Gen. Compr. Charts can be used for:

1. Fixing the state of a real gas
2. Evaluating how closely the real gas behaves like an ideal gas

\[ \begin{align*}
\text{Given 2 parameters} & \quad \rightarrow \text{Find the other 2 from the charts} \\
p, v, T, \gamma & \\
\text{through} & \\
\Pr, \nu', \Tr, \gamma & = \frac{pv}{RT}
\end{align*} \]

Example

Determine the pressure, in kPa, of nitrogen (N\(_2\)) at -80\(^\circ\)C and a specific volume of 0.0045 m\(^3\)/kg using Gen. Compr. data, and compare with values obtained using the ideal gas model.

(Solution: next page)

Ideal Gas Model

**Condition:** when \( p \to 0 \) or \( T \to \infty \)

\( Z \to 1 \) (test)

**Specifications:**

\[ \begin{align*}
pv & = RT \\
\bar{u} & = \bar{u}(T) \quad \text{Internal energy depends on gas temp. only.} \\
h & = h(T) = \bar{u}(T) + RT
\end{align*} \]

For ideal gases:

\[ \begin{align*}
C_v(T) & = \frac{du}{dt}, \quad C_p(T) = \frac{dh}{dT}, \quad \bar{C}_p = C_v + R \\
\text{Combined with} \quad k & = \frac{C_p(T)}{C_v(T)}, \quad \bar{C}_p(T) = C_v(T) + \bar{R} \\
C_p(T) & = \frac{kR}{k-1} \\
C_v(T) & = \frac{R}{k-1}
\end{align*} \]
Solution:

Nitrogen (N₂) at \( T = -80^\circ C = 193 \, K \), \( \nu = 0.0045 \, m^3/kg \)

Using data from Table A-1

\[ T_R = \frac{T}{T_c} = \frac{193}{126} = 1.53 \]

\[ \nu_R = \frac{\nu}{RT_c / P_c} \]

\[ = \frac{(0.0045 \, m^3/kg)(33.9 \, bars)}{\left(\frac{8.314 \, kJ}{28.01 \, kg \cdot K}\right)(126 \, K)} \left(\frac{10^5 \, N/m^2}{1 \, bar}\right) \left(\frac{1 \, kJ}{10^3 \, N \cdot m}\right) \]

\[ = 0.408 \]

Thus, from Fig A-2; \( P_R \approx 3.1 = \frac{P_{\text{chart}}}{P_c} \)

\[ P_{\text{chart}} = (3.1)(33.9 \, bars)\left(\frac{100 \, kPa}{1 \, bar}\right) = 10,509 \, kPa \]

Using the ideal gas model

\[ P_{\text{ideal gas}} = \frac{RT}{\nu} \]

\[ = \left(\frac{8.314}{28.01}\right)(193)\left(\frac{1}{0.0045}\right) \left(\frac{10^2}{10^2}\right) = 127.3 \, bars \]

\[ = 12,730 \, kPa \]

**COMMENT:** Assuming that the chart value is correct, the ideal gas model over-predicts the pressure by about 21%.
There are 3 ways to evaluate \(\Delta U\) or \(\Delta h\) for ideal gases:

1. \[ U_2 - U_1 = \int_{T_1}^{T_2} C_v(T) dT \quad \text{if} \quad T_2 - T_1 \text{ is small} \]
   \[ C_v(T_2 - T_1) \]
   \[ h_2 - h_1 = \int_{T_1}^{T_2} C_p(T) dT \approx C_p(T_2 - T_1) \]

2. \[ U_2 - U_1 = \int_{T_1}^{T_2} C_v(T) dT = R \int_{T_1}^{T_2} \left( \frac{C_p}{R} \right) dT \]
   \[ \alpha + \beta T + \gamma T^2 + \delta T^3 + \varepsilon T^4 \]
   \[ \text{Eqn. (3.48)} \]
   \[ \left( \frac{C_p}{R} \right) \]
   Values of constants \(\alpha, \beta, \ldots\) are listed in A-21

For monatomic gases (Ar, Ne, He):
\[ \alpha = \frac{5}{2}, \quad \beta = \gamma = \delta = \varepsilon = 0 \]
Hence
\[ C_p = \frac{5}{2} \]

3. "Ideal Gas Tables" A-22 A-23

Given \(T\) → Find \(U\), \(h\) or \(h, U\)

(Do not worry about \(p, \nu\) listed in the tables for now. They are NOT \(p, \nu\)).

Example 3.8

\[ T_1 = 540^\circ R \quad p_1 = 1 \text{atm} \]
Compression, during which heat transfer \(Q = 20 \text{Btu}\)

\[ T_2 = 840^\circ R \]

Find \(W = ? \) (Btu)

Assumptions:
1) Closed system.
2) \(\Delta KE = \Delta PE = 0\)
3) Air is modeled as ideal gas

1st Law:
\[ \Delta U = Q - W \]

\[ W = Q - \Delta U = Q - m(C_2 - C_1), \quad Q = -20 \text{Btu} \]

\[ A-22E \]

\[ \text{At } T_1 = 540^\circ R \Rightarrow C_1 = 92.04 \text{ Btu/} \text{lb} \]
\[ \text{At } T_2 = 840^\circ R \Rightarrow C_2 = 143.98 \text{ Btu/} \text{lb} \]

\[ W = 123.9 \text{ Btu} \] (Done on the system)
Polytropic Processes

\[ pV^n = \text{const} \]

under assumption of Quasi-Equilibrium, \( W = \int p\,dv \) can be integrated:

\[ \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^n \]

\[ W = \int_1^2 p\,dV = \left\{ \begin{array}{ll}
\frac{\beta V_2 - \beta V_1}{1-n} & (\text{if } n \neq 1), \\
\rho V_1 \ln \frac{V_2}{V_1} & (\text{if } n = 1)
\end{array} \right. = m \rho V_1 \ln \frac{V_2}{V_1} \]

Special cases:

- Isometric, \( V = \text{const} \rightarrow W = 0 \). \((n = \infty)\)
- Isobaric, \( p = \text{const} \rightarrow W = m \rho (V_2 - V_1) \). \((n = 0)\)
- Isothermal, \( T = \text{const} \rightarrow pV = \text{const} \) \(\text{(for ideal gas)} \) or \((n = 1)\)

* Isentropic, Entropy \( S = \text{const} \rightarrow Q = 0 \). \((n = k)\)

* To be discussed later.

Specific Heat Ratio

Polytropic Processes for ideal gases: Further idealization

\[ \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \]

\[ \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} \]

\[ W = \int_1^2 p\,dV = \left\{ \begin{array}{ll}
m \frac{\beta R (T_2 - T_1)}{1-n} & (\text{if } n \neq 1), \\
m R T_1 \ln \frac{V_2}{V_1} & (\text{if } n = 1)
\end{array} \right. \]

Use these relationships in problems involving ideal gases (e.g. Air) and polytropic processes!

Major Skills from Chapter 3:

- Water/Refrigerants
- Steam Tables
- Air
- Ideal Gas Tools