mae 422

Problem Set 3 (PS3) Due Tuesday February 7

1. Derive in detail for 1 –D flow with heat transfer, the Rayleigh Flow equations, 3.78 and 3.81.

2. At a point in an isentropic flow, T=280 K, P=100 kPa, v=100 m/sec. What are the stagnation properties of this flow ? To what value would the pressure have to be reduced to achieve sonic flow in the duct ?

3. What friction factor is required to accelerate a flow from 300 K, 100 kPa, 6.94 m/sec to sonic in a constant area duct with a diameter of .5 m and a length of 6000 m ?

3.9 and 3.10 in the Anderson Text.

PS 3.1

energy equation $q+h_1+\frac{u_1^2}{2}=h_2+\frac{u_2^2}{2}$

momentum equation $p_1+\rho_1 u_1^2=p_2+\rho_2 u_2^2$

sonic velocity $a^2=\gamma RT$

ideal gas $p=\rho RT$

since by definition, $u^2=a^2\ M^2$,

and for an ideal gas, $a^2=\gamma RT=\frac{\gamma p}{\rho}$

substituting into the momentum equation,

$$p_2-p_1=\rho_1 u_1^2-\rho_1 u_1^2=\rho_1 a_1^2 M_1^2-\rho_1 a_2^2 M_2^2$$

$$p_2-p_1=\rho_1\frac{\gamma p_1}{\rho_1}M_1^2-\rho_2\frac{\gamma p_2}{\rho_2}M_2^2$$

dividing by p_1,

$$\frac{p_2}{p_1}-1=\rho_1\frac{\gamma p_1}{\rho_1 p_1}M_1^2-\rho_2\frac{\gamma p_2}{\rho_2 p_1}M_2^2$$

$$\frac{p_2}{p_1}\left(1+\gamma M_2^2\right)=1+\gamma M_1^2$$

$$\frac{p_2}{p_1}=\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\quad(3.78)$$

continuity $\rho_1 u_1=\rho_2 u_2$

for an ideal gas, $\rho=\frac{p}{RT}$

substituting into continuity,

$$\frac{p_1}{RT_1}u_1=\frac{p_2}{RT_2}u_2$$

$$\frac{T_2}{T_1}=\frac{p_2 u_2}{p_1 u_1}$$

from definition of Mach Number

$$\frac{u_2}{u_1}=\frac{M_2 a_2}{M_1 a_1}=\frac{M_2}{M_1}\left(\frac{\gamma RT_2}{\gamma RT_1}\right)^{\frac{1}{2}}$$

substituting,

$$\frac{T_2}{T_1}=\frac{p_2}{p_1}\frac{M_2}{M_1}\left(\frac{\gamma RT_2}{\gamma RT_1}\right)^{\frac{1}{2}}$$

$$\left(\frac{T_2}{T_1}\right)^{\frac{1}{2}}=\frac{p_2}{p_1}\frac{M_2}{M_1}$$

$$\frac{T_2}{T_1}=\left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2\left(\frac{M_2}{M_1}\right)^2\quad(3.81)$$

PS 3.2

$$a_1 = \sqrt{\gamma RT} = \sqrt{1.4 \times .287 \times 1000 \times 280} = 335.4 \text{ m/sec}$$

$$M_1 = \frac{100}{335.4} = .3$$

$$@ M = .6, \frac{p_O}{p_1} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} = 1.276 \text{ or Table A.1}$$

$$p_O = 1.276 \times 100 = 127.6 \text{ kPa}$$

$$\frac{T_O}{T} = 1.072$$

$$T_O = 1.072 \times 280 = 300.16 \text{ K}$$

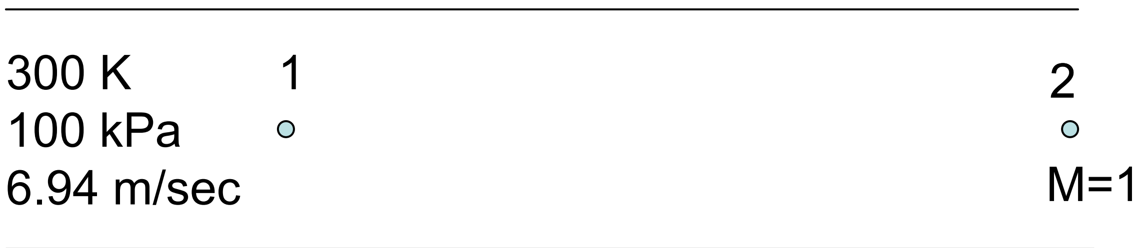
$$@ M = 1, \quad \frac{p_O}{p_2} = 1.893$$

