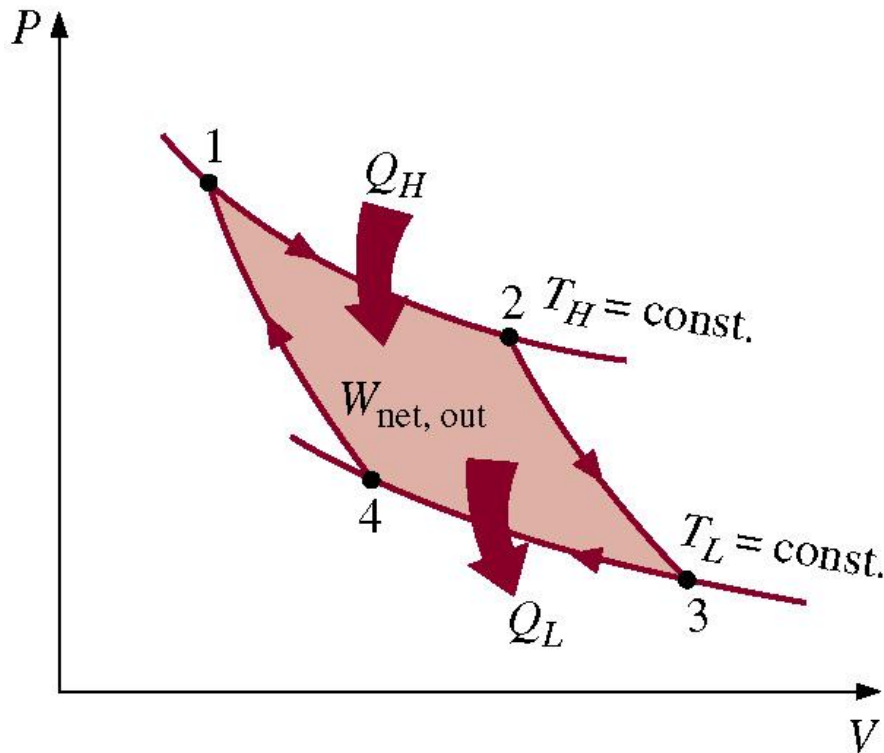


**Fig. 9.2** General Cycle Approximated by Large Number of Carnot Cycles.

## Clausius Inequality

$$\oint \frac{Q}{T} \leq 0$$



for a Reversible Cycle

$$\left( \frac{Q_h - Q_l}{Q_h} \right) = \left( \frac{T_h - T_l}{T_h} \right)$$

$$1 - \frac{Q_l}{Q_h} = 1 - \frac{T_l}{T_h}$$

$$\frac{Q_h}{T_h} - \frac{Q_l}{T_l} = 0$$

$$\oint \frac{Q}{T} = 0$$

for an Irreversible Cycle

$$\left( \frac{Q_h - Q_l}{Q_h} \right) < \left( \frac{T_h - T_l}{T_h} \right)$$

$$1 - \frac{Q_l}{Q_h} < 1 - \frac{T_l}{T_h}$$

$$\frac{Q_h}{T_h} - \frac{Q_l}{T_l} < 0$$

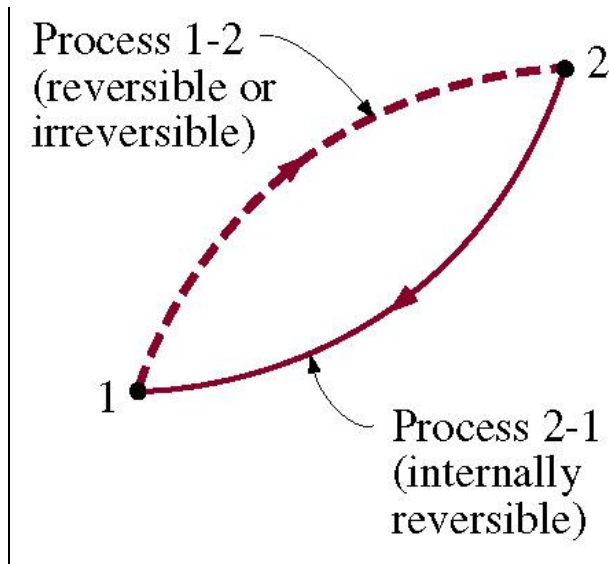
$$\oint \frac{Q}{T} < 0$$

# Entropy Definition and Change

$$\oint \frac{Q}{T} \leq 0 \quad \text{Clausius Inequality}$$

for a cycle composed of a reversible and an irreversible process

$$\oint \frac{Q}{T} = \int_{2 \text{ irrev}}^1 \frac{Q}{T} + \int_{1 \text{ rev}}^2 \frac{Q}{T} \leq 0$$



DEFINE A PROPERTY  $S$  ENTROPY

$$\int_{1 \text{ rev}}^2 \frac{Q}{T} = S_2 - S_1$$

$$\int_{2 \text{ irrev}}^1 \frac{Q}{T} + S_2 - S_1 \leq 0$$

$$\int_{2 \text{ irrev}}^1 \frac{Q}{T} + ds \leq 0$$

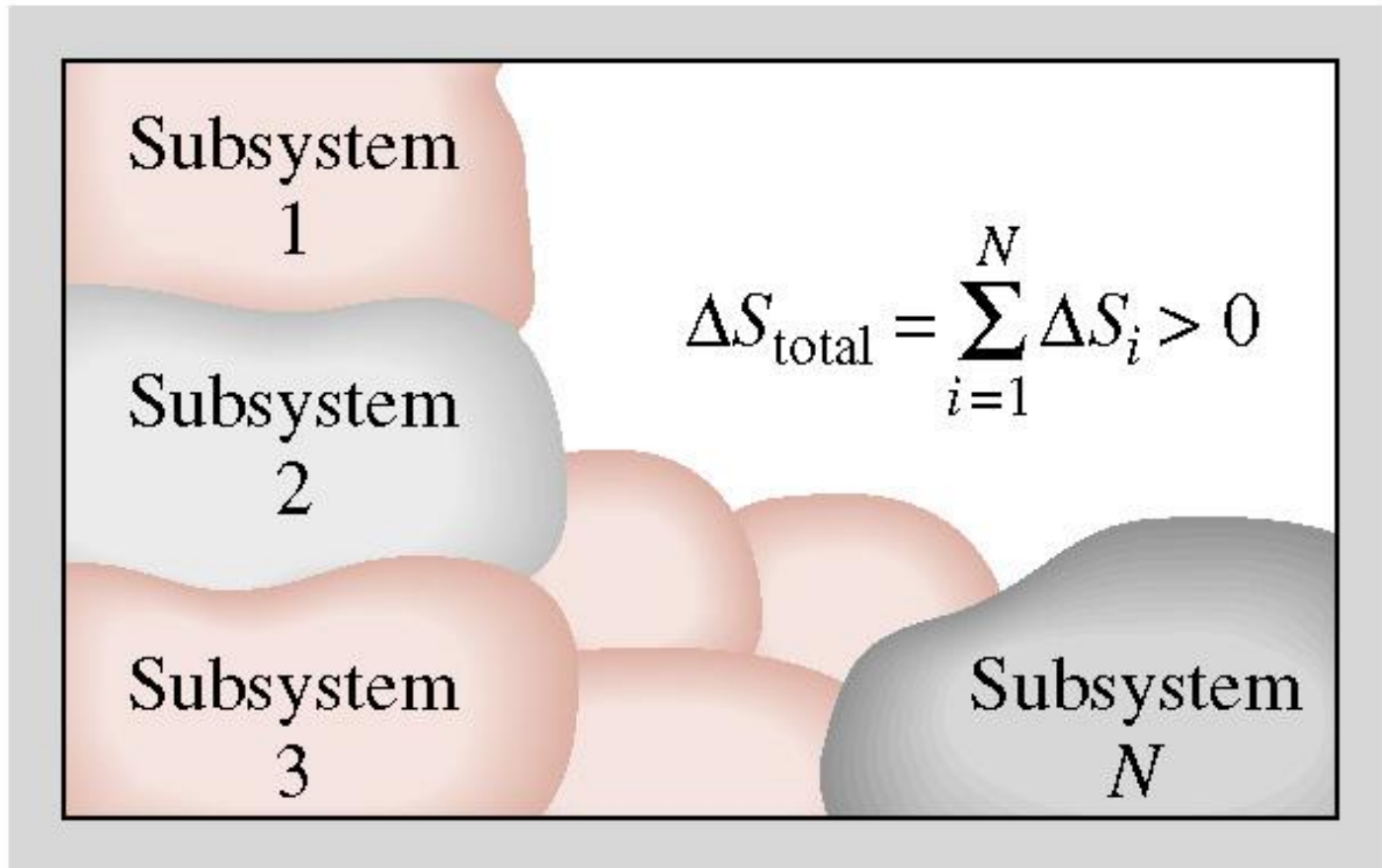
$$ds \geq \int_{2 \text{ irrev}}^1 \frac{Q}{T}$$

