Ch. 4  Control Volume Energy Analysis  
(Apply 1st Law to Open System)

How to analyze mass flow
1) Follow fixed amount (control mass) that is moving or preferably—
2) Control Volume (fixed region in space), account for the energy accompanying the mass that enters/exit the boundary

Mass can cross the boundary!
Energy can cross the boundary

How? \[ \begin{align*}
- & Q \\
- & W \\
- & \text{Energy carried by the mass flow}
\end{align*} \]

Purpose: 1) Develop the eqn. for C.V:
Conservation of mass
Conservation of energy
2) Apply them

Note:
- Be mindful of assumptions/simplifications for the problem at hand
- Draw diagram, mark control volume
- Apply energy/mass balance
- Find properties (fix states) at inlets/exits (using property data, ideal gas model etc.)
1. Conservation of Mass for C.V.
(Mass Balance)

\[
\frac{d \text{mcv}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e
\]

- If both faucet & drain are closed:
  \[
  \frac{d \text{mcv}}{dt} = 0 \quad \text{(closed system, no mass across boundary of C.V.)}
  \]
- If faucet is on, drain is closed:
  \[
  \text{mcv} \uparrow \quad \frac{d \text{mcv}}{dt} = \dot{m}_i
  \]
- If faucet is off, drain is open:
  \[
  \text{mcv} \downarrow \quad \frac{d \text{mcv}}{dt} = -\dot{m}_e
  \]
- Both are on:
  \[
  \frac{d \text{mcv}}{dt} = \dot{m}_i - \dot{m}_e
  \]

\text{Time rate of change of mass stored within the C.V. @ time } t
\text{Total rate of mass flowing in across all inlet @ time } t
\text{Total rate of mass flowing out across all exits @ time } t

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"steady state": Every property inside the C.V. is independent of time
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\text{Water is actually replaced constantly}

\[
\frac{d \text{mcv}}{dt} = 0 = \sum_i \dot{m}_i - \sum_e \dot{m}_e
\]

\[
\sum \dot{m}_i = \sum \dot{m}_e
\]

\text{Faucet flow rate = drain flow rate}

How to represent $\dot{m}$ in terms of velocity distribution at the exit?

1) Idealized (Simplified) case:

\[ \dot{m} = \frac{\text{mass}}{\text{time}} = \lim_{\Delta t \to 0} \frac{\rho(\Delta A)}{\Delta t} = \rho AV \]

or
\[ \dot{m} = \left( \frac{AV}{V} \right) \text{(1D Flow)} \]

2) Velocity is not uniform across, $\vec{v}$ may not be normal to surface, or may contain swirling (3D Flow):

\[ \dot{\mathbf{m}} = \int_A \rho \mathbf{V} n \, dA = \int_A \rho \mathbf{V} \cdot \mathbf{n} \, dA \]

\[ \text{normal component} \]
Example

Steam at 120 bars, 520°C enters a turbine operating at a steady state with a volumetric flow rate of 460 m³/min. 22% of the entering mass flow exits at 10 bars, 220°C, with a velocity of 20 m/s. The rest exits at another location with a pressure of 0.06 bars, quality of 88.2%, and velocity of 500 m/s. Determine the dia. of each exit duct, in m.

Given:

\[ P_1 = 120 \text{ bars} \]
\[ T_1 = 520 \degree C \]
\[ (AV)_1 = 460 \text{ m}^3/\text{min} \]

Find:

\[ d_2 = ? \]
\[ d_3 = ? \]

\[ P_2 = 10 \text{ bars} \]
\[ T_2 = 220 \degree C \]
\[ V_2 = 20 \text{ m/s} \]

\[ m_2 = 0.22 m_1 \]
\[ m_2 = 10 \text{ bars} \]
\[ T_2 = 220 \degree C \]
\[ V_2 = 20 \text{ m/s} \]

Assumption:

Steady State

Key: Mass balance for steady state; From mass flow rate → get volumetric fl. rate → area at exits

\[ \frac{dm}{dt} = m_1 - m_2 - m_3 \]

\[ m = \frac{(AV)}{u_i} \quad u_i: \text{steamtable (A-4) } \frac{P_i}{T_i} \rightarrow u_i = 0.02781 \text{ m}^3/\text{kg} \]

\[ m_1 = \frac{460 \text{ m}^3/\text{min}}{0.02781 \text{ m}^3/\text{kg} \left( \frac{1 \text{ min}}{60 \text{ s}} \right)} = 275.7 \text{ kg/s} \]

\[ m_2 = 0.22 m_1 = 60.65 \text{ kg/s} \]

\[ m_3 = m_1 - m_2 = 0.78 m_1 = 215.1 \text{ kg/s} \]
2. Conservation of Energy for C.V.

