FORCES

SURFACE FORCES: Forces in the fluid which act on a solid or imaginary surface. These forces are specified in terms of an intensity or stress(force per unit area).

Local: Pressure
Shear (friction) stress
Total or Resultant: Lift, Drag, etc.

BODY FORCES: Forces that act on the mass of the fluid. We will only deal with gravity. Body forces are given in terms of force per unit volume.

γ = Specific Weight = Weight/Volume
= lbs/cubic feet or newtons/cubic m

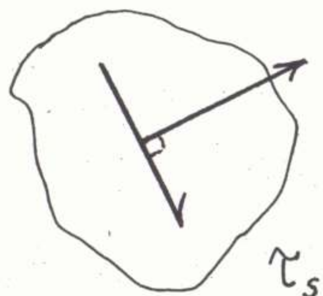
Force = γ x volume

ρ = Density = mass/volume
= slugs/cubic ft or kgs/cubic m

$\gamma = \rho g$

STRESS

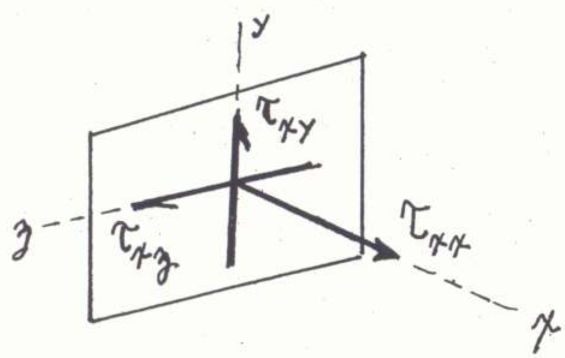
STRESS: Force per unit area of surface – N/m**2.
Can be normal and tangential components.



$$\tau_n = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A} = \text{Normal}$$

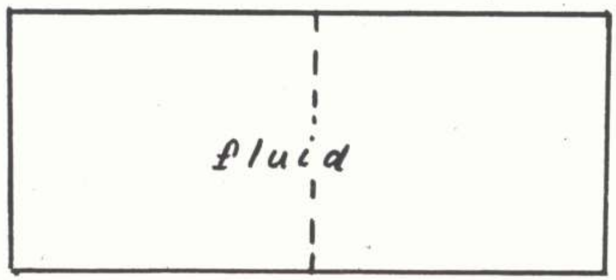
$$\tau_s = \lim_{\Delta A \to 0} \frac{\Delta F_s}{\Delta A} = \text{Shear}$$

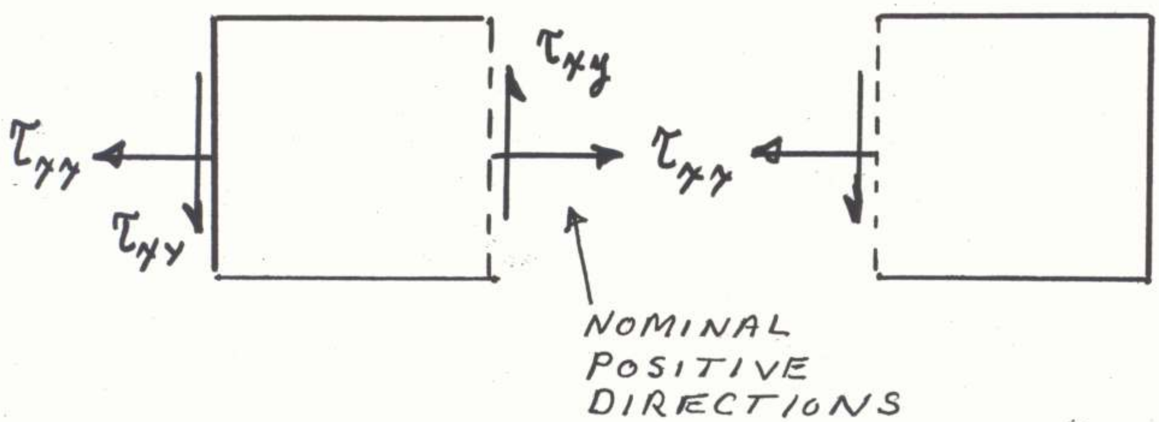
The normal component is unique. The tangent component requires an angle or can be given as two components.

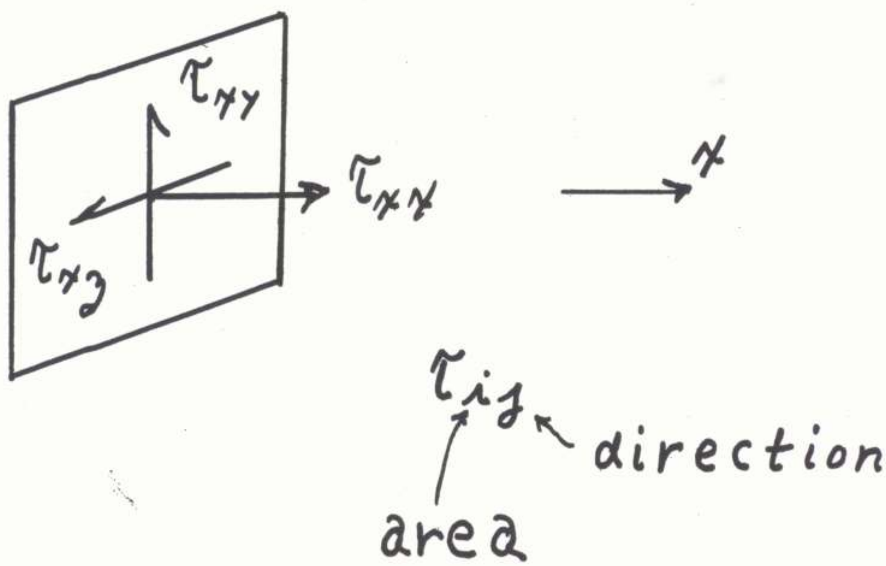


FOR A GIVEN ORIENTATION THERE ARE THREE INDEPENDENT COMPONENTS OF STRESS.