$$ds \geq \int_{\text{irrev}} \frac{Q}{T}$$

$$ds = \int_{\text{rev}} \frac{Q}{T}$$

$$Q = \int_{\text{reversible internally reversible}} T ds$$

(Isolated)



Total Entropy Change,  $S$ , for an isolated system

For a reversible system,  $S = 0$

For an irreversible system,  $S > 0$

**Completely Reversible Process** - System and surroundings returned to the original state

**Internally Reversible** - System returned to the original state.

**Externally Reversible** - Surroundings returned to the original state.

Heat transfer across a temperature difference is an irreversible process.  
 1 lb/sec of steam is condensed at 65 C by R-134a boiling at 60 C.

$$Q_{R-134a} = Q_{\text{steam}} = m_{\text{steam}} \times h_{\text{fg, steam @ } 65^\circ \text{C}} = 1 \text{ kg/sec} \times 2345.4 \text{ kJ/kg}$$

$$Q_{R-134a} = m_{R-134a} \times h_{\text{fg, R-134a @ } 60^\circ \text{C}}$$

$$m_{R-134a} = \frac{2345.4 \text{ kJ/sec}}{139.1 \text{ kJ/kg}} = 16.86 \text{ kg/sec}$$

$$S_{\text{steam}} = m_{\text{steam}} \times (s_2 - s_1) = 1 \text{ kg/sec} \times (.8937 - 7.8296) = -6.9359 \text{ kJ/kg}^\circ \text{Ksec}$$

$$S_{R-134a} = m_{R-134a} \times (s_2 - s_1) = 16.86 \text{ kg/sec} \times (.90696 - .48920) = +7.0434 \text{ kJ/kg}^\circ \text{Ksec}$$

$$S_{\text{ISOLATED SYSTEM}} = S_{R-134a} + S_{\text{steam}} = +7.0434 - 6.9359 = +.1075 \text{ kJ/kg}^\circ \text{Ksec}$$

Heat transfer across a 0 temperature difference is a reversible ( an also impossible) process. 1 lb/sec of steam is condensed at 60 C by R-134a boiling at 60 C.

For idealized heat transfer at a constant 60° C

$$Q_{R-134a} = Q_{\text{steam}} = m_{\text{steam}} \times h_{\text{fg}} \text{ steam @ } 60^\circ \text{ C} = 1 \text{ kg/sec} \times 2357.7 \text{ kJ/kg}$$

$$Q_{R-134a} = m_{R-134a} \times h_{\text{fg}} \text{ R-134a @ } 60^\circ \text{ C}$$

$$m_{R-134a} = \frac{2357.7 \text{ kJ/sec}}{139.1 \text{ kJ/kg}} = 16.9496 \text{ kg/sec}$$

$$S_{\text{steam}} = m_{\text{steam}} \times (s_2 - s_1) = 1 \text{ kg/sec} \times (.8313 - 7.9082) = -7.0769 \text{ kJ/kg}^\circ \text{K sec}$$

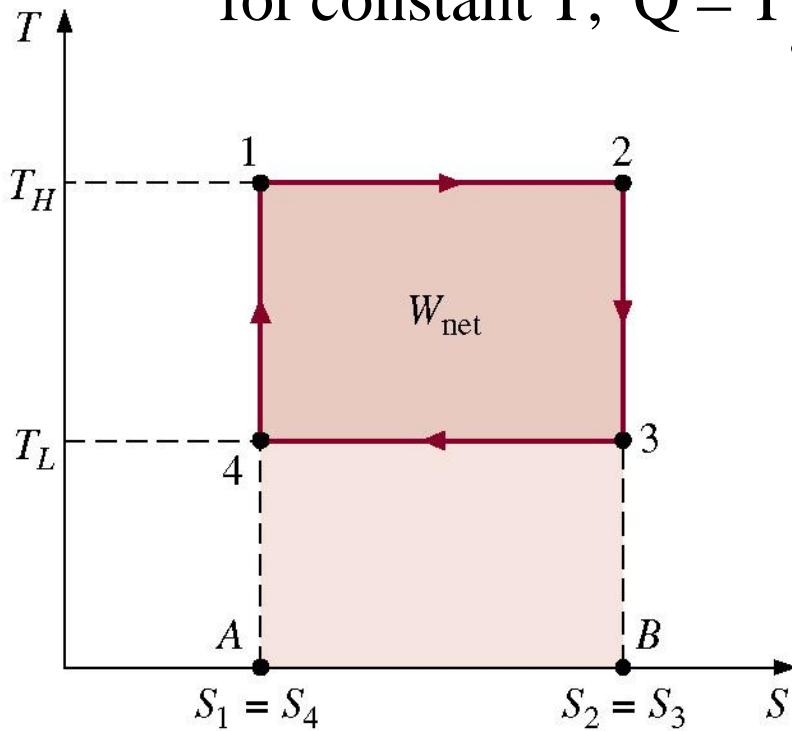
$$S_{R-134a} = m_{R-134a} \times (s_2 - s_1) = 16.9496 \text{ kg/sec} \times (.90669 - .4892) = +7.08137 \text{ kJ/kg}^\circ \text{K sec}$$

$$S_{\text{ISOLATED SYSTEM}} = S_{R-134a} + S_{\text{steam}} = +7.0769 - 7.0762 = .0007 \text{ kJ/kg}^\circ \text{K sec}$$