Closed sys. 
\[ E_2 - E_1 = Q - W \]

Open Sys. (C.V.)
\[ \dot{m}_1 \rightarrow \text{C.V.} \rightarrow \dot{m}_2 \]

Energy balance must account for energy transfer across boundary due to mass flow.
\[ \frac{\Delta E}{\Delta t} = Q - W + \sum E_{in} - \sum E_{out} \]

\( \dot{m} \) (specific energy \( u, \text{k.e., p.e.} \))

Extra term due to flow:
* Energy required to push fluid into or out of a C.V. is called "flow work" or "flow energy"

\[ p \nu \text{ (per unit mass based)} \]

Specific volume

Total energy of a flowing fluid:
\[ p \nu + u + \text{k.e.} + \text{p.e.} \]
\[ = h + \frac{1}{2} V^2 + g z \text{ (kJ/kg)} \]

1st Law for C.V. (Conservation of Energy):
\[ \frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum \dot{m}_i (h_i + \frac{1}{2} V_i^2 + g z_i) - \sum \dot{m}_o (h_e + \frac{1}{2} V_e^2 + g z_e) \]

Conservation of mass:
\[ \frac{d\dot{m}_{cv}}{dt} = \sum \dot{m}_i - \sum \dot{m}_o \]
To find \( \dot{m}_2 \):

\[ \dot{m}_2 = \frac{(AV)_2}{V_2} \]

\[ u_2 = \frac{\text{Fix state} @ P_2, T_2}{[A-4]} \text{ (spht. vp)} \rightarrow 21675 \text{ m}^3/\text{kg} \]

\[ A_2 = \frac{\dot{m}_2 u_2}{V_2} = \frac{(60.65 \text{kg/s})(21675 \text{ m}^3/\text{kg})}{20 \text{ m/s}} = 657 \text{ m}^2 \]

\[ A_2 = \frac{\pi d_2^2}{4} \]

\[ d_2 = \sqrt{\frac{4A_2}{\pi}} = 0.915 \text{ m} \]

\[ \dot{m}_3 = \frac{(AV)_3}{V_3} \quad u_3 \quad \frac{\text{Sat. Table}[A-3]}{G P_3 = .06 \text{ bar}, X_3 = .862} \quad u_f_3 + X_3 (u_g - u_f) \quad = 20.463 \text{ m}^3/\text{kg} \]

\[ A_3 = \frac{\dot{m}_3 u_3}{V_3} = 8.803 \text{ m}^3 \]

\[ d_3 = \sqrt{\frac{4A_3}{\pi}} = 2.35 \text{ m} \]
Steady State: \[ \frac{dE_v}{dt} \to 0, \quad \frac{dm_v}{dt} \to 0 \]

Steady State with 1-inlet 1-exit:
\[ \dot{m}_i = \dot{m}_o = \dot{m} \quad (\text{cons. mass}) \]
\[ 0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_i - h_2) + \frac{V_i^2 - V_2^2}{2} + g (z_i - z_2) \right] \]

**Application**

1. Nozzles / Diffusers

   A passage to change flow speed.

   - **Subsonic Nozzle**
     - \( h_i, V_i \)
     - \( h_2, V_2 \)
     - Accelerate flow speed
     - \( V_2 > V_i \) (\( \dot{m}_i = \dot{m}_2 \))
     - \( \rho_AV_i = \rho_AV_2 \)

   - **Subsonic diffuser**
     - \( h_i, V_i \)
     - \( h_2, V_2 \)
     - Decelerate

   **No work other than "flow work" pu. which is included in \( h \) \( \Rightarrow \dot{W}_{cv} = 0 \)**

   **Usually** \( z_i \approx z_2 \)

   **especially for gases**

   \( \Rightarrow g (z_i - z_2) = 0 \)

   **Often with insulation**

   \( \dot{Q}_{cv} \approx 0 \)

   So energy balance is reduced to

   \[ 0 = (h_i - h_2) + \frac{V_i^2 - V_2^2}{2} \]

**Q. How does pressure \( p_2 \) compared to \( p_1 \)?**

**(Hint: Bernoulli’s principle)**

In fact, energy balance for this special case is reduced to Bernoulli's principle if we further assumes the internal energy does not change.

\[ p + \frac{1}{2} \rho V^2 = \text{const} \]
2. Turbine: To develop work output

\[ W_t > 0 \]

Supht. Vpr. \[ \Rightarrow \]

Exhaust

High T

low T

High p

Low p

High V

Low V

\( h = pv + u \)

Increases with \( T \) & \( p \)

Hence, high \( h \)

"Expands" to a lower pressure

Example:

Steam develops a power output of 1000 kW

Note:

\[ h_1 \neq h_2 \quad V_1 \neq V_2 \quad \text{only if} \quad (s_1 - s_2) \approx 0 \]