STRESS NOTATION

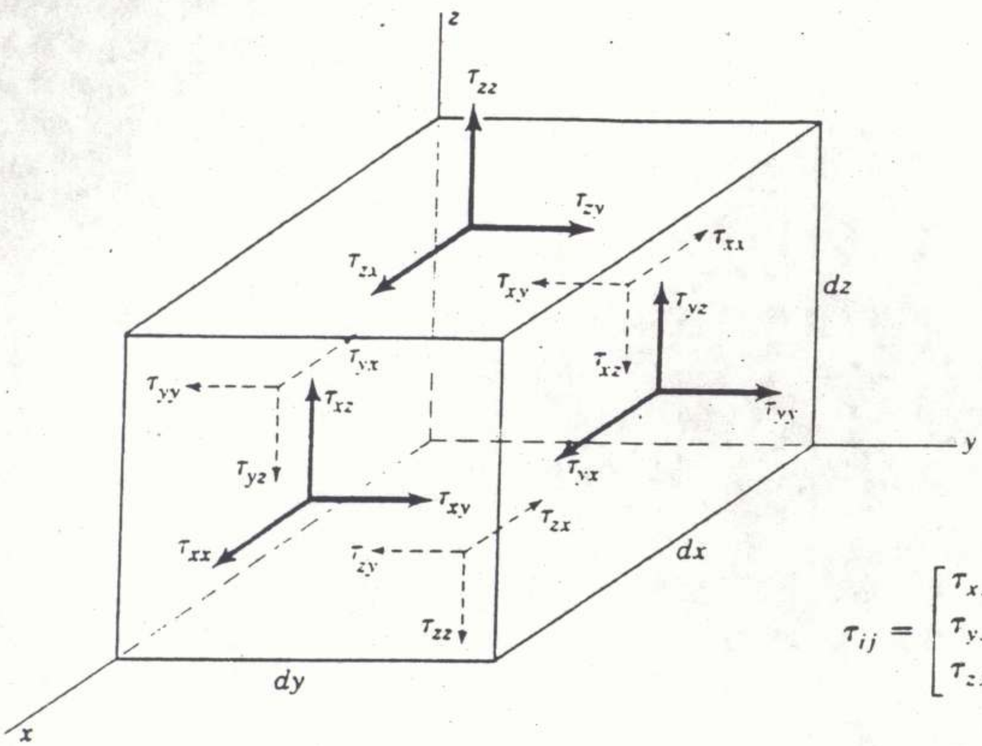






STRESS TENSOR

STATE OF STRESS: In general requires three components (normal and two shears) on each of three mutually perpendicular surfaces for a total of nine components.



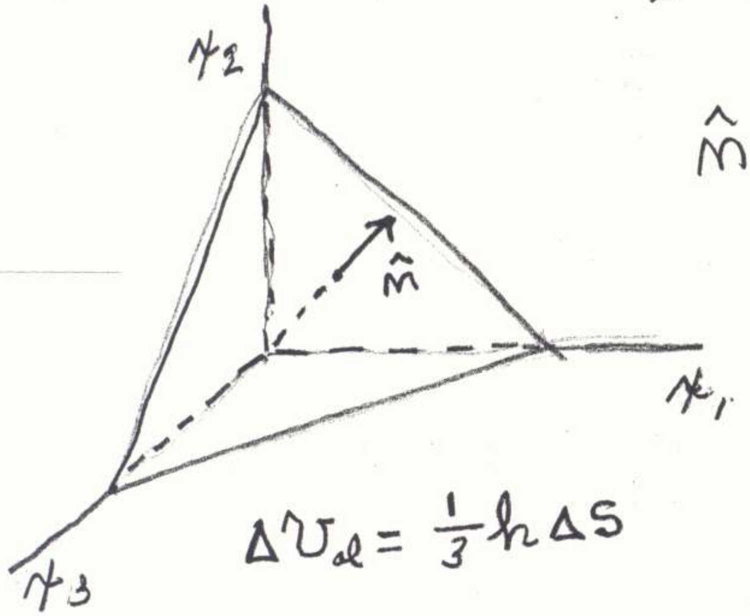
$$\tau_{ij} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix}$$

Stress is a Second Order Tensor, has nine components.

Vector: First Order Tensor, has three components.
Velocity: Vx, Vy, Vz

Scalar: Zero Order Tensor, has magnitude only.

Stress Vector $T_i^{(n)}$



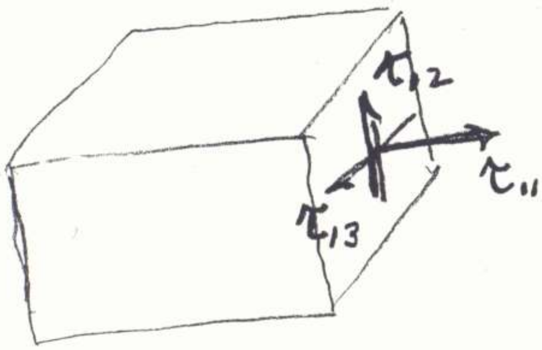
$$\hat{n} = n_1 \hat{i}_1 + n_2 \hat{i}_2 + n_3 \hat{i}_3$$

$$\Delta \vec{S} = \hat{n}\, \Delta S$$

$$\Delta S_1 = \Delta \vec{S} \cdot \hat{i}_1 = n_1 \Delta S$$

$$\Delta S_i = n_i \Delta S$$

$$\Delta V_{el} = \frac{1}{3} h \Delta S$$



$$\vec{T}^{(1)} = \sigma_{11} \hat{i}_1 + \sigma_{12} \hat{i}_2 + \sigma_{13} \hat{i}_3$$

$$\vec{T}^{(2)} =$$

$$\vec{T}^{(3)} =$$

$$\sum \vec{F} = \frac{D \vec{V}\, dm}{Dt} = \frac{D\vec{V}}{Dt} dm = \rho \vec{a} \Delta V$$

$$\vec{T}^{(n)} \Delta S - \vec{T}^{(1)} \Delta S_1 - \vec{T}^{(2)} \Delta S_2 - \vec{T}^{(3)} \Delta S_3 + \rho \Delta V \vec{g} = \rho \vec{a} \Delta V$$

$$\overrightarrow{\overrightarrow{T}}^{(M)} - \vec{T}^{(1)} m_1 - \vec{T}^{(2)} m_2 - \vec{T}^{(3)} m_3 + \frac{1}{3}\rho h \vec{g} = \frac{1}{3}\rho h \vec{a}$$