# Temperature Entropy Property Diagrams

$$dS = \int \frac{Q}{T}$$

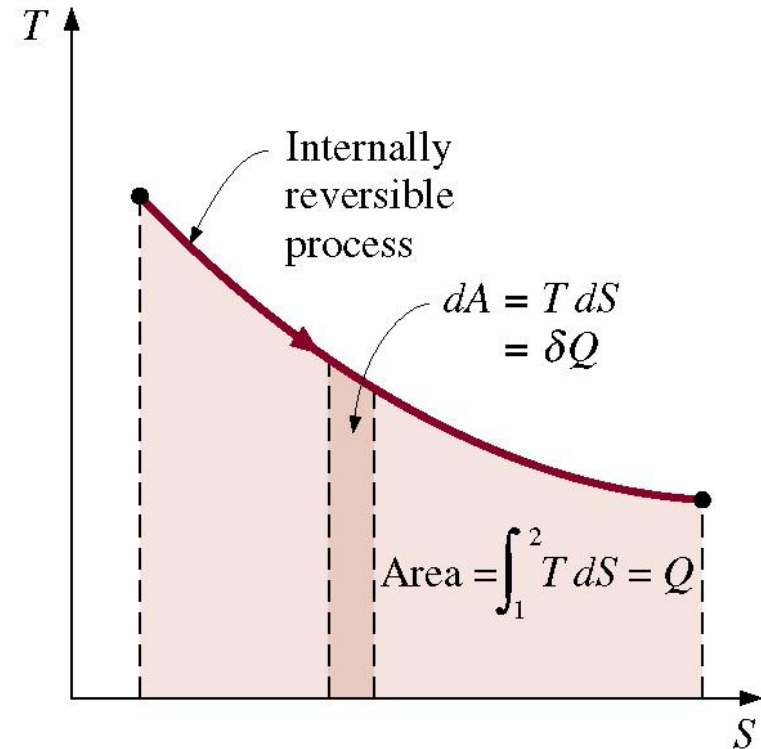
for constant T,  $Q = T \int dS = T_{\text{constant}} \Delta S$



$$Q_{\text{in}} = T_H \Delta S$$

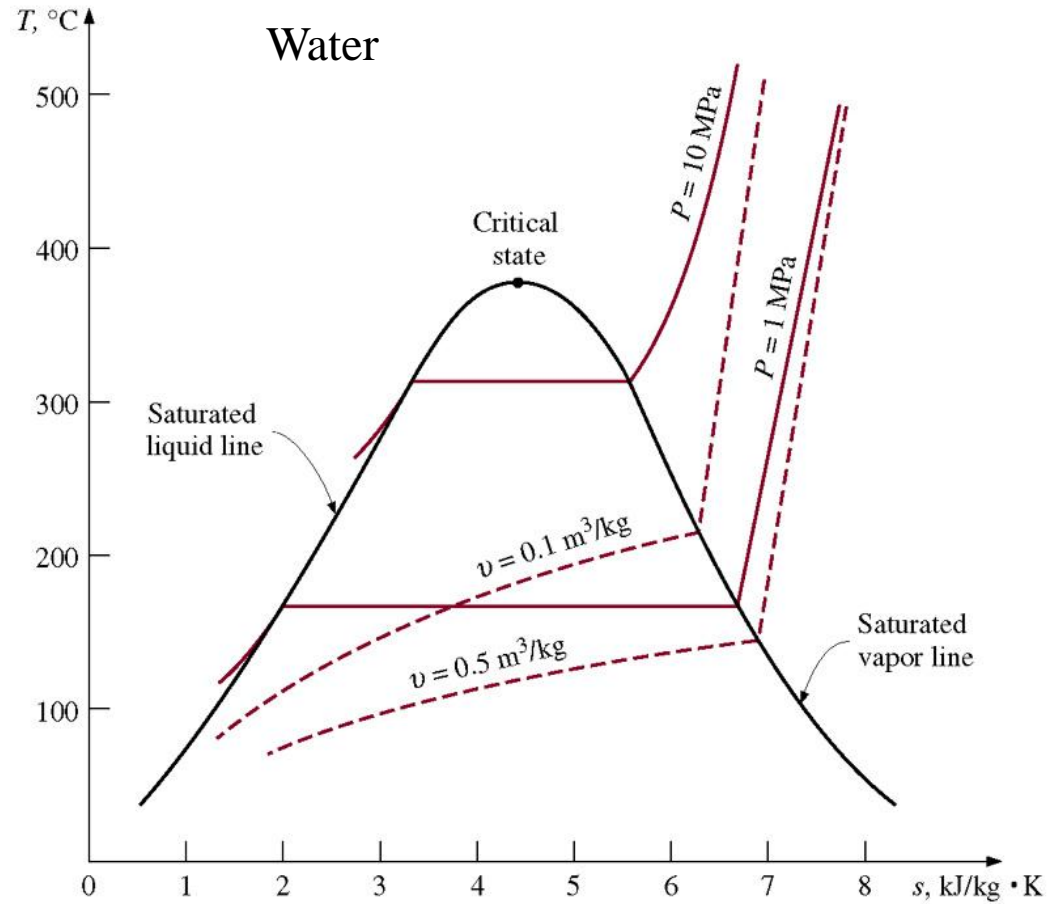
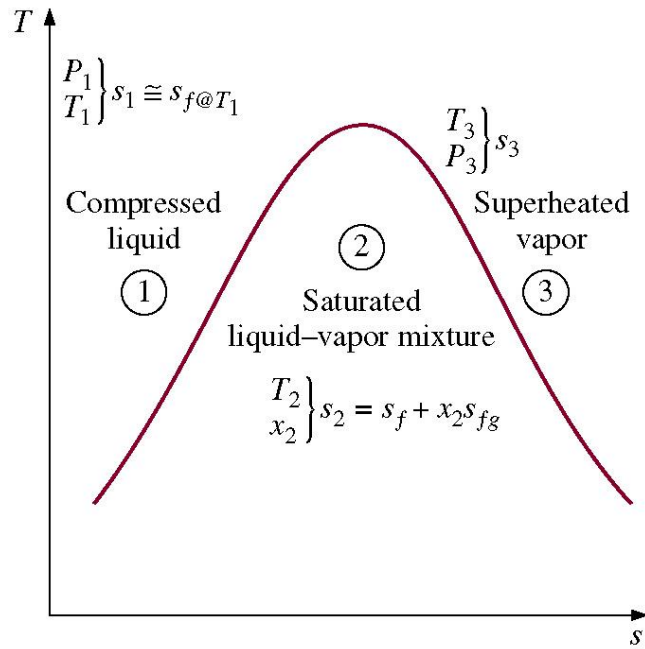
$$Q_{\text{out}} = T_L \Delta S$$

