation
Current density due to both an electric field and a concentration gradient

\[ \tilde{J}_n = qn \mu_n E + qD_n \frac{dn}{dx} \]

\[ \tilde{J}_p = qp \mu_p E - qD_p \frac{dp}{dx} \]

\[ \tilde{J} = \tilde{J}_n + \tilde{J}_p \]

**Extrinsic semiconductor** (doped with an electron donor)

- Without thermal excitation
- With thermal excitation

**Energy bands**

- Intrinsic semiconductor
- Extrinsic semiconductor (doped with an electron donor)

**Extrinsic semiconductor** (doped with an electron acceptor)

- Without thermal excitation
- With thermal excitation

**Intrinsic semiconductor** - A semiconductor in which properties are controlled by the element or compound that makes the semiconductor and not by dopants or impurities.

**Extrinsic semiconductor** - A semiconductor prepared by adding dopants, which determine the number and type of charge carriers.

**Doping** - Deliberate addition of controlled amounts of other elements to increase the number of charge carriers in a semiconductor.
Intrinsic semiconductor

Extrinsic semiconductor (doped with an electron acceptor)

Defect semiconductor (excess semiconductor Zn_{1+x}O)

Zn^{+} ion serves as an electron donor.

Defect semiconductor (deficit semiconductor Ni_{1-x}O)

Ni^{3+} ion serves as an electron acceptor.

TABLE 18.7 The donor and acceptor energy gaps (in electron volts) when silicon and germanium semiconductors are doped

<table>
<thead>
<tr>
<th>Dopant</th>
<th>Silicon</th>
<th>Germanium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_d$</td>
<td>$E_a$</td>
</tr>
<tr>
<td>P</td>
<td>0.045</td>
<td>0.0120</td>
</tr>
<tr>
<td>As</td>
<td>0.049</td>
<td>0.0127</td>
</tr>
<tr>
<td>Sb</td>
<td>0.039</td>
<td>0.0096</td>
</tr>
<tr>
<td>B</td>
<td>0.045</td>
<td></td>
</tr>
<tr>
<td>Al</td>
<td>0.057</td>
<td></td>
</tr>
<tr>
<td>Ga</td>
<td>0.065</td>
<td></td>
</tr>
<tr>
<td>In</td>
<td>0.160</td>
<td></td>
</tr>
</tbody>
</table>
Energy bands of Ni_{1-x}O

Consider an n-type semiconductor being illuminated.

Illumination increases conduction electrons and holes by equal number, since electrons and holes are generated in pairs.

Thus, the minority carrier concentration (p_n) is affected much more than the majority carrier concentration (n_n).

### Table 18-6: Properties of commonly encountered semiconductors

<table>
<thead>
<tr>
<th>Semiconductor</th>
<th>Bandgap (eV)</th>
<th>Mobility of Electrons ((\mu_n))</th>
<th>Mobility of Holes ((\mu_p))</th>
<th>Dielectric Constant ((\varepsilon))</th>
<th>Resistivity ((\Omega\cdot\text{cm}))</th>
<th>Density (g/cm³)</th>
<th>Melting Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon (Si)</td>
<td>1.11</td>
<td>1360</td>
<td>480</td>
<td>11.8</td>
<td>25 \times 10³</td>
<td>2.33</td>
<td>1415</td>
</tr>
<tr>
<td>Germanium (Ge)</td>
<td>0.67</td>
<td>3000</td>
<td>3000</td>
<td>16.0</td>
<td>40</td>
<td>5.32</td>
<td>996</td>
</tr>
<tr>
<td>Gallium Arsenide (GaAs)</td>
<td>0.71</td>
<td>5000</td>
<td>5000</td>
<td>12.2</td>
<td>4 \times 10³</td>
<td>5.31</td>
<td>1228</td>
</tr>
<tr>
<td>Diamond</td>
<td>&gt;5.50</td>
<td>1800</td>
<td>1500</td>
<td>5.7</td>
<td>10^3</td>
<td>3.52</td>
<td>&gt;4000</td>
</tr>
<tr>
<td>SiC</td>
<td>2.00</td>
<td>3000</td>
<td>3000</td>
<td>10.4</td>
<td>5.80</td>
<td>230</td>
<td></td>
</tr>
</tbody>
</table>

Source: Data from C. A. Harper, Ed., *Handbook of Materials and Processes for Electronics*, McGraw-Hill Book Company, NY, 1979. This value is above our upper limit of 2 eV used to define a semiconductor. Such a limit is somewhat arbitrary. In addition, most commercial devices involve impurity levels that substantially change the nature of the band gap (see Chapter 17).
- Temperature Effect - When the temperature of a metal increases, thermal energy causes the atoms to vibrate
- Effect of Atomic Level Defects - Imperfections in crystal structures scatter electrons, reducing the mobility and conductivity of the metal

\[ \Delta\rho = \alpha \Delta T \]

where \( \alpha \) = temperature coefficient of electrical resistivity
Matthiessen’s rule –
The resistivity of a metallic material is given by the addition of a base resistivity that accounts for the effect of temperature, and a temperature independent term that reflects the effect of atomic level defects, including impurities forming solid solutions.
Effect of Processing and Strengthening

For a metal, $s$ decreases with increasing temperature because $\mu$ decreases with increasing temperature.

