MAE 482/582 Problems, PK Mallick, Chapt 1

For all material replacement or comparison problems, let the geometric variables of length, width, height, etc. **remain** as variables L, w, h, etc. Insert actual values **only in the last step.**

1.2 The material of a tension member is changed from SAE 4340 Steel to a unidirectional HM carbon fiber (Gr)/epoxy.

a) Calculate the ratio of cross-sectional areas, axial stiffness and weights of these two members *for equal load-carrying capacities*.

b) Calculate the ratio of cross-sectional areas, load-bearing capacity, and weights of these two members *for equal axial stiffness.*

HINT: Load bearing = strength . area; Axial stiffness (per length) = modulus . area

1.3 Calculate ratio of heights and weights for three rectangular beams made of Al 6061, Ti 6-4, and HS unidirectional Gr/Ep so as to enforce:

a) equal bending stiffness

c) equal bending moment capacity.

All beams have the same length and width. Let Al be the "standard" material.

HINT: beam stiffness = $Ebh^3/12$, moment capacity (single material) = $Sbh^2/6$

1.4 Calculate the ratio of flexural (bending) stiffnesses of two beams of equal weight, one made of 4340 steel and the other of unidirectional HM Gr/Ep composite. Density is ρ. The beams have the same width, but different heights. Do the particular boundary conditions of the beams (cantilever, simple supports, etc.) have any influence on this calculation?

1.5 A round steel beam with DIA = 10 mm is to be replaced by a unidirectional fiber-reinforced beam of equal length. The composite beam should have a natural frequency of bending vibration 25% higher than the steel beam it replaces. Compare the weight of HM Gr, HS Gr, and Kevlar fiber composites and select the minimum weight replacement. See Table 1.1.

Note that the natural frequency of beam bending vibration W_n is:

$W_n = C\,(EI/mL^4)^{1/2}$. where C is a constant dependent on the beam support conditions, m is mass per beam length, L is beam length.

1.6 An SAE 1010 steel panel of thickness 0.06 in is replaced with an E-glass/polyester SMC panel with equal flexural stiffness. Calculate the ratio of weight and cost of the two panels. Note that the panel stiffness is $Eh^3/12(1-\upsilon^2)$, where h is thickness and υ is the Poisson's ratio. The following properties are known:

	Steel	SMC
Modulus, 10^6 psi	30	3

Specific weight lbm/in^3	0.284	0.067
Cost	0.30	0.80

Suggest an alternative design (other than a flat plate) by which the wall thickness of the SMC panel can be reduced *without* lowering its flexural stiffness.

1.7 In order to reduce material cost, an engineer uses a hybrid beam instead of an all Gr fiber beam. Both beams have the same dimensions and have rectangular cross-section. The hybrid beam contains Gr fibers in the outer layers and either E-glass or Kevlar 49 fibers in the core. Costs of these materials are as follows:

Gr/epoxy $25/lbm E-glass/Ep $1.20/lbm Kevlar 49/Ep $8.00/lbm.

Find the ratio of the bending stiffnesses for the hybrid *beams relative to the all Gr/Ep beam* if the total Gr/Ep thickness in the hybrid beam is equal to the core thickness. That is, thickness of Gr/Ep only is h, thickness of hybrid layers is h/4,h/2,h/4 for Gr/Ep, core, Gr/Ep layers.

This ratio of bending stiffnesses should be expressed in terms of the E values for their components. What can be done to make the two stiffness equal? Find the ratio of the costs for the two beams.

GRAD STUDENTS: Let the first beam have thickness h_1, and the hybrid layer thicknesses be $h_1/4, 2h_2, h_1/4$. Set up the (cubic) equation in h_2/h_1 to match the flexural stiffnesses.

For both parts, the required equation for flex stiffness is: $EI = b\,\Sigma\, E_i\,(t_i^3/12 + t_i\, z_i^2)$ where z_i is the distance from the midplane of the beam to the middle of the layer and t_i is the layer thickness.

1.8 The shear modulus G of steel and a quasi-isotropic (in the plane) Gr/Ep is 78 and 17 GPa, respectively. The mean diameter D of a thin-walled steel torque tube is 25 mm and its wall thickness t is 3 mm. The torsional stiffness of a thin-walled tube is $k \equiv M_t/\theta = (\pi/4)Gtd^3$. Calculate:

1) the mean diameter of a composite tube with the same torsional stiffness and wall thickness as the steel tube.
2) The wall thickness of a composite tube that has the same torsional stiffness and mean diameter as the steel tube.
3) The ratio of weights for steel tube/composite tube in each of the above cases, assuming equal length for both tubes.

1.9 Using the information from Problem 1.8, design a composite torque tube that is 30% lighter than the steel tube but has the same torsional stiffness. What is the ratio of axial stiffness of these two tubes if their axial modulus is E ?