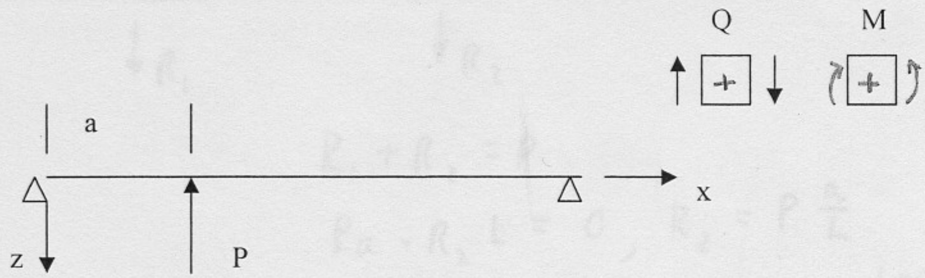
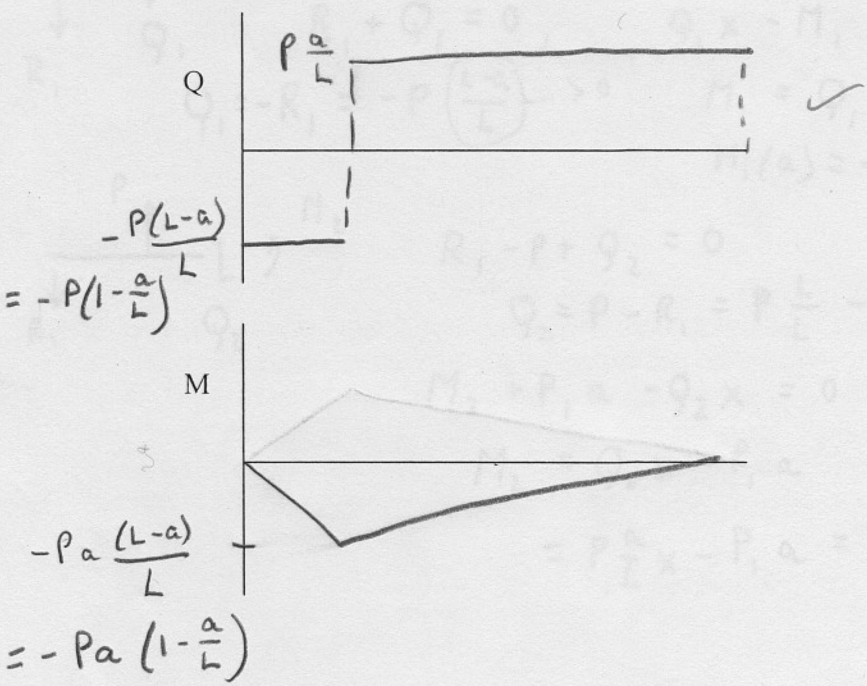
MAE 482/582 Test #3 Name Answers (print)
Closed Book 105/115 points
16 Dec 2004; time allowed 1:50; **show all work.**
Professor R. C. Wetherhold

1. Consider the use of the elementary beam Euler-Bernoulli beam theory for the laminated beam of length "L" shown below.



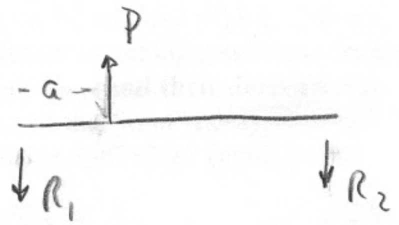
a) Calculate and draw the shear and moment diagrams. **Signs count;** label the important points. Hint: suggest you solve for the reactions first. It would be nice if your answer would simplify to the expected result if a=L/2.
(2@4 points)



(blank page)

Note - per instructions, show all work

(otherwise, at least -2 off)

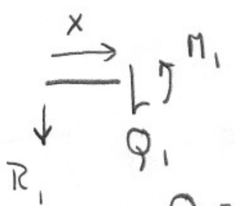


$$R_1 + R_2 = P$$

$$Pa - R_2 L = 0, \quad R_2 = P\frac{a}{L}$$

$$\Rightarrow R_1 = P - R_2 = P\frac{L}{L} - P\frac{a}{L} = P\frac{L-a}{L}$$

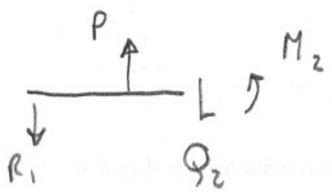
$(a = L/2 ? \Rightarrow R_1 = P\,1/2$ ✓$)$



$$R_1 + Q_1 = 0, \qquad Q_1 x - M_1 = 0$$

$$Q_1 = -R_1 = -P\left(\frac{L-a}{L}\right) \quad \frac{L-a}{L} > 0 \qquad M_1 = Q_1 x = -P\left(\frac{L-a}{L}\right)x \quad \frac{L-a}{L} > 0$$