$$W_{\text{net}} = Q_{\text{in}} - Q_{\text{out}} = (T_H - T_L) \Delta S$$





# Temperature Entropy Property Diagrams



## Entropy Change of an Ideal Gas

$$\delta q = du + \delta w \quad \text{First Law}$$

$$\delta q = Tds \quad \text{Second Law}$$

$$Tds = du + pdv$$

$$ds = \frac{du}{T} + \frac{pdv}{T}$$

$$\text{For an ideal gas:} \quad du = c_v dT$$

$$\frac{p}{T} = \frac{R}{v}$$

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v}$$

$$s_2 - s_1 = c_v \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{v_2}{v_1} \right)$$

$$h = u + pv$$

$$dh = du + pdv + vdp$$

$$du = dh - pdv - vdp$$

$$\text{Substituting into } Tds = du + pdv$$

$$Tds = dh - pdv - vdp + pdv$$

$$Tds = dh - vdp$$

$$ds = \frac{dh}{T} - \frac{v}{T} dp$$

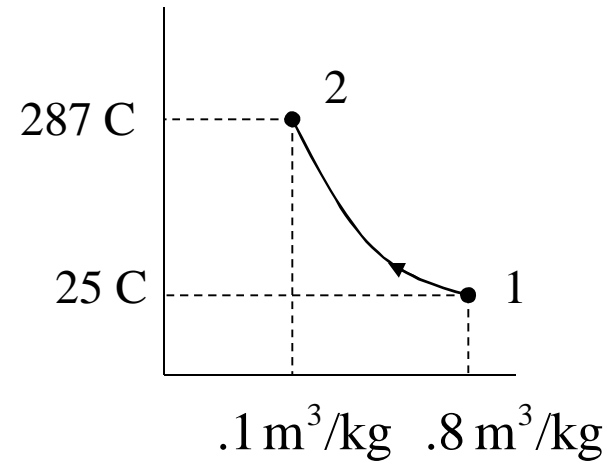
$$\text{For an ideal gas,} \quad \frac{dh}{T} = c_p \frac{dT}{T}$$

$$\frac{v}{T} dp = R \frac{dp}{p}$$

$$ds = c_p \frac{dT}{T} - R \frac{dp}{p}$$

$$s_2 - s_1 = c_p \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{p_2}{p_1} \right)_{10}$$

**Oxygen at .8 cubic meters/kg and 25 C is compressed in a piston cylinder to .1 cubic meter/kg and 287 C. What is the entropy change of the oxygen?**



$$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$$

$$s_2 - s_1 = .69 \frac{\text{kJ}}{\text{kg}^\circ\text{K}} \ln\left(\frac{273.15^\circ\text{K} + 287^\circ\text{C}}{273.15^\circ\text{K} + 25^\circ\text{C}}\right) + .259 \frac{\text{kJ}}{\text{kg}^\circ\text{K}} \ln\left(\frac{.1 \text{ m}^3/\text{kg}}{.8 \text{ m}^3/\text{kg}}\right)$$

$$s_2 - s_1 = -.105 \frac{\text{kJ}}{\text{kg}^\circ\text{K}}$$

**Steam at 200 C and 200 kPa condenses in a 20 liter closed radiator with both the inlet and exit valves closed to a temperature of 80 C. Determine the entropy change of the steam.**

$$@ P = 200 \text{ kPa}, \quad T = 200^\circ \text{C}$$

$$v_1 = 1.08049 \text{ m}^3/\text{kg}$$

$$s_1 = 7.5081 \text{ kJ/kg}$$

$$@ T = 80^\circ \text{C}$$

$$v_f = .001029 \text{ m}^3/\text{kg}, \quad v_g = 3.4053 \text{ m}^3/\text{kg}$$

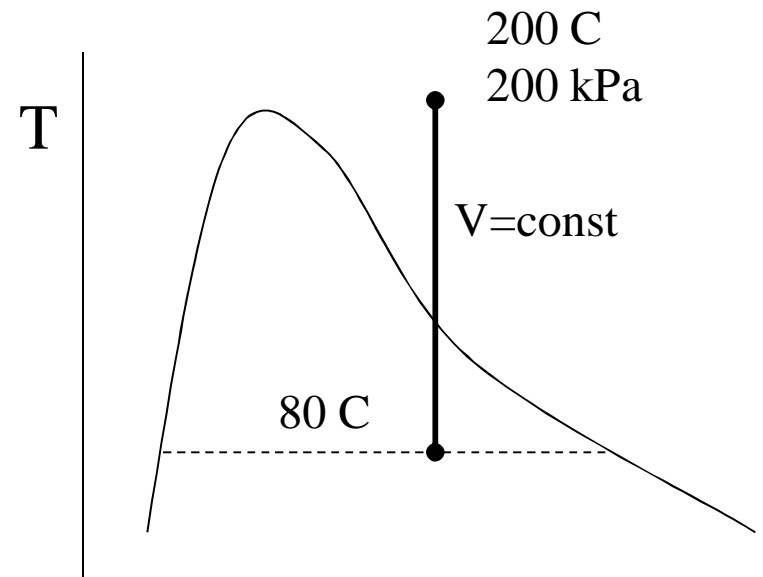
$$V_1 = V_2$$

$$v_1 = v_2$$

$$x = \frac{v_2 - v_f}{v_{fg}} = \frac{1.08049 - .001029}{3.4053 - .001029} = .317$$

$$s_2 = s_f + x \times s_{fg} = 1.0756 + .317 \times 6.5355 = 3.1479$$

Cooled, closed steam radiator



$$m = \frac{V_1}{v_1} = \frac{.02 \text{ m}^3}{1.08049 \text{ m}^3/\text{kg}} = .0185 \text{ kg}$$

$$S_2 - S_1 = m \times (s_2 - s_1)$$

$$S_2 - S_1 = .0185 \times (3.1479 - 7.5081)$$

$$S_2 - S_1 = -.0807 \frac{\text{kJ}}{\text{K}}$$

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## Isentropic Adiabatic Process

$$Q = dU + W \quad \text{First Law}$$

$$\text{Adiabatic process} \quad Q = 0$$

$$dU + W = 0$$

$$c_v dT + pdv = 0$$

$$\text{for an ideal gas} \quad T = \frac{pv}{R}, \quad dT = \frac{pdv}{R} + \frac{vdp}{R}$$

$$\frac{c_v}{R} pdv + \frac{c_v}{R} vdp + pdv = 0$$

$$\left( \frac{c_v}{R} + 1 \right) pdv + \frac{c_v}{R} vdp = 0$$

$$\left( \frac{1}{k-1} + 1 \right) \frac{dv}{v} + - \left( \frac{1}{k-1} \right) \frac{dp}{p} = 0$$

integrating

$$k \ln v + \ln p = \text{constant}$$

$$pv^k = \text{constant} \quad \text{Adiabatic}$$

$$\text{Isentropic, } s = 0$$

Ideal Gas

$$pv^k = \text{constant}$$

$$p_1 v_1^k = p_2 v_2^k$$

$$\frac{p_2}{p_1} = \left( \frac{v_1}{v_2} \right)^k$$

substitute from  $pv = RT$

$$\frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{k-1} = \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}}$$

$$s_2 - s_1 = c_p \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{p_2}{p_1} \right)$$

$$s_2 - s_1 = c_p \ln \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} - (c_p - c_v) \ln \left( \frac{p_2}{p_1} \right)$$