$h \to 0$

$$\overrightarrow{\overrightarrow{T}}^{(M)} = \vec{T}^{(1)} m_1 + \vec{T}^{(2)} m_2 + \vec{T}^{(3)} m_3 = \vec{T}^{(k)} m_k$$

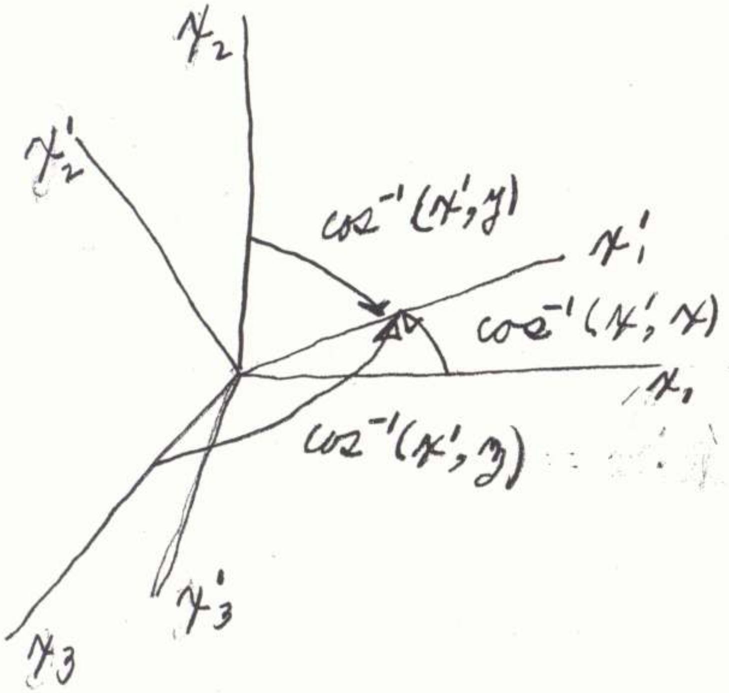
$$T_1^{(M)} = \sigma_{11} m_1 + \sigma_{21} m_2 + \sigma_{31} m_3$$

$$T_2^{(M)} = \sigma_{12} m_1 + \sigma_{22} m_2 + \sigma_{32} m_3$$

$$T_3^{(M)} = \sigma_{13} m_1 + \sigma_{23} m_2 + \sigma_{33} m_3$$

$$\boxed{T_i^{(M)} = \sigma_{ji} m_j}$$

Rotation of Coordinate axes



$$a_{ij} = \begin{pmatrix} \cos(x',x) & \cos(x',y) & \cos(x',z) \\ \cos(y',x) & & \\ & & \end{pmatrix}$$

$$= \begin{pmatrix} \hat{i}'\cdot\hat{i} & \hat{i}'\cdot\hat{j} & \hat{i}'\cdot\hat{k} \\ \hat{j}'\cdot\hat{i} & & \\ \hat{k}'\cdot\hat{i} & & \end{pmatrix}$$

$$x_1' = x_1 a_{11} + x_2 a_{12} + x_3 a_{13}$$

$$x_2' =$$

$$x_i' = a_{ij} x_j$$

$$T_i^{(n)} = n_j \sigma_{ji} \qquad T_i^{(x_1')} = a_{1j} \sigma_{ji}$$

$$T_i^{(x_1')} \cdot \hat{i}_1' = \tau_{x_1' x_1'} = a_{i1} a_{1j} \sigma_{ji} = \tau_{11}'$$

$$T_i^{(x_1')} \cdot \hat{i}_2' = \tau_{x_1' x_2'} = a_{i2} a_{1j} \sigma_{ji} = \tau_{12}'$$

$$T_i^{(x_1')} \cdot \hat{i}_3' = \tau_{x_1' x_3'} = a_{i3} a_{1j} \sigma_{ji} = \tau_{13}'$$

$$\tau_{kl}' = a_{kj} a_{il} \sigma_{ji}$$

Obeys Transformation Rule — ∴ Stress is a tensor

Stress Tensor

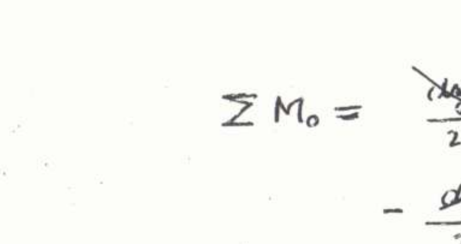
$$\tau_{ij} \Rightarrow \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix}$$

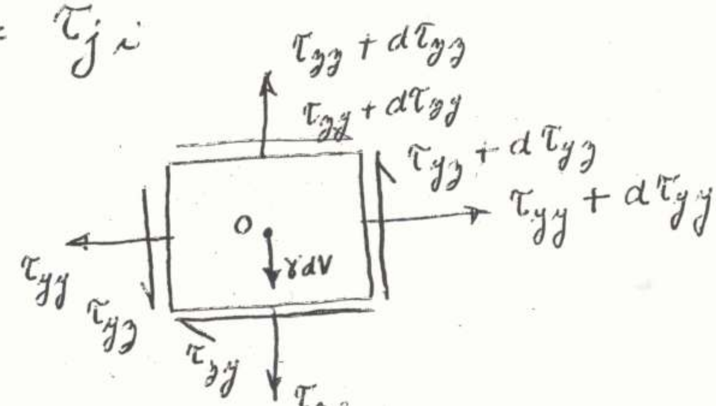
These stresses due not represent the average stresses on the representative block faces, but the local stress at a point.

