Problem Set 9 (PS 9) Due Tuesday March 28

9.1 Compute the lift, drag and lift/drag ratio for a flat plate positioned a 5 angles of attach from 0 to 25 degrees in an air flow at M= 2.5, 1 atm, 270 K. What happens to the flow at an angle of 30 degrees? Qualitatively how does this affect the lift and drag?

9.2 A diamond shaped air foil with a 15 degree half angle, a thickness of .5 ft and a 1 ft span is positioned at an angle of attack of 5 degrees in an air flow at M=2, p= 2116 lb/square foot. Calculate the lift, drag and lift drag ratio.

9.3 A supersonic flow with a Mach Number of 4 and a pressure of 1 atm is first deflected 10 degrees over an expansion corner and then deflected 10 degrees through a compression corner. The flow after the two corners is in the same direction as it was approaching the corners. Calculate the Mach Number and pressure of the flow downstream of these corners.
9.1 FLAT PLATE AT 5 DEGREE ANGLE OF ATTACK

UPPER SURFACE

isentropicTableA.1@M_1 = 2.5;

\[ \frac{P_{01}}{p_1} = 17.085, \frac{T_0}{T_1} = 2.2502 \]

P \_ MTableA.5@M_1 = 2.5

\[ \nu_1 = 39.12 \]

\[ \nu_3 = \nu_1 + \theta_2 = 39.12 + 5 = 44.1 \]

P \_ MTableA.5@\nu_3 = 44.1; M_3 = 2.7233

isentropicTableA.1@M_3 = 2.7233;

\[ \frac{P_{03}}{p_3} = 24.13, \]

\[ \frac{T_3}{T_1} = 2.4832 \]

\[ p_3 = 1.3799 \]

\[ T_3 = 296.3 \]

\[ \alpha = 5^\circ \]

LOWER SURFACE

M - \beta - \theta@M_1 = 2.5; \beta = 27.42

M_{in} = M_1 \sin \beta = 1.1514

normal shock Table A.2 @ M_{in} = 1.1514;

\[ \frac{p_2}{p_1} = 1.3799, \]

\[ \frac{T_2}{T_1} = 1.0974 \]

\[ p_2 = p_1 \times 1.3799 \times 1 = 1.3799, \]

\[ T_2 = 1.0974 \times 270 = 296.3 \]

Lift = \( (p_2 - p_1) \cos \alpha \) = .6693

Drag = \( (p_2 - p_1) \sin \alpha \) = .0586

Lift / Drag = 11.4
UPPER SURFACE

isentropic Table A.1 @ M = 2.6; \( \frac{p_0}{p_1} = 17.17 \)

P – M Table A.5 @ \( M_1 = 2.5; \) \( v_1 = 39.1 \)

at \( \alpha = 5, \) \( v = v_1 + \theta = 44.1 \)

P – M Table A.5 @ \( v = 44.1; \) \( M_1 = 2.7233 \)

isentropic Table A.1 @ M = 2.7233; \( \frac{p_0}{p_3} = 24.13 \)

\( p_3 = p_1 \frac{p_0}{p_3} = 1 \times 17.17/24.15 = .711 \)

\begin{tabular}{cccc}
\( \alpha \) & \( v \) & \( M \) & \( \frac{p_0}{p_3} \) & \( p_3 \) \\
0 & 39.1 & 2.50 & 17.17 & 1 atm \\
5 & 44.1 & 2.7233 & 24.13 & .7110 \\
10 & 49.1 & 2.9674 & 34.98 & .4885 \\
15 & 54.1 & 3.2368 & 52.19 & .3231 \\
20 & 59.1 & 3.5376 & 80.45 & .2123 \\
25 & 64.1 & 3.877 & 128.7 & .1328 \\
30 & 69.1 & 4.265 & 214.9 & .0795 \\
\end{tabular}

LOWER SURFACE

\( \alpha \) \( \beta \) \( M_{in} \) \( p_2 = \frac{p_2}{p_1} \times 1 \)

0
5 \( 27.42 \) \( 1.1514 \) \( 1.380 \)
10 \( 31.85 \) \( 1.3193 \) \( 1.864 \)
15 \( 36.94 \) \( 1.503 \) \( 2.468 \)
20 \( 42.89 \) \( 1.702 \) \( 3.211 \)
25 \( 50.25 \) \( 1.922 \) \( 4.143 \)
30 detached oblique shock \( M_2 \)
Lift = \((p_3 - p_2)\cos \theta\)
Drag = \((p_3 - p_2)\sin \theta\)