Steady state energy balance

\[ W_t = \dot{Q}_t + m \left[ (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} \right] \]

(may \( \approx 0 \) )
3. Compressors/Pumps:

To raise pressure of gas < liquid
\[ W_c < 0 \] (Need work input)

\[
\begin{align*}
\text{Inlet} & \quad \text{Exit} \\
\text{low } p & \quad \text{higher } p
\end{align*}
\]

"Compresses"

Example:
Air is compressed

Note:
\[ W_c < 0 \]
\[ h_1 \neq h_2 \]
\[ g(z_i, z_f) \]
\[ \approx 0 \text{ for compressor} \]
\[ \text{But is often significant for pumps} \]
\[ \frac{V_i^2 - V_f^2}{2} \text{ might } \approx 0 \]

4. Heat Exchangers:

Multiple streams of fluid at various temp. to exchange heat.

The "Heat Exchange" happen inside C.V.
If outside is well insulated, \[ Q_{cv} = 0 \]
\[ \dot{W}_{cv} = 0, \quad \Sigma (z_i - z_f) \approx 0 \]

But h's are different!
Need to resort to c. of mass!
\[ 0 = \dot{m}_1 (h_1 - h_2) + \dot{m}_3 (h_3 - h_4) \]
How to Solve Problems

1. Assumptions must be appropriate. (steady state, Q=0)
   List them all!

2. Draw system diagram, mark boundary of C.V.
   Identify locations of inlets/exits.

3. Give known data, preferably on the system diagram,
   Specify "Find ... = ? (units)"

4. Solution: Start w/ general equations.
   (Energy balance, mass balance)
   Simplify them by applying the assumptions.
   When you need to look up property tables, give
   the table numbers & known parameters used to
   retrieve the unknown numbers
   List all steps!
   (You’ll get partial credit even if answer
   is wrong.)

5. Carry units in your calculations! Note Signs!
   Be careful right from the start when you list "Given".
   Example: “Work input is 161.5 KJ per kg of air flowing”
   Given: \( \frac{W}{m} = \frac{\dot{W}}{m} = -161.5 \text{ KJ/kg} \)
   \( \dot{m} \) you might find this more useful
   Units must be homogeneous. If you encounter
   \( h_2 = h_1 + \frac{V_i^2}{2} = (1192.6 \text{ Btu/lb}) + (500 \text{ ft/s})^2/2 \)
   You need to work the 2nd term towards Btu/lb
   by multiplying
   \[
   \left( \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lb}} \right) \left( \frac{1 \text{ lb}}{32.2 \text{ ft lb/s}^2} \right)
   \]
Transient Analysis

Transient operation: State changes with time (e.g. startup or shut down of a compressor)

How to apply mass balance and energy balance for transient analysis?

\[
\frac{dm_{cv}}{dt} = 0, \quad \frac{dE_{cv}}{dt} = 0
\]

-Evaluate the finite change over a finite period of time (0-t) by integrating the equations.

Example. A well-insulated rigid tank of volume 0.5m$^3$ is initially evacuated. A hole develops in the wall, and air from the surroundings at 1 bar, 21°C flows in until the pressure in the tank reaches 1 bar. Determine the final temperature in the tank, in °C.

\[
\text{KNOWN: A hole develops in an initially evacuated tank and air flows in from the surroundings until the pressure becomes 1 bar. Heat transfer is negligible.}
\]

\[
\text{FIND: Determine the final temperature.}
\]

\[
\text{SCHEMATIC & GIVEN DATA:}
\]

\[
\text{ASSUMPTIONS: (1) For the control volume shown, } W_{cv} = 0. (2) Kinetic and potential energy effects are negligible. (3) Heat transfer can be neglected. (4) The air behaves as an ideal gas with constant specific heats.}
\]

\[
\text{ANALYSIS: The mass rate balance takes the form } \frac{dm_{cv}}{dt} = m_i. \text{ The energy rate balance reduces to}
\]

\[
\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - W_{cv} + m_i h_i
\]

The specific enthalpy at the inlet is constant. Thus, combining the mass and energy rate balances and integrating

\[
m_2 u_2 - m_2 u_2^0 = h_i (m_2 - m_1^0)
\]

or

\[
u_2 = h_i
\]

With

\[
h_i = u_i + p_i v_i = u_i + RT_i
\]

\[
u_2 = u_i + RT_i
\]

Introducing

\[
u_2 - u_1 = c_v (T_2 - T_i)
\]

\[
T_2 = \left( \frac{R}{c_v} + 1 \right) T_i
\]

From Table A-20; $c_v = 0.721 \text{ kJ/kg·K}$. Thus

\[
T_2 = \left[ \frac{(8.31 \cdot 289.7)}{0.721} + 1 \right] (294 \text{ K})
\]

\[
= 138 \text{ °C}
\]