It takes 9 quantities to specify the state of stress at a point, i.e., the three components on each of three mutually perpendicular planes.

$\Rightarrow$ The stress tensor is symmetric

$$\tau_{ij} = \tau_{ji}$$





$$\begin{aligned} \Sigma M_o = & \frac{dy}{2} \tau_{yz} \cancel{dx\, dz} + \frac{dy}{2} (\tau_{yz} + d\tau_{yz}) \cancel{dx\, dz} \\ & - \frac{dz}{2} \tau_{zy} \cancel{dx\, dy} - \frac{dz}{2} (\tau_{zy} + d\tau_{zy}) \cancel{dx\, dy} \\ = & \frac{1}{12} \rho (dy^2 + dz^2) \cancel{dx\, dy\, dz}\, \alpha \end{aligned}$$

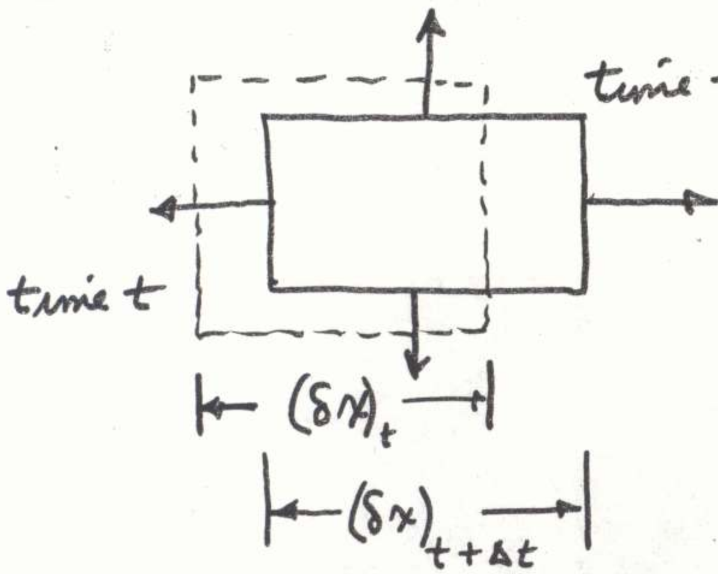
$\Sigma M_o \sim r^2 dm\, \alpha$

$$\tau_{yz} + \cancel{\tfrac{1}{2}\, d\tau_{yz}} - \tau_{zy} - \cancel{\tfrac{1}{2}\, d\tau_{zy}} = \cancel{\tfrac{1}{12}\, \rho \,(dy^2 + dx^2)\,\alpha}$$

$$\left.\begin{aligned} \therefore\ \tau_{yz} &= \tau_{zy} \\ \text{Also}\ \ \tau_{xy} &= \tau_{yx} \\ \tau_{xz} &= \tau_{zx} \end{aligned}\right\} \ \tau_{ij} = \tau_{ji}$$

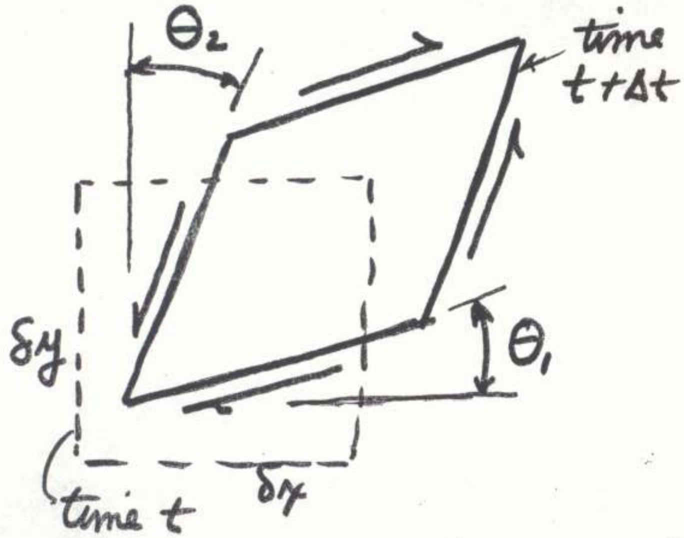
STRESS + STRAIN

NORMAL STRESS → ELONGATION



$$(\text{STRAIN})_x = \epsilon_{xx} = \frac{(\delta x)_{t+\Delta t} - (\delta x)_t}{(\delta x)_t}$$