$$M_1(a) = -P\frac{L-a}{L}a$$



$$R_1 - P + Q_2 = 0$$

$$Q_2 = P - R_1 = P\frac{L}{L} - P\frac{L-a}{L} = P\frac{a}{L}$$

$$M_2 + P_1 a - Q_2 x = 0$$

$$M_2 = Q_2 x - P_1 a \qquad \text{Note: } M_2(L) \equiv 0$$

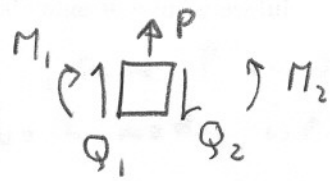
$$= P\frac{a}{L}x - P_1 a = P\left(\frac{a}{L}x - a\right) \qquad (\checkmark)$$

b) Say that the beam shown in part a) changes cross-section at x=a; the bending stiffness is $(E^bI)_1$ in the first segment and $(E^bI)_2$ in the second segment. Use the notation w_1 and w_2 for the displacement functions in the two intervals.
Undergraduates: give the required (4) b.c. and m.c. (a total of four) for this beam.
Graduates: Give all four matching conditions (m.c.) at x=a **in terms of the displacements w_1 and w_2 (and their derivatives).** Hint: $M= -(E^bI)w''$, $Q= -(E^bI)w'''$; consider equilibrium of the "joint" (x=a)

4@2 (undergrad) or 2@2+2@3 (grad) points

4@2 — UG: $w_1(0) = 0$, $w_2(0) = 0$, $w_1(a) = w_2(a)$, $w_1'(a) = w_2'(a)$

2@2 — G: $w_1(a) = w_2(a)$, $w_1'(a) = w_2'(a)$



2@3 —

$$M_1(a) = M_2(a) \Rightarrow -(EI)_1 w_1''(a) = -(EI)_2 w_2''(a)$$

$$Q_2 - P - Q_1 = 0 \Rightarrow -(EI)_1 w_1'''(a) - P + (EI)_2 w_2'''(a) = 0$$

c) It has been observed that failure can initiate inside a laminate. The critical event is when the local stress exceeds the strength, wherever that occurs first.
In the beam of problem 4.14, the laminate is [gr-ep/glass-ep]$_s$, all layers have same thickness "t". The gr-ep is 5 times stronger than the gl-ep. Formulate an expression for the ratio of the maximum stresses in each layer in general terms (e.g. in terms of bending curvature κ) and determine where the failure will occur. The ratio of the moduli for gr-ep:gl-ep is 6:1.
6 pts

$$\frac{\varepsilon_{gr}}{\varepsilon_{gl}} = \frac{\kappa(2t)}{\kappa(t)} = 2$$

$$\frac{\sigma_{gr}}{\sigma_{gl}} = \frac{E_{gr}\,\varepsilon_{gr}}{E_{gl}\,\varepsilon_{gr}} = 6 \cdot 2 = 12$$

stress is 12 times higher, strength is only 5 times higher ⇒ failure in gr-ep

d) This is similar to the HW problem 4.23. A beam can fail either from bending stresses (σ_x) or transverse shear (τ_{xz}). For a 3-point bending test, these maximum values

$$\frac{3}{2}\frac{PL}{bh^2}\ and\ \frac{3}{4}\frac{P}{bh}$$

for bending and shear, respectively. The ultimate strength in bending (normal stress) is "B" and the strength in shear is "S". Your co-worker has made a calculation that the required value of (L/h) to ensure a shear failure is given by (B/S)

Is the co-worker's calculation correct? Prove that this value does or does not lead to a shear failure. Hint: finding the critical value would be useful.

(8) pts

@$(L/h)_{cr}$, same P causes failure in both modes, i.e.

$$B = \frac{3}{2}\frac{PL}{bh^2} \qquad S = \frac{3}{4}\frac{P}{bh}$$

$$\text{or} \quad \frac{B\,\cancel{2}\cancel{b}h^{\cancel{2}}}{\cancel{3}L} = S\,\frac{\overset{2}{\cancel{4}}\cancel{b}\cancel{h}}{\cancel{3}} \qquad \frac{h}{L} = \frac{S}{B}2$$

$$\text{or } (L/h)_{cr} = \frac{B}{2S} < B/S\text{, i.e. } \frac{L}{h} > \left(\frac{L}{h}\right)_{cr}$$

→ Not correct, need $L/h < (L/h)_{cr}$ to ensure shear failure

e) A beam is modeled as a 1-dimensional object, but we know from plate theory that other effects are possible. Consider the existence of the D_{16} coupling stiffness; this couples bend and twist; this coupling is / is not unique to composites. Consider the existence of the D_{12} coupling stiffness; this coupling is/isn't unique to composites.
(3+2+2) pts

f) There are several methods for measuring the shear modulus in composites, such as the $(\pm 45)_n$ specimen in tension and the $(0/90)_n$ specimen in rail shear. We never seemed to bother with this testing for an isotropic material; why not?

could calculate $G = E/2(1+\nu)$

If we test a composite for strength using bending and tensile tests, I expect that the strength value from tensile tests will be bigger/smaller than that for bending tests.