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>(p_2)</th>
<th>(p_3)</th>
<th>Lift</th>
<th>Drag</th>
<th>\frac{Lift}{Drag}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.</td>
<td>1.</td>
<td></td>
<td></td>
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<tr>
<td>5</td>
<td>0.711</td>
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<td>0.669</td>
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<tr>
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<td>2.818</td>
<td>1.926</td>
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</tr>
<tr>
<td>25</td>
<td>0.1328</td>
<td>4.143</td>
<td>3.635</td>
<td>1.695</td>
<td>2.14</td>
</tr>
</tbody>
</table>
\[ \theta_1 = \varepsilon - \alpha \]
\[ \theta_2 = \varepsilon + \alpha \]
\[ \theta_3 = \theta_4 = 180 - (180 - 2\varepsilon) \]
\[ \theta_3 = 2\varepsilon \]
**Upper Surface – region 1 - 2 - 3**

\[ M_1 = 2.0 \]

| \( \theta_2 \) | 10 |
| \( \beta_2 \) | 39.31 |
| \( M_2 \) | 1.64 |
| \( p_2 / p_1 \) | 1.707 |
| \( p_2 \) | 3611.2 |
| \( F_2 \) | 3488.4 |

**P - M expansion**

\[ \nu_2 = 16.07 \]

**isentropic flow**

\[ p_{o2} / p_2 = 4.518 \]

\[ \theta_3 = 30 \]

\[ \nu_3 = \nu_2 + \theta_3 \]

\[ \nu_3 = 46.07 \]

**P - M expansion**

\[ M_3 = 2.816 \]

**isentropic flow**

\[ p_{o3} / p_3 = 27.8 \]

\[ p_3 = p_1 \frac{p_3}{p_{o3}} \frac{p_2}{p_1} \]

\[ p_3 = 586.8 \]

\[ F_3 = 566.8 \]

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**Lower Surface – regions 4 - 5 - 6**

\[ \theta_5 = 20. \]

\[ \beta_5 = 53.42 \]

\[ M_5 = 1.21 \]

\[ p_{5} / p_4 = 2.823 \]

\[ p_5 = 5973.5 \]

\[ F_5 = 5770.4 \]

\[ \nu_5 = 3.811 \]

\[ p_{o5} / p_5 = 2.458 \]

\[ \theta_6 = 30 \]

\[ \nu_6 = 33.81 \]

\[ M_6 = 2.28 \]

\[ p_{o6} / p_6 = 12.142 \]

\[ p_6 = 1209.2 \]

\[ F_3 = 1168. \]
9.2

Drag = \sum F_x = \left( F_2 \cos(90 - \theta_2) + F_5 \cos(90 - \theta_5) \right) \\
\quad - \left( F_3 \cos(90 - \theta_3 + \theta_2) + F_6 \cos(90 - \theta_6 + \theta_5) \right)

Drag = \sum F_x = \left( 3488.4 \cos(80) + 5770.4 \cos(70) \right) \\
\quad - \left( 566.8 \cos(70) + 1168 \cos(80) \right)

Drag = \sum F_x = \left( 605.7 - 1973.6 \right) - \left( 193.9 - 202.8 \right)

Drag = \sum F_x = 2182

Lift = \sum F_y = \left( F_2 \sin(90 - \theta_2) + F_5 \sin(90 - \theta_5) \right) \\
\quad - \left( F_3 \sin(90 - \theta_3 + \theta_2) + F_6 \sin(90 - \theta_6 + \theta_5) \right)

Lift = \sum F_y = \left( 3435. + 532.6 \right) - \left( 5422.21150.3 \right)

Lift = \sum F_y = 2605
isentropic flow @ $M_1 = 4$;
\[
p_{O_1} = \frac{151.83}{p_1} = \frac{p_{O_2}}{p_2} \quad \quad P - M @ M_1 = 4; \quad v_1 = 65.78
\]
\[
v_2 = v_1 + \theta_1 = 65.78 + 10 = 75.78
\]
\[
P - M @ v_2 = 75.78; \quad M_2 = 4.88
\]
isentropic flow @ $M_2 = 4.88$;
\[
\frac{p_{O_2}}{p_2} = 450.92
\]
\[
M - \beta - \theta @ M_2 = 4.88, \theta_2 = 10; \quad \beta = 19.65
\]
\[
M_{2n} = M_2 \sin \beta = 1.641
\]
normal shock @ $M_{2n} = 1.641$;
\[
M_{3n} = 0.6565, \quad \frac{p_3}{p_2} = 2.975
\]
\[
M_3 = \frac{M_{3n}}{\sin(\beta - \theta)} = 3.917
\]
isentropic flow @ $M_3 = 3.917$;
\[
\frac{p_{O_3}}{p_3} = 146.07
\]
\[
p_3 = p_1 \frac{p_3}{p_2} \frac{p_2}{p_{O_2}} \frac{p_{O_1}}{p_1} \quad \quad p_3 = 22.975 \frac{1}{450.82} 151.83 = 1 \text{ atm}
\]
\[
p_{O_1} - p_{O_3} = 151.83 - 146.07 = 5.8 \text{ atm}
\]