SHEAR STRESS → CHANGE IN ANGLE

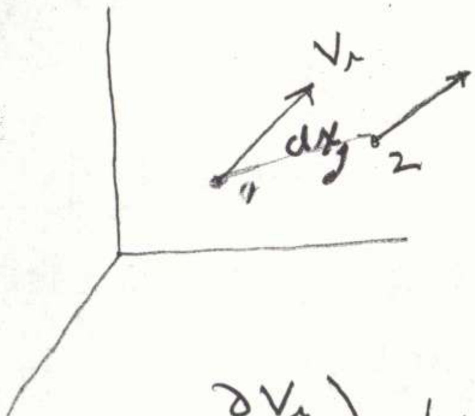


$$(\text{STRAIN})_{xy} = \epsilon_{xy} = \theta_1 + \theta_2$$

$$\tau_{xy} \propto \epsilon_{xy}$$

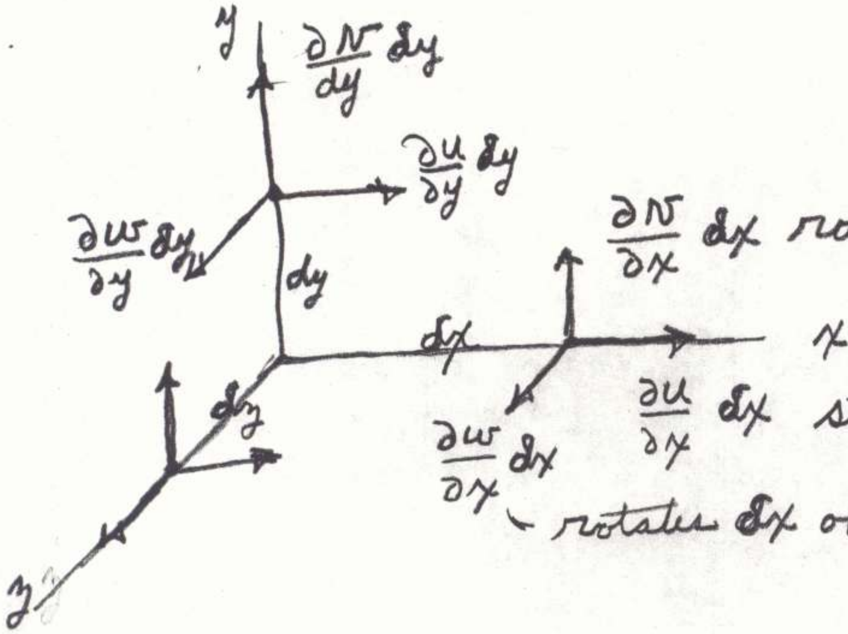
$$(\text{Rotation})_z = \frac{1}{2}(\theta_1 - \theta_2)$$

Strain



$$V_i + dV_i = V_i + \left.\frac{\partial V_i}{\partial x_j}\right)_1 dx_j$$

$\left.\frac{\partial V_i}{\partial x_j}\right)_1 dx_j$ is relative motion of point 2 with respect to 1.



$\frac{\partial v}{\partial x}dx$ rotates δx about z

$\frac{\partial u}{\partial x}dx$ stretches dx

rotates δx about y

Normal Strain

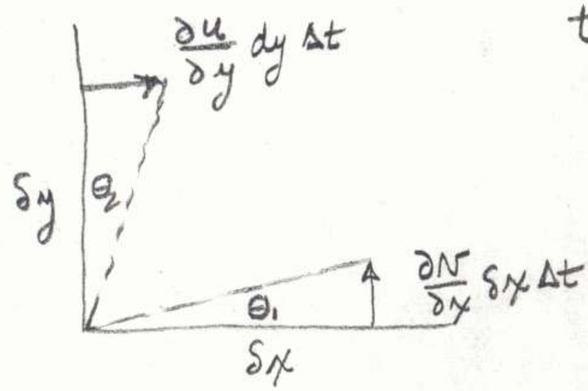
$$\epsilon_{xx} = \frac{(\delta x)_{t+\Delta t} - (\delta x)_t}{(\delta x)_t} = \frac{\frac{\partial u}{\partial x}\,\delta x\,\Delta t}{\delta x}$$

$$\dot{\epsilon}_{xx} = \frac{\partial u}{\partial x} \quad \text{strain rate}$$

$$\dot{\epsilon}_{yy} = \frac{\partial v}{\partial y} \qquad \dot{\epsilon}_{zz} = \frac{\partial w}{\partial z}$$

$$\text{Vol strain rate} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{\partial v_j}{\partial x_j}$$

Shear Strain



$$\tan\theta_1 \cong \theta_1 = \frac{\frac{\partial v}{\partial x}\,\delta x\,\Delta t}{\delta x}$$

$$\dot{\theta}_1 = \partial v/\partial x$$

$$\tan\theta_2 \cong \theta_2 = \frac{\frac{\partial u}{\partial y}\,\delta y\,\Delta t}{\delta y}$$

$$\dot{\theta}_2 = \frac{\partial u}{\partial y}$$

$$\text{Shear strain} = \theta_1 + \theta_2 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

$$\text{strain}_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

$$\text{rot}_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\dot{\epsilon}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) = \dot{\epsilon}_{ji}$$

$$\dot{\omega}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right) = -\dot{\omega}_{ji}$$

$$\dot{\omega}_{11} = \dot{\omega}_{22} = \dot{\omega}_{33} = 0$$

$$\frac{\partial v_i}{\partial x_j} = \dot{\epsilon}_{ij} + \dot{\omega}_{ij}$$

Stress-Strain

Newtonian fluid $\Rightarrow$ Stress linearly related to strain

$\tau_{ij} = C_{ijkl}\, \epsilon_{kl}$ 81 coefficients

ϵ_{kl} symmetric $C_{ijkl} = C_{ijlk}$

τ_{ij} symmetric $C_{ijkl} = C_{jikl}$

Fluid is isotropic = properties are independent of direction

C_{ijkl} = isotropic tensor

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

$$+ \gamma(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk})$$

$\gamma = 0$ to satisfy symmetries

$$\tau_{ij} = [\lambda \delta_{ij} \delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})]\, \epsilon_{kl}$$

$$= \lambda \epsilon_{kk} \delta_{ij} + \mu(\epsilon_{ij} + \epsilon_{ji})$$

$$= \lambda \frac{\partial v_k}{\partial x_k} \delta_{ij} + \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$$

$$\sigma_{ij} = -p\, \delta_{ij} + \tau_{ij}$$

$$\tau_{jj} = 0$$

$$\tau_{jj} = 3\lambda \frac{\partial v_k}{\partial x_k} + 2\mu \frac{\partial v_j}{\partial x_j} = 0 \Rightarrow \lambda = -\frac{2}{3}\mu$$

$$\therefore \quad \tau_{ij} = \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) - \frac{2}{3}\mu \frac{\partial v_k}{\partial x_k} \delta_{ij}$$