While testing a bending specimen to determine the modulus of a composite, we note that at shorter (L/h) values we get higher/lower values than at larger (L/h). We know that this

comes about because of additional deformation from (transverse) shear that has not been accounted for in our simple beam theory.
(3+2+2+3) pts

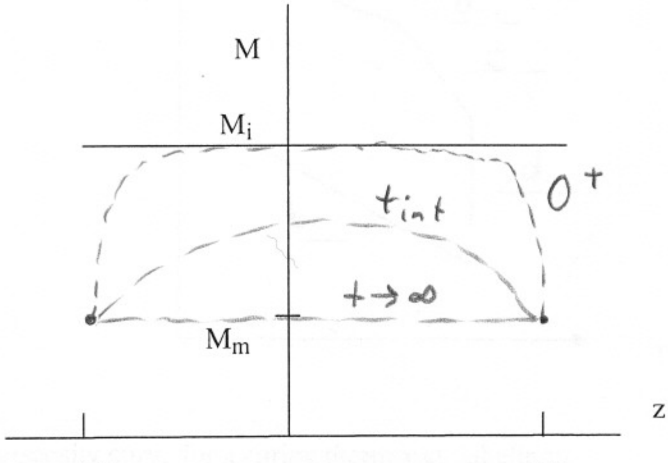
2.

a) Consider the moisture diffusion in the thickness direction, with the usual initial and boundary conditions.

$M(z,0) = M_i$ $\quad$ $t = 0,\ z\,\varepsilon\,[-h/2, h/2]$ $\quad$ (initial condition)

$M(\pm h/2, t) = M_m$ $\quad$ $t > 0$ $\quad$ (boundary condition)

Sketch on the figure below the three curves for: t=0+, t intermediate, $t \to \infty$

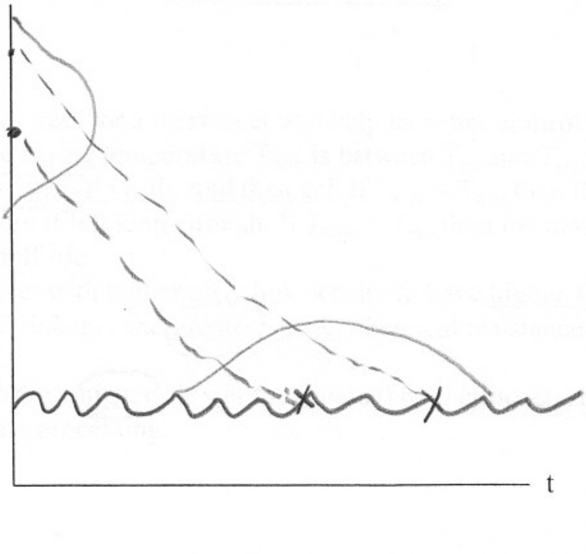


If we wish to calculate the details of the moisture-induced stresses, we must know M(z,t) /weight gain M(t). Common sense question—Compare the diffusion into a PMC of an organic solvent such as antifreeze or break fluid. When compared with water, the organic should have a lower/higher value of diffusion coefficient "D" and take a shorter/longer time to come to equilibrium. Water is a small molecule, the organics are larger, diffuse in more slowly.
(6+3@2) pts

brake fluid !

b) On the figure below, draw the load curve and a representative strength curve for a fatigue test. Show the failure point for the specimen. Indicate, by sketching two representative strength curves, how the distribution of initial strength leads to a distribution in fatigue lifetimes.
(6 points)

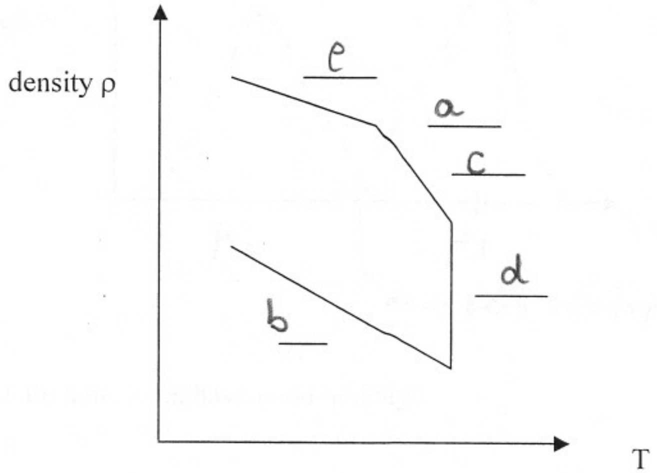
strength, applied stress



3. a) Label the segments/ points on the density curve below for a curing thermoset with the appropriate labels. Choose from:

   a) glass transition on cooldown
   b) heat-up expansion
   c) liquid cooling
   d) curing shrinkage
   e) glass cooling