$$s_2 - s_1 = c_p \frac{k-1}{k} \ln \left( \frac{p_2}{p_1} \right) - (c_p - c_v) \ln \left( \frac{p_2}{p_1} \right)$$

$$s_2 - s_1 = \ln \left( \frac{p_2}{p_1} \right) \left( c_p \frac{k-1}{k} - (c_p - c_v) \right)$$

$$s_2 - s_1 = \ln \left( \frac{p_2}{p_1} \right) \left( c_p \frac{\frac{c_p - c_v}{c_p} - \frac{c_v}{c_p}}{\frac{c_p}{c_v}} - (c_p - c_v) \right) = 0$$

adiabatic process,  $pv^k = \text{constant}$

is also constant entropy

$$Q = U + W \quad \text{First Law}$$

$$dQ = dU + dW$$

$$dQ = Tds \quad \text{Second Law}$$

$$dW = pdv \quad \text{Boundary Work}$$

$$du = c_v dT \quad \text{u property definition}$$

substituting,

$$Tds = du + pdv$$

$$h = u + pv \quad \text{h property definition,}$$

h is an exact differential

$$dh = du + dv + vdp$$

substituting,

$$Tds = dh - pdv - vdp + pdv$$

$$Tds = dh - vdp$$

$$\text{for } Q = Tds = 0$$

$$dh = vdp$$

### Example: water pumped from 10 psia to 30 psia

$$w = h_2 - h_1 = v(p_2 - p_1)$$

$$w = \frac{(30 \text{ psia} - 10 \text{ psia}) \times 144 \text{ psf/psi}}{62.4 \text{ lb/ft}^3}$$

$$w = \frac{2160 \frac{\text{lb}_f}{\text{ft}^2}}{62.4 \frac{\text{lb}_m}{\text{ft}^3} \frac{1}{\text{ft}}} = 34.6 \frac{\text{ft} \cdot \text{lb}_f}{\text{lb}_m}, \text{ (ft of fluid)}$$

$$w = 34.6 \frac{\text{ft} \cdot \text{lb}_f}{\text{lb}_m} \times \frac{1 \text{ BTU}}{778 \text{ ft} \cdot \text{lb}_f} = .0445 \text{ BTU/lb}_m$$

### Example: water pumped from 100 kPa to 300 kPa @ 100 C

$$w = v(p_2 - p_1)$$

$$w = .001043 \times (300 - 100)$$

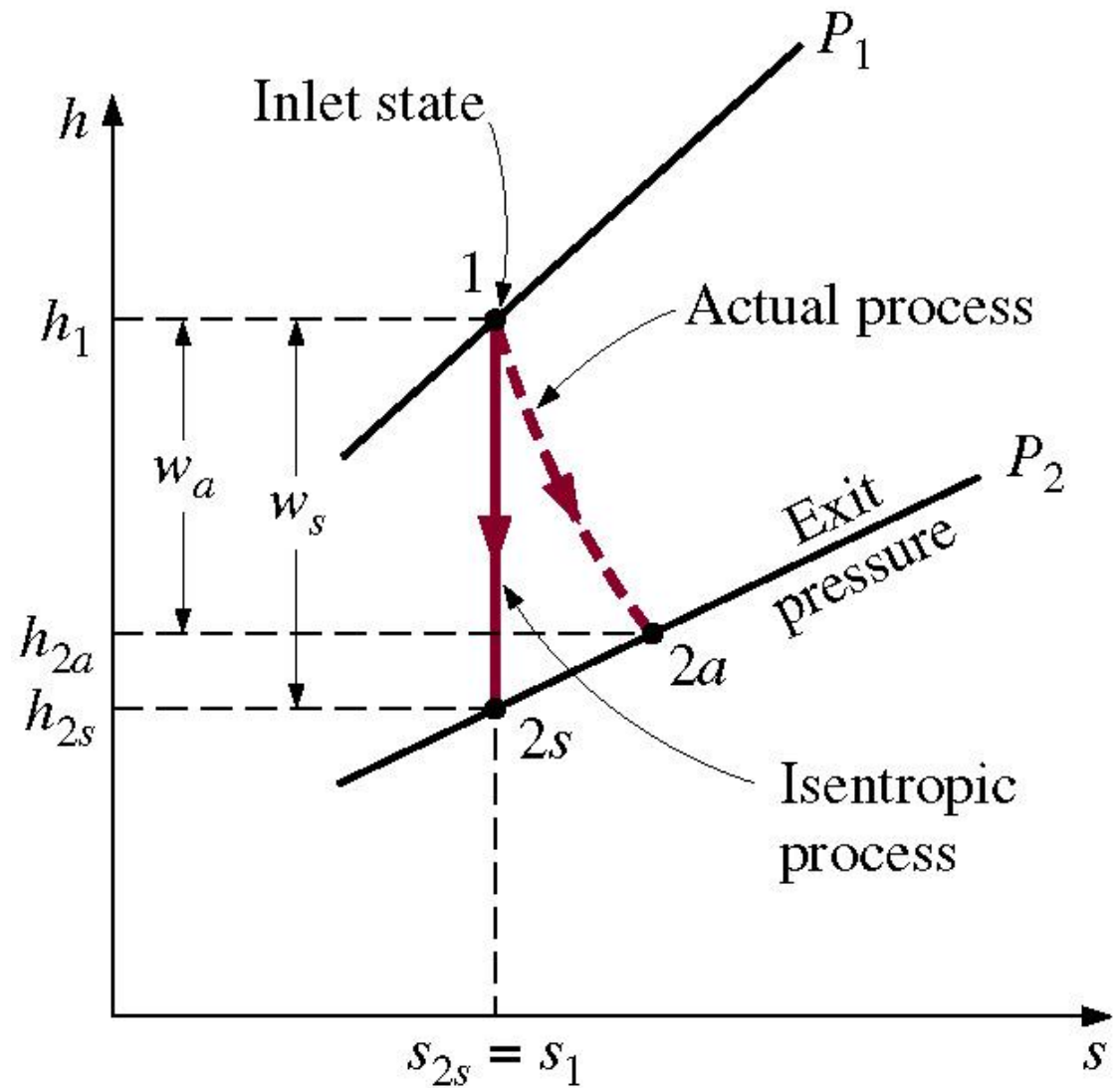
$$w = .2086 \frac{\text{m}^3}{\text{kg}} \text{ kPa}, \quad \text{kJ/kg} \quad 14$$

## Expansion Process

$$= \frac{\text{Actual Work}}{\text{Isentropic Work}}$$

$$= \frac{W_{\text{actual}}}{W_{\text{isentropic}}}$$

$$= \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$



**3 kg/sec of steam expands in an 90 % efficient turbine from 8 MPa, 500 C to 30 kPa. Determine the exit temperature and power of the turbine.**