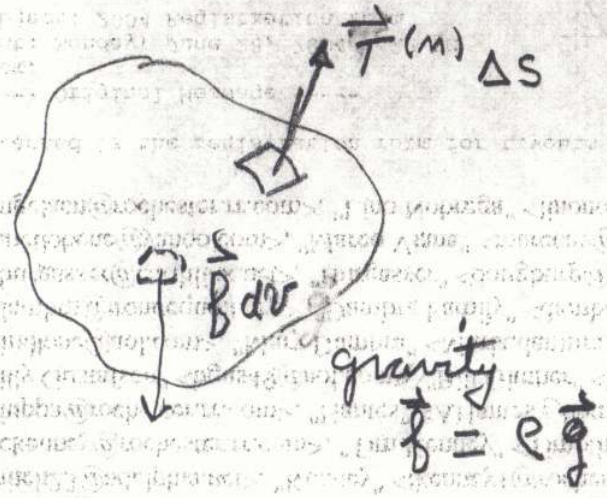
Momentum - Finite Volume

$$\sum \vec{F} = \frac{D}{Dt} \int \vec{V}\, dm$$

$$\frac{D}{Dt} \int \alpha\, dV = \frac{\partial}{\partial t} \int \alpha\, dv + \int \alpha \vec{V} \cdot \hat{n}\, dS$$

$$\alpha = \rho \vec{V}$$

$$\sum \vec{F} = \frac{\partial}{\partial t} \int \rho \vec{V}\, dv + \int \vec{V} \rho \vec{V} \cdot \hat{n}\, dS$$



$$\sum \vec{F} = \int \vec{T}^{(n)} dS + \int \vec{f}\, dV$$

Momentum – Differention

$$\sum \vec{F} = \frac{D}{Dt}\int \vec{v}\, dm$$

$$\frac{D}{Dt}\int \alpha\, dv = \int \frac{\partial \alpha}{\partial t}\, dv + \int \alpha\, \vec{v}\cdot\hat{n}\, dS$$

$$= \int \frac{\partial \alpha}{\partial t}\, dv + \int \nabla\cdot(\alpha\vec{v})\, dv$$

$$\alpha = \rho\vec{v}$$

$$\sum F_i = \int \overset{\sigma_{ji} n_j}{T_i^{(n)}}\, dS + \int f_i\, dv$$

$$= \int \frac{\partial \sigma_{ji}}{\partial x_j}\, dv + \int f_i\, dv$$

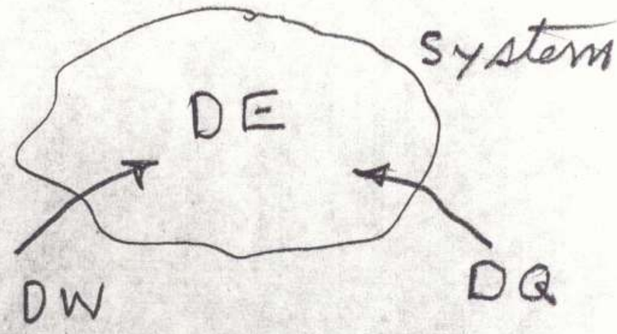
$$\int_V \left[\frac{\partial \rho v_i}{\partial t} + \frac{\partial}{\partial x_j}\rho v_i v_j - \frac{\partial \sigma_{ji}}{\partial x_j} - f_i \right] dv = 0$$

Since the volume is arbitrary, the integrand $= 0$

$$\frac{\partial \rho v_i}{\partial t} + \frac{\partial}{\partial x_j}\rho v_j v_i = \rho\frac{\partial v_i}{\partial t} + \rho v_j \frac{\partial v_i}{\partial x_j} = \rho\frac{D v_i}{Dt}$$

$$= \frac{\partial \sigma_{ji}}{\partial x_j} + f_i$$

Energy Equation



$$\frac{DE}{Dt} = \frac{\not{D}W}{Dt} + \frac{\not{D}Q}{Dt}$$

E = point function
W, Q = path functions

$$E = \int \Big(\underbrace{e}_{\text{internal}} + \underbrace{\frac{V^2}{2}}_{\text{kinetic}} \Big) \rho \, d\mathcal{V}$$

$$\not{D}W = \int f_i V_i \, dt \, d\mathcal{V} + \int T_i^{(n)} V_i \, dt \, dS$$

$$\frac{\not{D}W}{Dt} = \int f_i V_i \, d\mathcal{V} + \int \sigma_{ji} n_j V_i \, dS$$

$$= \int f_i V_i \, d\mathcal{V} + \int \frac{\partial \sigma_{ji} V_i}{\partial x_j} \, d\mathcal{V}$$

$$\frac{\not{D}Q}{Dt} = -\int q_j n_j \, dS = -\int \frac{\partial q_j}{\partial x_j} \, d\mathcal{V}$$

Outward normal

$$\mathcal{E} = \rho\left(e + \frac{V^2}{2}\right)$$

$$\frac{D\mathcal{E}}{Dt} = \int \frac{\partial \mathcal{E}}{\partial t}\, dv + \int \mathcal{E}\,\vec{V}\cdot\hat{n}\, ds$$

$$= \int \frac{\partial \mathcal{E}}{\partial t}\, dv + \int \nabla\cdot(\mathcal{E}\vec{V})\, dv$$

$$\int \left[\frac{\partial \mathcal{E}}{\partial t} + \frac{\partial \mathcal{E} V_j}{\partial x_j} - f_i V_i - \frac{\partial \sigma_{ji} V_i}{\partial x_j} + \frac{\partial q_j}{\partial x_j} \right] dv = 0$$

$$\boxed{\begin{aligned} &\frac{\partial \rho\left(e + \frac{V^2}{2}\right)}{\partial t} + \frac{\partial \rho V_j\left(e + \frac{V^2}{2}\right)}{\partial x_j} \\ &\qquad = \frac{\partial \sigma_{ji} V_i}{\partial x_j} + f_i V_i - \frac{\partial q_j}{\partial x_j} \\ &= \rho\, \frac{\partial (e + V^2/2)}{\partial t} + \rho V_j\, \frac{\partial (e + V^2/2)}{\partial x_j} = \rho\, \frac{D}{Dt}(e + V^2/2) \end{aligned}}$$

