Problem Set 4 (PS 4) Due Tuesday February 14

Ps 4.1 The Shuttle space craft begins to enter the atmosphere at M=25. Calculate, assuming an ideal gas and 1.4 specific heat ratio, the temperature at the stagnation point of the vehicle and the free stream velocity over the space craft at a location in the reentry path where M= 20 and T=200 K. Are these estimates accurate? If not why and are they high or low ? Where in the trajectory path would you expect the highest stagnation point temperature ?

PS 4.2 A normal shock is located at the inlet to a duct. The air approaches the duct at M=3, 400 R and .9 atm. The duct is 2 ft long and 1 ft in diameter and has a friction factor of .005. What is the pressure, the temperature and the velocity at the end of the duct ?

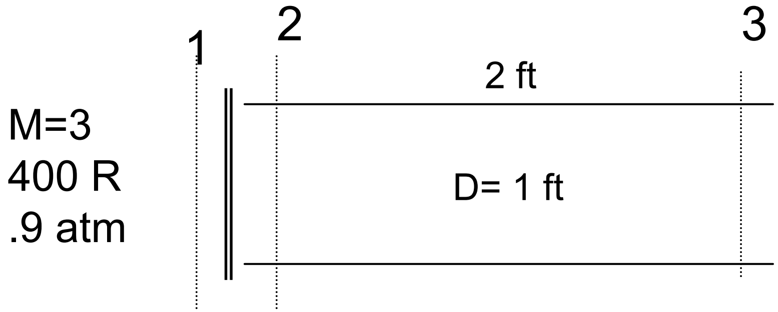
PS 4.3 The conditions of air at the exit of a duct which is .03 m diameter, 50 m long and has a friction factor of .005, are M = .4, 1 atm, and 270 K. Assuming 1D flow and an ideal gas with a specific heat ratio of 1.4 calculate the mach number, temperature and pressure at the duct inlet.

4.1 assume a normal shock in front of the space craft

M	estimated T_1	Table A.2 $\frac{T_2}{T_1}$	T_2
25	20 K	122.65	2453 K
10	230 K	20.39	4690 K
		Table A.1	
.5	290 K	1.050	305 K

The M= 25 point is not the highest temperature. The highest temperature occurs at some lower elevation on the reentry path. A 1 D shock assumes an ideal gas model. The gas around the space craft at high temperature is a plasma and its properties are not well predicted by the ideal gas model.

4.2



$$\text{Table A.2 @ } M_1 = 3,$$

$$M_2 = .4752,\ \frac{p_2}{p_1} = 10.33,\ \frac{T_2}{T_1} = 2.679$$

$$\left(\frac{4fL}{D}\right)_{duct} = \frac{4 \times .005 \times 2}{1} = .04$$

$$\text{Table A.4 @ } M_2 = .4752$$

$$\left(\frac{4fL^*}{D}\right)_2 = .1295,\ \frac{p_2}{p^*} = 2.255,\ \frac{T_2}{T^*} = 1.1512$$

$$\left(\frac{4fL^*}{D}\right)_3 = \left(\frac{4fL^*}{D}\right)_2 - \left(\frac{4fL}{D}\right)_{duct}$$

$$\left(\frac{4fL^*}{D}\right)_3 = .1295 - .01 = .0895$$

$$\text{Table A.4 @} \left(\frac{4fL^*}{D}\right)_3 = .0895$$

$$M_3 = .782,\ \frac{p_3}{p^*} = 1.3219,\ \frac{T_3}{T^*} = 1.0693$$

$$\frac{T_3}{T_1} = \frac{T_3}{T^*}\frac{T^*}{T_2}\frac{T_2}{T_1}$$

$$T_3 = T_1 \times 1.0693 \times \frac{1}{1.1512} \times 2.679$$

$$T_3 = 995.4\ R$$

$$a_3 = \sqrt{\gamma\, g\, R\, T} = \sqrt{1.4 \times 53.35 \times 32.2 \times 995.4}$$

$$a_3 = 1547.2 \text{ft/sec}$$

$$v_3 = M_3 \times a_3 = 1210 \text{ ft/sec}$$

$$\frac{p_3}{p_1} = \frac{p_3}{p^*}\frac{p^*}{p_2}\frac{p_2}{p_1}$$

$$p_3 = p_1 \times 1.3219 \times \frac{1}{2.255} \times 10.33$$

$$p_3 = 5.45 \text{ atm}$$

4.3

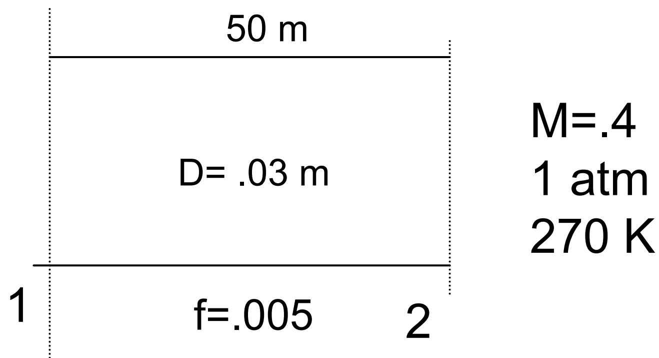


Table A.4 @ $M_2 = .4$

$$\left(\frac{4fL^*}{D}\right)_2 = 2.308,\ \frac{p_2}{p^*} = 2.696,\ \frac{T_2}{T^*} = 1.163$$

$$\left(\frac{4fL}{D}\right)_{duct} = \frac{4 \times .005 \times 50}{.03} = 33.33$$

$$\left(\frac{4fL^*}{D}\right)_1 = \left(\frac{4fL^*}{D}\right)_2 + \left(\frac{4fL}{D}\right)_{duct}$$

$$\left(\frac{4fL^*}{D}\right)_1 = 2.308 + 33.33 = 35.64$$

Table A.4 @ $\left(\frac{4fL^*}{D}\right)_1 = 35.64$

$$M_1 = .135,\quad \frac{p_1}{p^*} = 8.1266,\ \frac{T_1}{T^*} = 1.1955$$

$$\frac{T_1}{T_2} = \frac{T_1}{T^*}\frac{T^*}{T_2}$$

$$T_1 = T_2\left(1.1955 \times \frac{1}{1.163}\right) = 277.5\ \text{K}$$

$$p_1 = p_2 \times \left(8.1266 \times \frac{1}{2.696}\right) = 3.014\ \text{atm}$$