7.1 Air is flowing in a duct configured like Figure 4.19 in the text with $\theta_1 = 10$ degrees, $\theta_2 = 20$ degrees and with Mach Number of 3 in the flow approaching the corner. What is the temperature, pressure and velocity in region 3 after the second reflection?

7.3 Plot 6 points of a pressure deflection diagram for flows over a wedge with $M = 2.6$. Include some points for both a weak and strong shock.

7.2 Air is flowing through a duct with corners configured like Figure 4.23 in the text with $\theta_1 = 10$ degrees, $\theta_2 = 20$ degrees and with Mach Number of 3 in the flow approaching the corners. What are the temperatures, pressure and velocities in the regions 4 and 4'?

7.4 Derive in detail showing all steps equation 4.35 in the test, the governing equation for Prandtl-Meyer flow. In what ways is this equation restricted?
\[ \theta - M - \beta \text{ @ } M_1 = 3, \theta = 10 \]
\[ \beta = 27.38 \]
\[ M_{1n} = M_1 \sin \beta = 1.38 \]
\[ \text{Table A.2 @ } M_{1n} = 1.38 \]
\[ M_{2n} = .748 \]
\[ p_2/p_1 = 2.054 \]
\[ T_2/T_1 = 1.242 \]
\[ M_2 = M_{2n}/\sin(\beta - \theta) = 2.504 \]

\[ \theta - M - \beta \text{ @ } M_2 = 2.504, \theta = 30 \]

won't reflect. properties indeterminate by 1D methods
7.2

$\theta - M - \beta$ Chart @ $M_1 = 3$, $\theta_2 = 20 \implies \beta = 37.76$

$M_{1n} = m_1 \times \sin \beta = 1.8372$

Table A.2 @ $M_{1n} = 1.8372$

$M_{n2} = .6078$, $p_2/p_1 = 3.783$, $T_2/T_1 = 1.5596$

$M_2 = \frac{M_{2n}}{\sin(\beta - \theta)} = 1.994$

$\theta - M - \beta$ Chart @ $M_1 = 3$, $\theta_3 = 10 \implies \beta = 32.24$

$M_{1n} = M_1 \times \sin \beta = 1.3798$

Table A.2 @ $M_{1n} = 1.3798$

$M_{3n} = .7484$, $p_3/p_1 = 2.0545$

$M_3 = \frac{M_{3n}}{\sin(\beta - \theta)} = 2.505$
\[ \theta_2 = 20, \quad M_2 = 1.994 \]

\[
\begin{align*}
\theta_4' & \quad \beta & \quad M_{2n} = M_2 \sin \beta & \quad \frac{p_4'}{p_2} & \quad p_4' = p_1 \frac{p_4}{p_2} \frac{p_2}{p_1} & \quad \Phi = \theta_2 - \theta_4' & \quad \frac{T_4'}{T_3} & \quad M_4.
\end{align*}
\]

<table>
<thead>
<tr>
<th>( \theta_4' )</th>
<th>( \beta )</th>
<th>( M_{2n} )</th>
<th>( \frac{p_4'}{p_2} )</th>
<th>( p_4' = p_1 \frac{p_4}{p_2} \frac{p_2}{p_1} )</th>
<th>( \Phi = \theta_2 - \theta_4' )</th>
<th>( \frac{T_4'}{T_3} )</th>
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<td>1.7050</td>
<td>6.450</td>
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</tr>
</tbody>
</table>

\[ \theta_3 = 10, \quad M_2 = 2.505 \]

\[
\begin{align*}
\theta_4 & \quad \beta & \quad M_{3n} = M_3 \sin \beta & \quad \frac{p_4}{p_3} & \quad p_4 = p_1 \frac{p_4}{p_3} \frac{p_3}{p_1} & \quad \Phi = \theta_2 - \theta_4 & \quad \frac{T_4}{T_3} & \quad M_4.
\end{align*}
\]