Mechanical Energy Equation

$$\rho \frac{DV_i}{Dt} = \frac{\partial \sigma_{ji}}{\partial x_j} + f_i = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ji}}{\partial x_j} + f_i$$

$$\rho \frac{DV^2/2}{Dt} = \frac{\partial V_i \sigma_{ji}}{\partial x_j} + V_i f_i - \sigma_{ji} \frac{\partial V_i}{\partial x_j}$$

$$\rho \frac{DV^2/2}{Dt} = \frac{\partial V_i \tau_{ji}}{\partial x_j} + V_i f_i + p \frac{\partial V_j}{\partial x_j} - \underbrace{\tau_{ji} \frac{\partial V_i}{\partial x_j}}_{\text{Dissipation of mechanical energy}}$$

$$\rho \frac{De}{Dt} = \underbrace{-\frac{\partial q_j}{\partial x_j}}_{\text{heat transfer}} \underbrace{- p\frac{\partial v_j}{\partial x_j}}_{p\,dV \text{ work}} + \underbrace{\tau_{ji}\frac{\partial v_i}{\partial x_j}}_{\text{dissipation}}$$

$\tau_{ji}\frac{\partial v_i}{\partial x_j}$ Dissipation of mechanical energies by friction $\Rightarrow$ appears as source term in the thermal energy equation and can increase e or result in a heat transfer.

$$\tau_{ji}\frac{\partial v_i}{\partial x_j} = \left[\mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) - \frac{2}{3}\mu\frac{\partial v_k}{\partial x_k}\delta_{ij}\right]\frac{\partial v_i}{\partial x_j}$$

$$= \left[2\mu\epsilon_{ij} - \frac{2}{3}\mu\frac{\partial v_k}{\partial x_k}\delta_{ij}\right](\epsilon_{ij} + \omega_{ij})$$

$$= 2\mu\epsilon_{ij}\epsilon_{ij} - \frac{2}{3}\mu\left(\frac{\partial v_k}{\partial x_k}\right)^2 > 0$$

NAVIER STOKES EQUATIONS

$$\sigma_{ij} = -p\,\delta_{ij} + \tau_{ij}$$

friction

average normal stress = pressure

Newtonian Fluid

$$\tau_{ij} \propto \text{Strain}_{ij}$$

Stokes Viscosity Law

$$\tau_{ij} = \mu\left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i}\right) + \lambda\,\frac{\partial V_k}{\partial x_k}\,\delta_{ij}$$

$$\lambda = -\frac{2}{3}\mu \quad \text{so that} \quad \frac{1}{3}\sigma_{jj} = -p$$

$$\rho\,\frac{DV_i}{Dt} = \rho\left(\frac{\partial V_i}{\partial t} + V_j\,\frac{\partial V_i}{\partial x_j}\right)$$

$$= -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i}\right) - \frac{2}{3}\mu\,\frac{\partial V_k}{\partial x_k}\,\delta_{ij}\right]$$

$$\tau_{xx} = \mu\left(\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial w}{\partial z}\right) \qquad \tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right.$$

$$\tau_{yy} = \mu\left(-\frac{2}{3}\frac{\partial u}{\partial x} + \frac{4}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial w}{\partial z}\right) \qquad \tau_{xz} = \tau_{zx} = \mu\left(\frac{\partial u}{\partial z} + \cdots\right.$$

$$\tau_{zz} = \mu\left(-\frac{2}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial u}{\partial y} + \frac{4}{3}\frac{\partial w}{\partial z}\right) \qquad \tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial v}{\partial z} + \cdots\right.$$

SUMMARY

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}\rho V_j = 0$$

$$\rho\left(\frac{\partial V_i}{\partial t} + V_j\frac{\partial V_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i}\right) - \frac{2}{3}\mu\frac{\partial V_k}{\partial x_k}\right]$$

$$\rho c_v\left(\frac{\partial T}{\partial t} + V_j\frac{\partial T}{\partial x_j}\right) = -\frac{\partial}{\partial x_j}\left(k\frac{\partial T}{\partial x_j}\right) - p\frac{\partial V_j}{\partial x_j}$$

$$+\mu\left[\left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i}\right)\frac{\partial V_i}{\partial x_j} - \frac{2}{3}\cancel{\mu}\left(\frac{\partial V_j}{\partial x_j}\right)^2\right]$$

dissipation

$p = \rho R T$ perfect gas

6 Equations ρ, V_i, P, T unknown

Also c_v, k, μ are functions of temperature

Component equations

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x}$$

$$+\frac{\partial}{\partial x}\left[\mu\left(\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial w}{\partial z}\right)\right]$$

$$+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right]$$

$$+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right]$$

SIMPLIFYING SITUATIONS

STEADY FLOW $\quad \partial/\partial t = 0$

$$\frac{\partial \rho V_j}{\partial x_j} = 0$$

$$\rho V_j \frac{\partial V_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i}\right) - \frac{2}{3}\mu \frac{\partial V_k}{\partial x_k}\delta_{ij}\right]$$

$$\rho c_v V_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j}\left(k \frac{\partial T}{\partial x_j}\right) - p\frac{\partial V_j}{\partial x_j} + \Phi$$

Could integrate unsteady equations to steady state. This is often done, however, it is usually faster to iterate steady equations to convergence if only steady state is desired.