$$\left(\frac{p_O}{p_2}\right) = \left(\frac{p_O}{p_1}\right) \frac{p_1}{p_2}$$

$$1.893 = 1.276 \frac{p_1}{p_2}$$

$$p_2 = \frac{1.276 p_1}{1.893} = .674 \times 100 = 67.4 \text{ kPa}$$

<u>PS 3.3</u>



$$a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \times .287 \times 1000 \times 300} = 347.2$$

$$M_1 = \frac{6.94}{347.2} = .02$$

$$@\, M_1 = .02, \quad \frac{4fL^*}{D} = 1778 \quad \text{(3.107) or Table A.4}$$

$$f = \frac{1778 \times .5}{4 \times 6000} = .037$$

3.9

$$q = \frac{4.5\times10^8 \text{ ft lbf}}{\text{slug}}\,\frac{1 \text{ slug}}{32.2 \text{ lb fuel}}\,\frac{1 \text{Btu}}{778 \text{ ft lbf}}\,\frac{.06 \text{ lbm fuel}}{\text{lbm air}}\,\frac{1 \text{ lb air}}{1.06 \text{ lb products}}$$

$$q = 1016.8\,\frac{\text{Btu}}{\text{lb products}}$$

$$q = c_p\left(T_{O2} - T_{O1}\right)$$

$$\frac{q}{c_p T_{O1}} = \frac{T_{O2}}{T_{O1}} - 1$$

$$1 + \frac{q}{c_p T_1 \frac{T_{O1}}{T_1}} = \frac{T_{O2}}{T_{O1}}$$

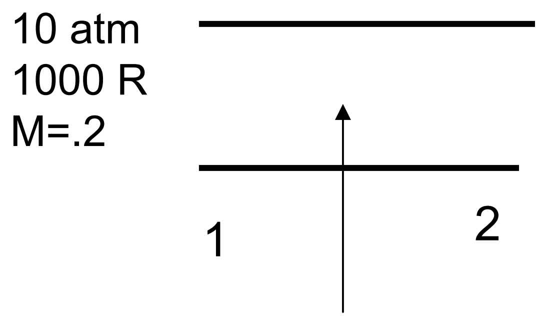
$$1 + \frac{q}{c_p T_1 \frac{T_{O1}}{T_1}} = \frac{T_{O2}}{T_O^*}\,\frac{T_O^*}{T_{O1}} \quad (a)$$

Table A.1 @ $M = .2$

$$\frac{T_{O1}}{T_1} = 1.008$$

Table A.3 @ $M = .2$

$$\frac{T_{O1}}{T_O^*} = .1736, \frac{p_1}{p^*} = 2.273, \frac{T_1}{T_O^*} = 2.066$$



$$q = \frac{4.5\times10^8 \text{ ftlbf}}{\text{slug}}$$

substituting into (a),

$$1 + \frac{q}{c_p T_1 \frac{T_{O1}}{T_1}} = \frac{T_{O2}}{T_O^*}\,\frac{T_O^*}{T_{O1}}$$

$$1 + \frac{1016.8}{.24\times1000\times1.008} = \frac{T_{O2}}{T_O^*}\,\frac{1}{.1736}$$

$$\frac{T_{O2}}{T_O^*} = .903$$

$$\text{Table A.3} @ \frac{T_{O2}}{T_O^*} = .903,$$

$$M = .96, \frac{p_2}{p^*} = 1.433, \frac{T_2}{T^*} = .9895$$

$$p_2 = \frac{p_2}{p^*}\,\frac{p^*}{p_1}\,p_1 = 1.433\,\frac{1}{2.273}\times10 = 6.30 \text{ atm}$$

$$T_2 = \frac{T_2}{T^*}\,\frac{T^*}{T_1}\times T_1 = .9895\,\frac{1}{.2066}\times1000 = 4789 \text{ R}$$

3.10

Table A.3 @ $M_2 = 1, \frac{T_{O2}}{T_O^*} = 1$

From 3.9,

$$1 + \frac{q}{c_p T_1 \frac{T_{O1}}{T_1}} = \frac{T_{O2}}{T_O^*}\frac{T_O^*}{T_{O1}} \quad (a)$$

$$1 + \frac{q}{.24 \times 1000 \times 1.008} = 1 \times \frac{1}{.1736}$$

$$q = 1151.6 \text{ Btu/lb products}$$

$$q = \frac{4.5 \times 10^8 \times AF}{(1 + AF) \times 778 \times 32.2} = 1151.5 \text{ Btu/lb products}$$

$$AF = .0685$$