Pt	T	p	h	s
1	500°C	8 MPa	3399.5	6.7266
1'		30 kPa	2267.27	6.7266
2	69.09°C	30 kPa	2308.9	

$$\text{@30kPa } x = \frac{s_1 - s_g}{s_{fg}} = \frac{6.7266 - .9441}{6.8234} = .847$$

$$h_1' = h_f + x \times h_{fg}$$

$$h_1' = 289.27 + .847 \times 2335.3 = 2267.27 \text{ kJ/kg}$$

$$= \frac{W_{\text{actual}}}{W_{\text{reversible}}} = \frac{h_1 - h_2}{h_1 - h_1'}$$

$$W = x \times (h_1 - h_1')$$

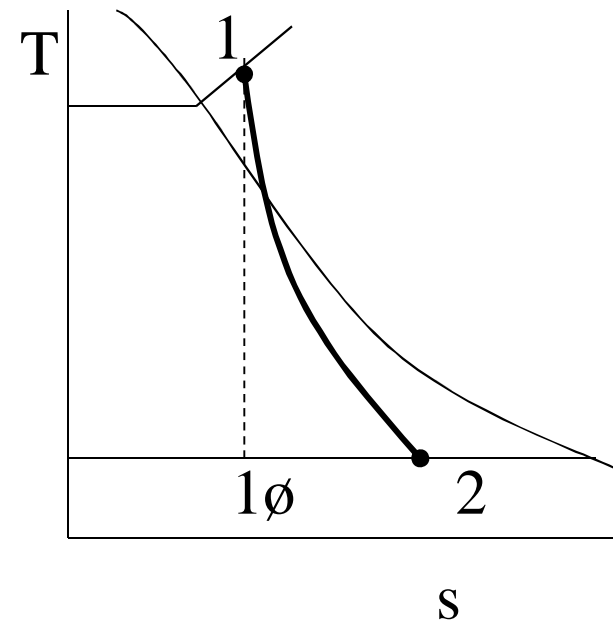
$$W = .9 \times (3399.5 - 2267.27) = 1019. \text{ kJ/kg}$$

$$h_2 = h_1 - W = 3399.5 - 1019. = 2380.5$$

$h_2$  is less than  $h_g$  @ 30 kPa  $\Rightarrow$  two phase

$$T_2 = T_{\text{saturation}} @ 30 \text{ kPa} = 69.09 \text{ C}$$

$$\text{Power} = m \times W = 3 \text{ kg/sec} \times 1019. \text{ kJ/kg} = 3057 \text{ KW}$$





**What is the work done by 3 kg/sec of nitrogen expanded from 900 kPa, 50 C to 100 kPa at an 85% efficiency.**

$$\frac{T_{1is}}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}}$$

$$T_{1is} = T_1 \left( \frac{100}{900} \right)^{.2857}$$

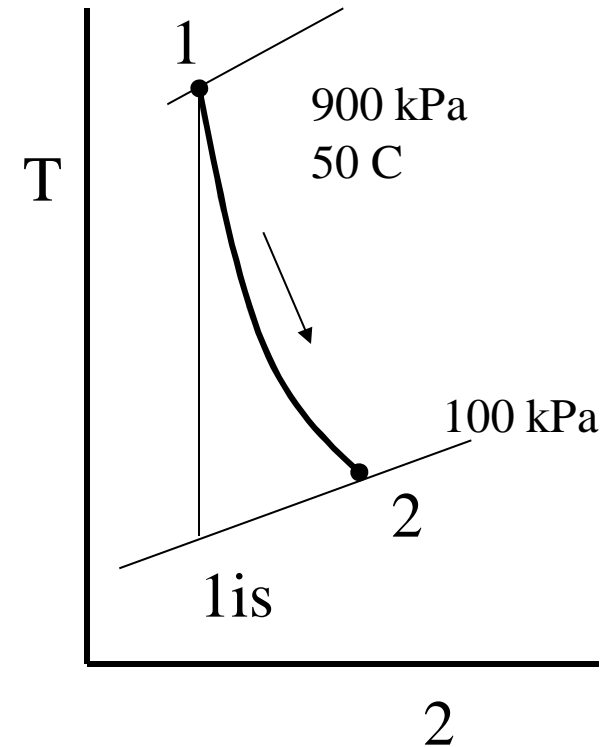
$$T_{1is} = (50 + 273)(.5338)$$

$$T_{1is} = 172.4^\circ \text{ K}$$

$$= \frac{W_{\text{actual}}}{W_{\text{isentropic}}} = .85$$

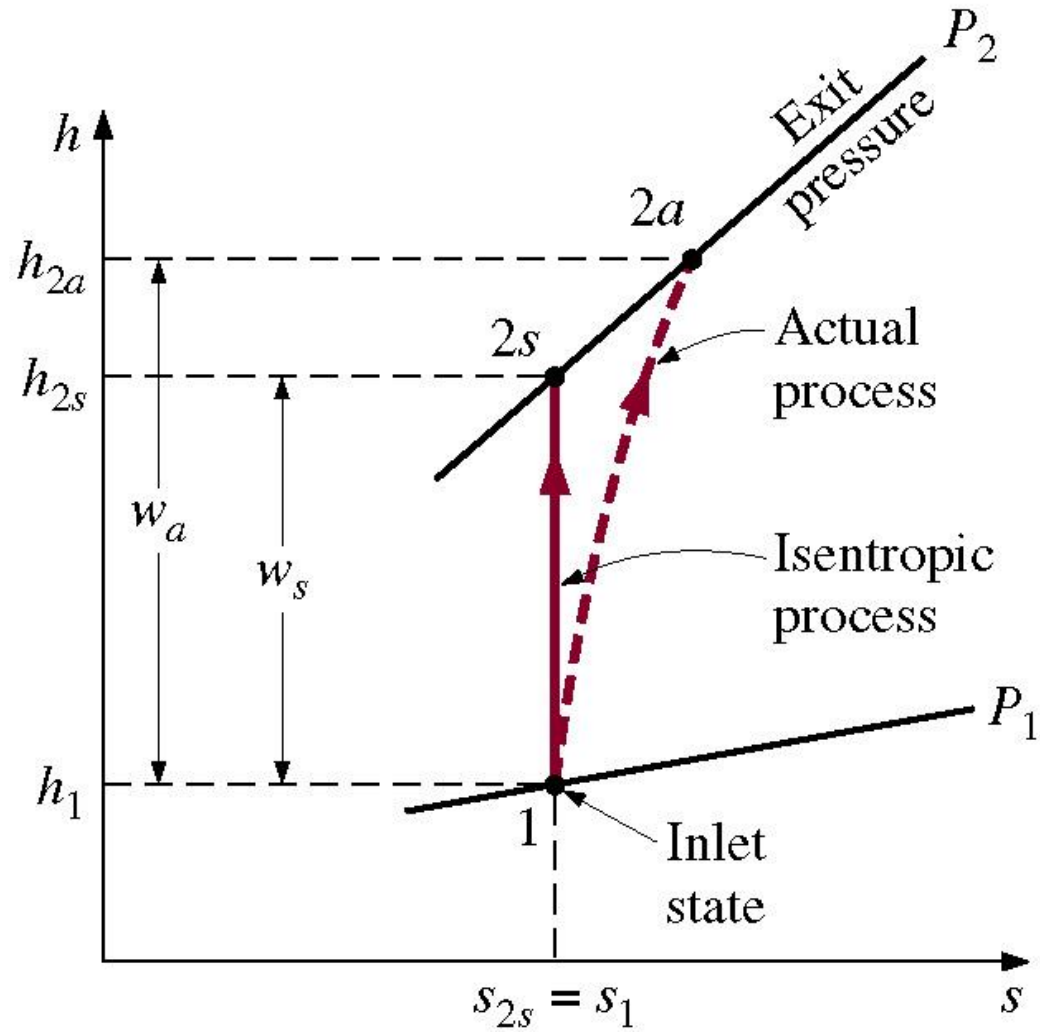
$$W_{\text{actual}} = .85 \times 3 \text{ kg/sec} \times 1.039 \times (323^\circ \text{ K} - 172.4^\circ \text{ K})$$

