11.1 Air flowing in a 1 foot square duct at 100 ft/sec, 530 R and 100 psi is expanded to choked conditions through a converging nozzle. What is the area, velocity, mass flow, volume flow and Mach Number at the nozzle throat? If the throat area is kept constant and the duct pressure is increased to 125 psi and then to 150 psi how do the throat velocity, volume flow, mass flow and pressure change?

11.2 Air is expanded from the duct conditions of problem 11.1 in a converging diverging nozzle to a back pressure of 70 psi. What is the exit area of the diverging portion of the nozzle? What would happen if the back pressure downstream of the diverging portion exit were above and below 70 psi.

11.3 With a back pressure of 80 psi in the nozzle of problem 11.2, with the throat area kept the same, at what area ratio (A shock/ A*) would a shock form?
isentropic @ $M_1 = 1$

\[
\frac{T_O}{T^*} = 1.200, \quad \frac{p_O}{p^*} = 1.839, \quad \frac{\rho_O}{\rho^*} = 1.577
\]

\[
T_t = \frac{T_t}{T_O} \frac{T_O}{T_1} = \frac{1}{1.2} \times 1.0016 \times 530 = 442.2
\]

\[
a_t = \sqrt{1.4 \times 32.2 \times 53.35 \times 442.2} = 1031.1
\]

\[
V_t = 1031.3 \text{ft} / \text{sec}
\]

\[
Q_t = A_t \times V_t = .1532 \times 1031.3 = 157.1 \text{ft}^3 / \text{sec}
\]

\[
T_t = \frac{T_t}{T_O} \frac{T_O}{T_1}
\]

\[
p_t = \frac{p_t}{p_O} \frac{p_O}{p_1} = \frac{1}{1.577} \times 1.0055 \times 100
\]

\[
p_t = 63.76 \text{psi}
\]

\[
m_t = Q_t V_t = 157.1 \times .3239 = 50.89 \text{lb} / \text{sec}
\]

\[
a_1 = \sqrt{\gamma RT}
\]

\[
a_1 = \sqrt{1.4 \times 32.2 \times 530 \times 53.35}
\]

\[
a_1 = 1129 \text{ft} / \text{sec}
\]

\[
M_1 = \frac{V_t}{a_1} = 100/1129 = .0886
\]

\[
\rho_1 = \frac{p}{RT} = \frac{100 \times 144}{53.35 \times 530} = .509 \text{lb/ft}^3
\]

isentropic @ $M_1 = .0886$

\[
\frac{p_O}{p_1} = 1.0055, \quad \frac{T_O}{T_1} = 1.0016, \quad \frac{\rho_O}{\rho_1} = 1.0039
\]

\[
\frac{A_1}{A_t} = \frac{A_t}{A^*} = 6.5625
\]

\[
A_t = \frac{A_1}{6.5625} = .1523 \text{ ft}^2
\]
\[ \frac{m_1}{m_2} = \frac{(\rho AV)_1}{(\rho AV)_2} = \frac{\rho_1}{\rho_2} = \frac{p_1}{p_2} \frac{R_2 T_2}{RT_1} \]

@ \( p_1 = 125 \text{psi} \)
\[ p_t = \frac{p_t}{p_o} \frac{p_o}{p_1} = \frac{1}{1.577} \times 1.0055 \times 125 = 63.76 \text{ psi} \]

\( Q_t = 157.1 \text{ ft}^3/\text{sec} \)

@ \( p_1 = 150 \text{ psi} \)
\[ m = 50.89 \frac{150}{100} = 76.33 \text{ lb/sec} \]
\[ p_t = \frac{p_t}{p_o} \frac{p_o}{p_1} = \frac{1}{1.577} \times 1.0055 \times 150 = 95.64 \text{ psi} \]

\( Q_t = 157.1 \text{ ft}^3/\text{sec} \)
At lower pressures exit approaches sonic

\[ p_e = 70 \text{ psi} \]

\[ \frac{A_1}{A_t} = \frac{A_1}{A^*} = 6.5625 \]

\[ A_t = \frac{A_1}{6.5625} = .1523 \text{ ft}^2 \]

\[ \frac{p_o}{p_e} = \frac{1.0055}{1.0016} \times \frac{100}{70} = 1.4364 \]

\[ \frac{A_e}{A^*} = 1.068, \quad M_e = .74 \]

\[ A_e = \frac{A_e}{A^*} \frac{A^*}{A_1} A_1 = 1.068 \times \frac{1}{6.5625} = .1627 \text{ ft}^2 \]

11.2

100 ft/sec
530 R
100 psi
1 ft^2

definition

11.3

no shocks would occur in the nozzle