For a semiconductor, $s$ increases with increasing temperature because $n$ and/or $p$ increases with increasing temperature.
For a semiconductor

\[ n \propto e^{-E_g/2kT}, \]

where \( E_g \) = energy band gap between conduction and valence bands,
\( k \) = Boltzmann’s constant,
\( T \) = temperature in K.

The factor of 2 in the exponent is because the excitation of an electron across \( E_g \) produces an intrinsic conduction electron and an intrinsic hole.

Taking natural logarithms,

\[ s = s_o e^{-E_g/2kT}. \]
\[ \ln s = \ln s_o - \frac{E_g}{2kT}. \]

Changing the natural logarithms to logarithms of base 10,

\[ \log s = \log s_o - \frac{E_g}{(2.3)2kT}. \]

Thermistor –

A semiconductor device that is particularly sensitive to changes in temperature, permitting it to serve as an accurate measure of temperature.

Conductivity of an ionic solid

\[ \sigma = q n \mu_C + q n \mu_A = q n (\mu_C + \mu_A), \]

where \( n \) = number of Schottky defects per unit volume
\( \mu_C \) = mobility of cations,
\( \mu_A \) = mobility of anions.

An n-type semiconductor

\[ n = n_i + n_e, \]

where \( n \) = total concentration of conduction electrons,
\( n_i \) = concentration of intrinsic conduction electrons,
\( n_e \) = concentration of extrinsic conduction electrons.

\[ D \rightarrow D^+ + e^-, \]
\[ n_e = N_D +, \]
\[ n_i \propto e^{-E_g/2kT}, \]
\[ n_e \propto e^{-E_D/kT}, \]
\[ n_i < < n_e. \]
\[ p = p_i. \]
Before donor exhaustion
\[ n_i \ll n_e \]

No extrinsic holes, thus
\[ p = p_i \]

However,
\[ p_i = n_i \]

Thus,
\[ p = n_i \]

At high temperatures (i.e., donor exhaustion),
\[ n \cong n_i \]

\[ n \cong n_e \]
\[ p \cong 0 \]
\[ \sigma = qn \mu_n + qp \mu_p \]
\[ \sigma \cong qn \mu_n \]

Arrhenius plot of log conductivity vs. 1/T, where T is temperature in K.

Extrinsic semiconductor (doped with an electron donor)
A p-type semiconductor

\[ p = p_i + p_e \]

where \( p \) = total concentration of conduction holes
\( p_i \) = concentration of intrinsic holes,
\( p_e \) = concentration of extrinsic holes.

\[ A + e^- \rightarrow A^- \]
\[ A \rightarrow A^- + h^+ \]
\( p_e = N_{A^-} \)
\( p_i \propto e^{-E_g/2kT} \)
\( p_e \propto e^{-E_A/kT} \)

\[ p_i \ll p_e \]
before acceptor saturation

\[ n = n_i \]
\[ n = p_i \]
\[ p \equiv p_e \]
\[ n \equiv 0 \]
before acceptor saturation

The mass-action law
Product of \( n \) and \( p \) is a constant for a particular semiconductor at a particular temperature
Intrinsic semiconductor
\[ n = n_i = p_i = p \, . \]
\[ np = n_i^2 \, . \]
\[ n_i = 1.5 \times 10^{10} \text{ cm}^{-3} \text{ for Si} \]
\[ n_i = 2.5 \times 10^{13} \text{ cm}^{-3} \text{ for Ge} \, . \]

This equation applies whether the semiconductor is doped or not.

Consider an n-type semiconductor.
\[ n \equiv n_e = N_{D+} \]
\[ N_{D+} = N_D \text{ (Donor exhaustion)} \]
\[ n \equiv N_D \, . \]
\[ p = \frac{n_i^2}{n} = \frac{n_i^2}{N_D} \, . \]

The pn junction

Rectification

A pn junction at bias voltage \( V = 0 \)
• Diodes, transistors, lasers, and LEDs are made using semiconductors. Silicon is the workhorse of very large scale integrated (VLSI) circuits.

• Forward bias - Connecting a p-n junction device so that the p-side is connected to positive. Enhanced diffusion occurs as the energy barrier is lowered, permitting a considerable amount of current can flow under forward bias.

• Reverse bias - Connecting a junction device so that the p-side is connected to a negative terminal; very little current flows through a p-n junction under reverse bias.

• Avalanche breakdown - The reverse-bias voltage that causes a large current flow in a lightly doped p-n junction.

• Transistor - A semiconductor device that can be used to amplify electrical signals.

• Superconductivity - Flow of current through a material that has no resistance to that flow.

• Applications of Superconductors - Electronic circuits have also been built using superconductors and powerful superconducting electromagnets are used in magnetic resonance imaging (MRI). Also, very low electrical-loss components, known as filters, based on ceramic superconductors have been developed for wireless communications.
Figure 15.18
The resistivity of YBa$_2$Cu$_3$O$_7$ as a function of temperature, indicating a $T_c = 95$ K.

Figure 15.19
Unit cell of YBa$_2$Cu$_3$O$_7$. It is roughly equivalent to three distorted perovskite unit cells of the type shown in Figure 13.14.