<table>
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<tr>
<th>( \theta_4 )</th>
<th>( \beta )</th>
<th>( M_{3n} )</th>
<th>( \frac{p_4}{p_3} )</th>
<th>( p_4 = p_1 \frac{p_4}{p_3} \frac{p_3}{p_1} )</th>
<th>( \Phi = \theta_2 - \theta_4 )</th>
<th>( \frac{T_4}{T_3} )</th>
<th>( M_4 )</th>
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</table>

\[ \Phi = \text{angle of stream line, slip line to inlet} \]

\[ \theta_4' = \theta_2 - \Phi = 20 - 9.75 = 11.25 \]

\[ \theta_4 = \Phi + \theta_3 = 9.75 + 10 = 19.75 \]
\[
\frac{T_{4'}}{T_1} = \frac{T'_{4'} T_2}{T_2 T_1} = 1.1744 \times 1.5596
\]
\[
\frac{T_{4'}}{T_1} = 1.832
\]
\[
\frac{v_{4'}}{v_1} = \frac{M_{4'} \times a_{4'}}{M_1 \times a_1} = \frac{M_{4'}}{M_1} \sqrt{\frac{T_{4'}}{T_1}}
\]
\[
\frac{v_{4'}}{v_1} = \frac{1.6257}{3} \sqrt{1.832} = .7335
\]
\[
\frac{T_4}{T_1} = \frac{T_4 T_3}{T_3 T_1} = 1.4529 \times 1.2417
\]
\[
\frac{T_4}{T_1} = 1.8041
\]
\[
\frac{v_4}{v_1} = \frac{M_4 \times a_4}{M_1 \times a_1} = \frac{M_4}{M_1} \sqrt{\frac{T_4}{T_1}}
\]
\[
\frac{v_4}{v_1} = \frac{1.661}{3} \sqrt{1.8041} = .7437
\]
assume a range of $\theta$'s, $0 - \theta_{\text{max}}$

$\theta - M - \beta\, @\, M_1 =, \quad \theta = \beta_{\text{weak}} = \beta_{\text{strong}} = \theta$

Table A.2 @ $M_{1n} = \beta$

$M_{2n} =, \frac{p_2}{p_1} =$

$M_1 = 2.6$

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\beta$</th>
<th>$M_{1n}$</th>
<th>$\frac{p_2}{p_1}$</th>
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7.3

[Graph showing strong and weak shock waves with $\theta$ and $\frac{p_2}{p_1}$ axes, indicating $\theta_{\text{max}}$.]
vector change in velocity across the wave is normal to the wave

two sides: $V$ and $V + dV$
opposite angles: $90 - \mu - d\theta$ and $90 + \mu$

by the law of sines.

\[
\frac{V}{\sin(90 - \mu - d\theta)} = \frac{V + dV}{\sin(90 + \mu)}
\]

\[
\frac{V + dV}{V} = \frac{\sin(90 + \mu)}{\sin(90 - \mu - d\theta)}
\]
7.4

\[
\frac{V + dV}{V} = \frac{\sin(90 + \mu)}{\sin(90 - \mu - d\theta)}
\]

from trigonometric identities,

\[
1 + \frac{dV}{V} = \frac{\cos \mu}{\cos \mu \cos d\theta - \sin \mu \sin d\theta}
\]

small angle assumption, \( \sin d\theta = d\theta \), \( \cos d\theta = 1 \)

\[
1 + \frac{dV}{V} = \frac{1}{1 - d\theta \tan \mu}
\]

expand the right side as a series,

\[
1 + \frac{dV}{V} = 1 + (d\theta \tan \mu) + (d\theta \tan \mu)^2 + 
\]

ignoring the higher order terms

\[
\frac{dV}{V} = d\theta \tan \mu
\]

since \( \mu = \sin^{-1} \frac{1}{M} \), \( \tan \mu = \frac{1}{\sqrt{M^2 - 1}} \)

\[
d\theta = \sqrt{M^2 - 1} \frac{dV}{V} \quad (4.35)
\]