$$W_{\text{actual}} = 399. \text{ kJ}$$



# Compression Process

$$\begin{aligned}
 &= \frac{\text{Isentropic Work}}{\text{Actual Work}} \\
 &= \frac{W_{\text{isentropic}}}{W_{\text{actual}}} \\
 &= \frac{h_1 - h_{2s}}{h_1 - h_{2a}}
 \end{aligned}$$



**.3 cubic meters/sec of R-134a is compressed at 80% efficiency from saturated vapor at 120 kPa to 1 Mpa. What is the discharge temperature and work?**

@ 120 kPa,  $T_1 = -22.32^\circ\text{C}$

$$h_1 = h_g = 236.97 \text{ kJ/kg}$$

$$s_1 = s_g = .94779 \text{ kJ/kg}^\circ\text{K} = s_{1is}$$

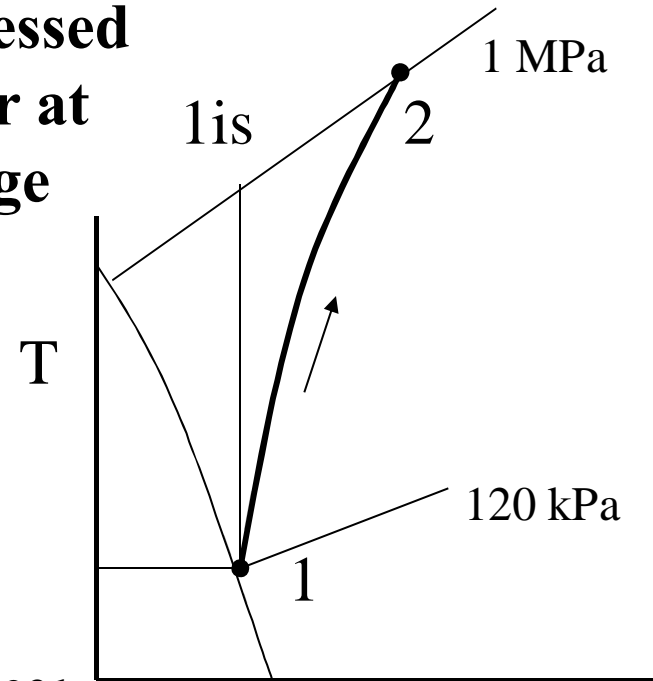
$$v_1 = v_g = .16212 \text{ m}^3/\text{kg}$$

$$m_1 = \frac{V}{v} = \frac{.3/60}{.16212} = .0308 \text{ kg/sec}$$

@ 1 MPa

Pt	T	h	s
	50	280.19	.9425
1is		278.92	.94799
	60	270.33	.9850

$$\text{entropy ratio} = \frac{.00549}{.0425} = .129$$



a)  $h_{1is} = 278.92$  by interpolation

$$= \frac{h_{1is} - h_1}{h_2 - h_1} = .8$$

$$h_2 = h_1 + (h_{is} - h_1) / .8$$

$$h_2 = 236.97 + (278.92 - 236.97) / .8$$

$$h_2 = 289.4 \text{ kJ/kg}$$

$$T_2 = 57.7^\circ\text{C} \text{ by interpolation}$$

b)  $W = m(h_2 - h_1)$

$$W = .0308 \text{ kg/sec} \times (289.4 - 236.97)$$

$$W = 1.615 \text{ kJ/sec} = 1.615 \text{ KW}$$

**Air is compressed from 15 psia, 60 F to 30 psia at 85% efficiency. What is the actual discharge temperature?**

$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}}$$

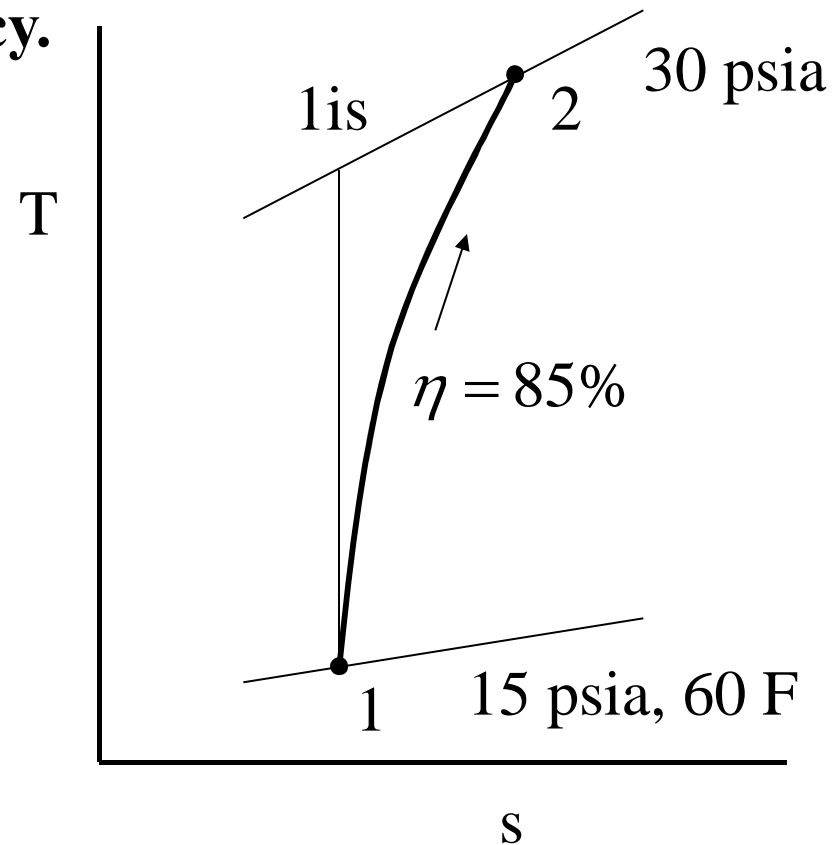
$$T_{1is} = T_1 \left( \frac{30}{15} \right)^{.2857}$$

$$T_{1is} = 520 \times 1.219 = 633.9^\circ \text{R}$$

$$= \frac{h_{1is} - h_1}{h_2 - h_1} = \frac{c_p (T_{1is} - T_1)}{c_p (T_2 - T_1)}$$