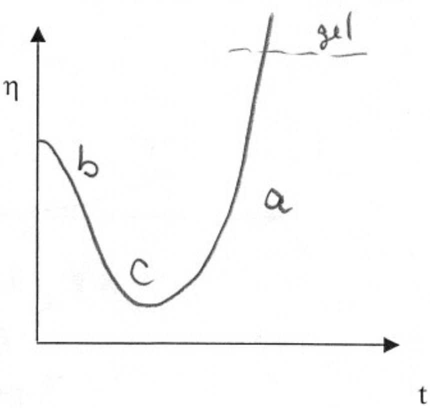
   (5@2 points)



b) Draw the viscosity curve for a curing thermoset, labeling:

   a) curing dominated
   b) heating dominated
   c) optimum processing

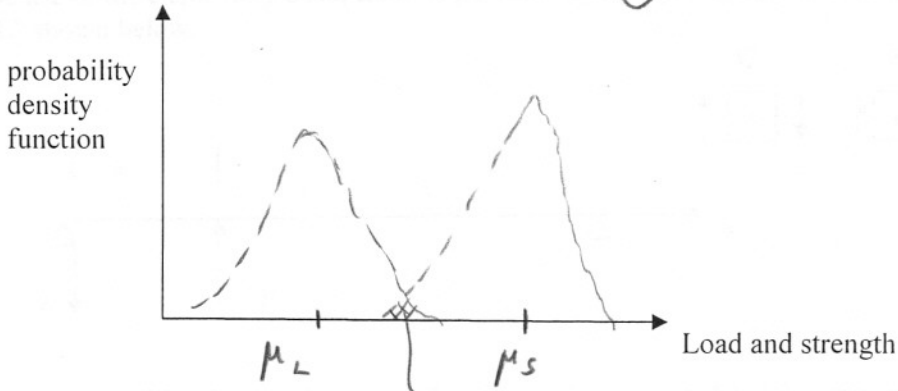
   (6 points—a free gift!)



c) Understanding the reaction process for a thermoset will help us better control the manufacturing process. If the curing temperature T_{cure} is between T_{gg} and $T_{g\infty}$, then the matrix will first gel and then vitrify / vitrify and then gel. If $T_{cure} < T_{g\infty}$, then the matrix will / will not complete its cure if left long enough. If $T_{room} > T_{gg}$, then the matrix will / will not have an "infinite" shelf life.
In general, I expect a thermoset with higher crosslink density to have higher / lower toughness, more / less cure shrinkage, and greater / lesser chemical resistance.

Thermoplastics, in general, have a higher / lower toughness than thermosets, and have a higher / lower viscosity during processing.
(8@2 points)

4. Consider that the load L and the strength S are random variables. Sketch on the probability density graph below distributions for L and S and indicate why there will be a non-zero probability of failure for any FS (factor of safety), where FS = μ_S / μ_L. Is it possible that two cases with the same FS could have different probabilities of failure? Y/n
(6+2) pts

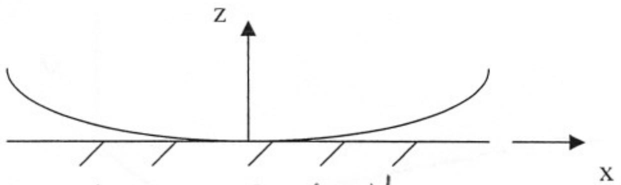


non-zero overlap $\Rightarrow P_f > 0$

Undergraduates: you are done here; have good holidays.

Graduate Students

5. Consider a beam which lies on a surface as shown. The origin lies at the **center** of the beam. The beam as shown is under a steady-state (time invariant) thermal loading which produces M^T_x.
   Using $M_x = D\kappa - M^T_x$ with $\kappa = -w''$, solve for the beam deflection w(x).
   (8 points)



NO mechanical load, $\forall x$

$$M_x \equiv 0$$

$$D(-w'') = M_x^T$$

$$-Dw' = M_x^T x + C_1$$

$$-Dw = M_x^T \frac{x^2}{2} + C_1 x + C_2$$

conditions: $\left.\begin{aligned} w(0) &= 0 \\ w_1'(0) &= 0 \end{aligned}\right\} \Rightarrow -Dw = M_x^T \frac{x^2}{2}$