Ps 4.1 The Shuttle space craft begins to enter the atmosphere at M=25. Calculate, assuming an ideal gas and 1.4 specific heat ratio, the temperature at the stagnation point of the vehicle and the free stream velocity over the space craft at a location in the reentry path where M=20 and T=200 K. Are these estimates accurate? If not why and are they high or low? Where in the trajectory path would you expect the highest stagnation point temperature?

PS 4.2 A normal shock is located at the inlet to a duct. The air approaches the duct at M=3, 400 R and .9 atm. The duct is 2 ft long and 1 ft in diameter and has a friction factor of .005. What is the pressure, the temperature and the velocity at the end of the duct?

PS 4.3 The conditions of air at the exit of a duct which is .03 m diameter, 50 m long and has a friction factor of .005, are M=.4, 1 atm, and 270 K. Assuming 1D flow and an ideal gas with a specific heat ratio of 1.4 calculate the mach number, temperature and pressure at the duct inlet.
4.1 assume a normal shock in front of the space craft

<table>
<thead>
<tr>
<th>M</th>
<th>estimated $T_1$</th>
<th>Table A.2 $\frac{T_2}{T_1}$</th>
<th>$T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>20 K</td>
<td>122.65</td>
<td>2453 K</td>
</tr>
<tr>
<td>10</td>
<td>230 K</td>
<td>20.39</td>
<td>4690 K</td>
</tr>
<tr>
<td>.5</td>
<td>290 K</td>
<td>1.050</td>
<td>305 K</td>
</tr>
</tbody>
</table>

Table A.1

The M= 25 point is not the highest temperature. The highest temperature occurs at some lower elevation on the reentry path. A 1 D shock assumes an ideal gas model. The gas around the space craft at high temperature is a plasma and its properties are not well predicted by the ideal gas model.
4.2

M = 3
400 R
.9 atm

2 ft

D = 1 ft

Table A.2 @ \( M_1 = 3 \),

\[
\frac{p_2}{p_1} = 10.33, \quad \frac{T_2}{T_1} = 2.679
\]

\[
\left( \frac{4fL}{D} \right)_{\text{duct}} = 4 \times \frac{0.005 \times 2}{1} = 0.04
\]

Table A.4 @ \( M_2 = 0.4752 \)

\[
\left( \frac{4fL^*}{D} \right)_2 = 0.1295, \quad \frac{p_2}{p^*} = 2.255, \quad \frac{T_2}{T^*} = 1.1512
\]

\[
\left( \frac{4fL^*}{D} \right)_3 = \left( \frac{4fL^*}{D} \right)_2 - \left( \frac{4fL}{D} \right)_{\text{duct}}
\]

\[
\left( \frac{4fL^*}{D} \right)_3 = 0.1295 - 0.01 = 0.0895
\]

Table A.4 @ \( \left( \frac{4fL^*}{D} \right)_3 = 0.0895 \)

\[
M_3 = 0.782, \quad \frac{p_3}{p^*} = 1.3219, \quad \frac{T_3}{T^*} = 1.0693
\]

\[
\frac{T_3}{T_1} = \frac{T_3}{T^*} \frac{T^*}{T_2} \frac{T_2}{T_1}
\]

\[
T_3 = T_1 \times 1.0693 \times \frac{1}{1.1512} \times 2.679
\]

\[
T_3 = 995.4 \text{ R}
\]

\[
a_3 = \sqrt{\frac{\gamma g R T}{1.4 \times 53.35 \times 32.2 \times 995.4}}
\]

\[
a_3 = 1547.2 \text{ ft/sec}
\]

\[
v_3 = M_3 \times a_3 = 1210 \text{ ft/sec}
\]

\[
\frac{p_3}{p_1} = \frac{p_3}{p^*} \frac{p^*}{p_2} \frac{p_2}{p_1}
\]

\[
p_3 = p_1 \times 1.3219 \times \frac{1}{2.255} \times 10.33
\]

\[
p_3 = 5.45 \text{ atm}
\]
4.3

50 m

D = .03 m

M = .4

1 atm

270 K

Table A.4 @ \( M_2 = .4 \)

\[
\left( \frac{4fL^*}{D} \right)_2 = 2.308, \quad \frac{p_2}{p^*} = 2.696, \quad \frac{T_2}{T^*} = 1.163
\]

\[
\left( \frac{4fL}{D} \right)_{duct} = \frac{4 \times .005 \times 50}{.03} = 33.33
\]

\[
\left( \frac{4fL}{D} \right)_1 = \left( \frac{4fL^*}{D} \right)_2 + \left( \frac{4fL}{D} \right)_{duct}
\]

\[
\left( \frac{4fL^*}{D} \right)_1 = 2.308 + 33.33 = 35.64
\]

Table A.4 @ \( \left( \frac{4fL^*}{D} \right)_1 = 35.64 \)

\( M_1 = .135, \quad \frac{p_1}{p^*} = 8.1266, \quad \frac{T_1}{T^*} = 1.1955 \)

\[
\frac{T_1}{T_2} = \frac{T_1}{T^*} \frac{T^*}{T_2}
\]

\[
T_1 = T_2 \left( 1.1955 \times \frac{1}{1.163} \right) = 277.5 \text{ K}
\]

\[
p_1 = p_2 \times \left( 8.1266 \times \frac{1}{2.696} \right) = 3.014 \text{ atm}
\]