$$T_2 = T_1 + \left( \frac{633.9 - 520}{.85} \right) = 654.^\circ \text{R}$$

$$T_2 = 194^\circ \text{F}$$



## W=0 Expansion Process

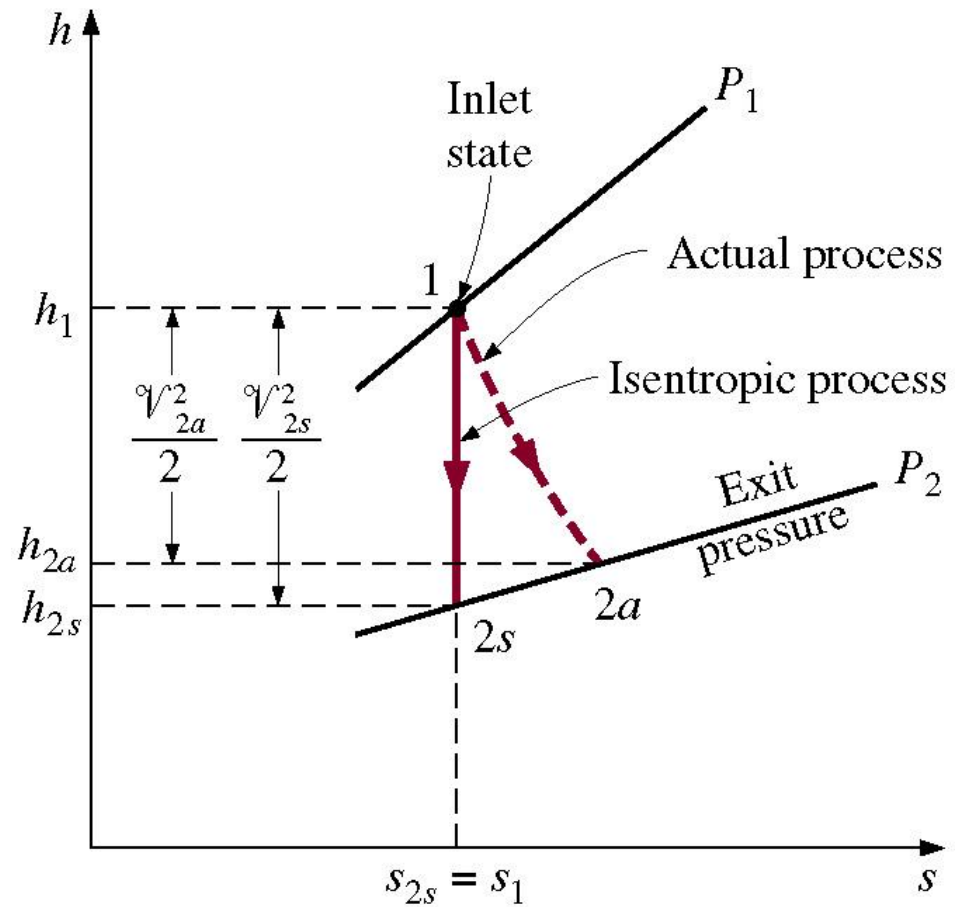
Steady Flow Energy Equation

$H = \text{constant}$

$$h_1 = h_{2\text{actual}} + \frac{V_{2\text{actual}}^2}{2}$$

$$h_1 = h_{2\text{isentropic}} + \frac{V_{2\text{isentropic}}^2}{2}$$

$$= \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{V_{2a}^2}{V_{2s}^2}$$



## AIR TABLE - Table A-22, A-22E

a) variable specific heat for u, h and s. Table base 0° F, 0° C

$$h = \int c_p(T) dT \quad \text{Section 3.7, page 113}$$

$$u = \int c_v(T) dT$$

$$s = \int c_p \frac{dT}{T} + \int R \frac{dp}{p} \quad \text{Section 6.3.2, page 230}$$

b) does the p, v, T calculation with variable  $k = \frac{c_p}{c_v}$  for an isentropic process

$$\frac{p_1}{p_2} = \frac{(\text{table } p_r)_1}{(\text{table } p_r)_2} \quad \text{Section 6.7.2, page 256}$$

$$\frac{v_1}{v_2} = \frac{(\text{table } v_r)_1}{(\text{table } v_r)_2}$$

# Air undergoes an adiabatic, isentropic expansion from 900 Kpa, 1000 K to 100 kPa.

@ room temperature, 300 K, ideal gas values, Figure 4 - 24

$$c_p = 1.050, \quad k = 1.4$$

$$T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} = 1000 \left( \frac{1}{9} \right)^{\frac{1.4-1}{1.4}} = 533.8^\circ \text{K}$$

@ 1000° K constant values, Figure 4 - 24

$$c_p = 1.142, \quad k = 1.336$$

$$T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} = 1000 \left( \frac{1}{9} \right)^{\frac{1.336-1}{1.336}} = 575.5^\circ \text{K}$$

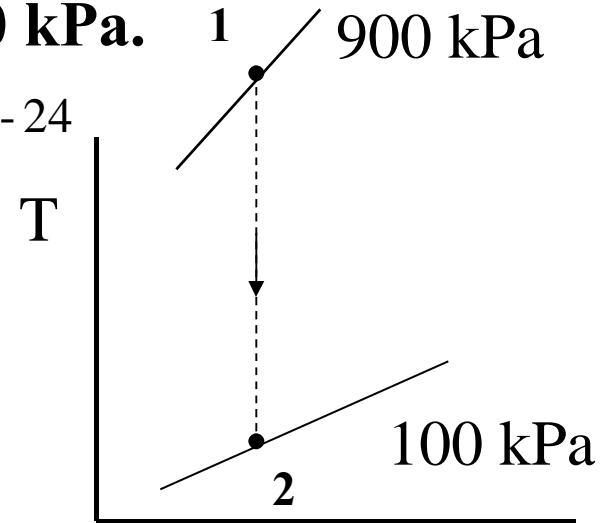
variable specific heat, Table A - 17

@ 1000° K,  $p_{r1} = 114.$ ,  $v_{r1} = 25.17$

$$\frac{p_2}{p_1} = \frac{p_{r2}}{p_{r1}}$$

$$p_{r2} = p_{r1} \left( \frac{p_2}{p_1} \right) = 114 \times \frac{100}{900} = 12.667$$

@  $p_{r2} = 12.667$ ,  $T_2 = 560^\circ \text{K}$



$$v_2 = \frac{RT}{p} = \frac{.286 \times 560.}{100 \text{ kPa}} = 1.6016 \text{ m}^3/\text{kg}$$

Table A - 17  $T_2 = 560^\circ \text{R}$ ,  $v_{r2} = 127.$

$$\frac{v_2}{v_1} = \frac{v_{r2}}{v_{r1}}$$

$$v_1 = v_2 \times \frac{v_{r1}}{v_{r2}} = 1.6016 \text{ m}^3/\text{kg} \times \frac{25.17.}{127.} = .3174 \text{ m}^3/\text{kg}$$

$$\text{Using Ideal Gas Law } v_1 = \frac{RT}{p} = \frac{.286 \times 100.}{900 \text{ kPa}}$$

$v_1 = .3178 \text{ m}^3/\text{kg}$  (almost no difference. Ideal gas law gives a good volume estimate)