INCOMPRESSIBLE FLOW

ρ = Constant $\quad (M \lesssim 0.3$ in gases$)$

$$\frac{\partial V_j}{\partial x_j} = 0$$

$$\rho\left(\frac{\partial V_i}{\partial t} + V_j \frac{\partial V_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i}\right)\right]$$

$$\rho c_v\left(\frac{\partial T}{\partial t} + V_j \frac{\partial T}{\partial x_j}\right) = \frac{\partial}{\partial x_j}\left(k \frac{\partial T}{\partial x_j}\right) + \Phi$$

$$\mu = \mu(T), \quad k = k(T)$$

INCOMPRESSIBLE, CONSTANT PROPERTIES

$$\frac{\partial V_j}{\partial x_j} = 0$$

$$\frac{\partial V_i}{\partial t} + V_j \frac{\partial V_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 V_i}{\partial x_j \partial x_j}$$

$$\frac{\partial T}{\partial t} + V_j \frac{\partial T}{\partial x_j} = \alpha \frac{\partial^2 T}{\partial x_j \partial x_j}$$

$\nu = \mu/\rho$ kinematic viscosity

$\alpha = k/\rho c_v$ thermal diffusivity

INVISCID, ADIABATIC $\Rightarrow$ EULER EQUATIONS

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho V_j}{\partial x_j} = 0$$

$$\rho \left(\frac{\partial V_i}{\partial t} + V_j \frac{\partial V_i}{\partial x_j} \right) = -\frac{\partial P}{\partial x_i}$$

$$\rho c_v \left(\frac{\partial T}{\partial t} + V_j \frac{\partial T}{\partial x_j} \right) = -p \frac{\partial V_j}{\partial x_j}$$

Jameson, A. "Computational Aerodynamics for Aircraft Design"
Science: Vol 245, July 28, 1989 pp 361-371

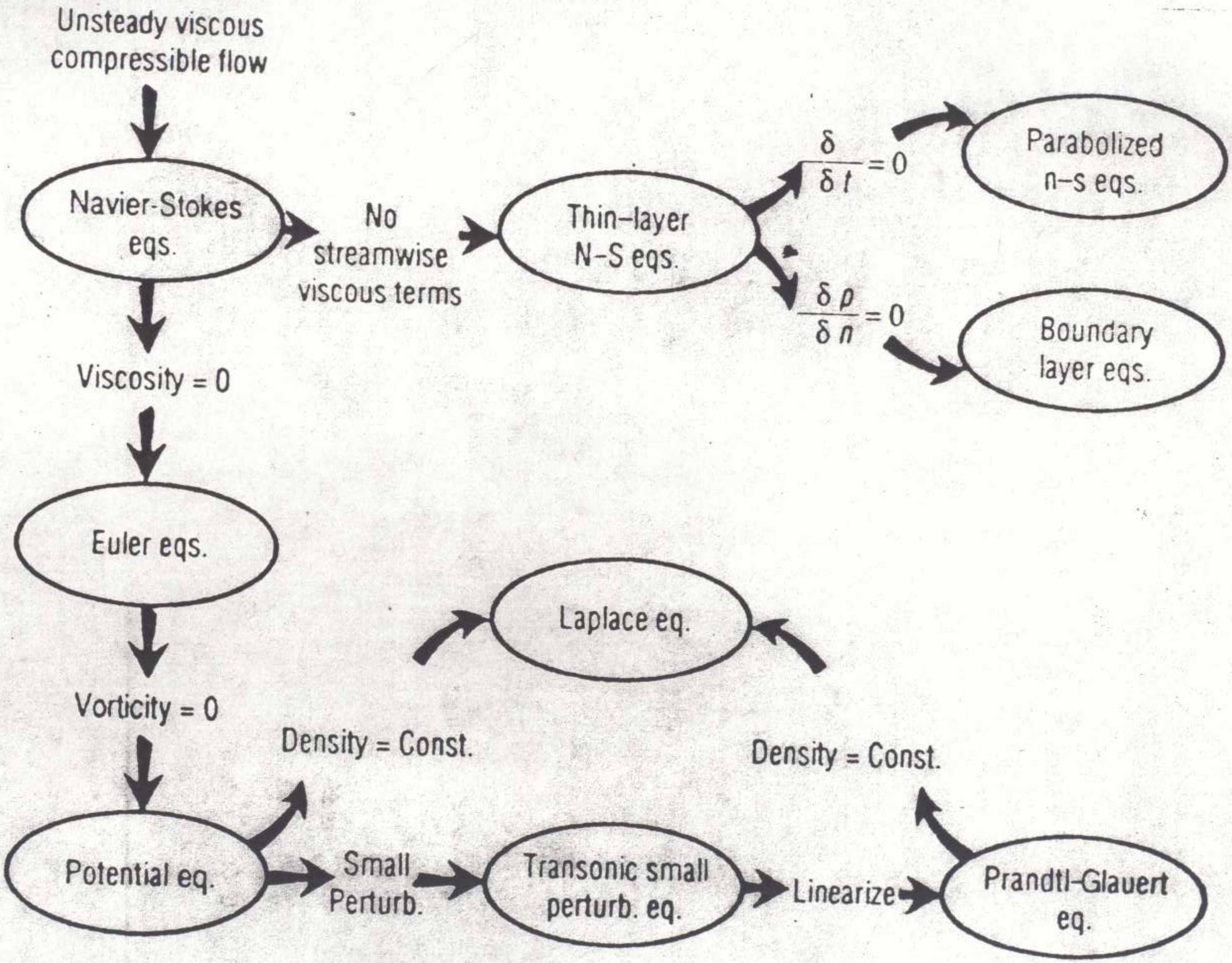


Fig. 2. Equations of fluid dynamics for mathematical models of varying complexity. (Supplied by Luis Miranda, Lockheed Corporation.)