1. Derive in detail for 1-D flow with heat transfer, the Rayleigh Flow equations, 3.78 and 3.81.

2. At a point in an isentropic flow, $T=280$ K, $P=100$ kPa, $v=100$ m/sec. What are the stagnation properties of this flow? To what value would the pressure have to be reduced to achieve sonic flow in the duct?

3. What friction factor is required to accelerate a flow from 300 K, 100 kPa, 6.94 m/sec to sonic in a constant area duct with a diameter of .5 m and a length of 6000 m?

3.9 and 3.10 in the Anderson Text.
energy equation \[ q + h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \]
momentum equation \[ p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \]
sonic velocity \[ a^2 = \gamma RT \]
ideal gas \[ p = \rho RT \]
since by definition, \[ u^2 = a^2 M^2, \]
and for an ideal gas, \[ a^2 = \gamma RT = \frac{\gamma p}{\rho} \]
substituting into the momentum equation, \[ p_2 - p_1 = \rho_1 u_1^2 - \rho_1 u_1^2 = \rho_1 a_1^2 M_1^2 - \rho_1 a_1^2 M_2^2 \]
\[ p_2 - p_1 = \frac{\rho_1 \gamma p_1}{\rho_1} M_1^2 - \frac{\rho_2 \gamma p_2}{\rho_2} M_2^2 \]
dividing by \( p_1, \) \[ \frac{p_2}{p_1} - 1 = \frac{\rho_1 \gamma p_1}{\rho_1 p_1} M_1^2 - \frac{\rho_2 \gamma p_2}{\rho_2 p_1} M_2^2 \]
\[ \frac{p_2}{p_1} \left( 1 + \gamma M_2^2 \right) = 1 + \gamma M_1^2 \]
\[ \frac{p_2}{p_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \] \( (3.78) \)
continuity \[ \rho_1 u_1 = \rho_2 u_2 \]
for an ideal gas, \[ \rho = \frac{p}{RT} \]
substituting into continuity, \[ \frac{p_1}{RT_1} u_1 = \frac{p_2}{RT_2} u_2 \]
\[ T_2 = \frac{p_2 u_2}{p_1 u_1} \]
from definition of Mach Number \[ \frac{u_2}{u_1} = \frac{M_2 a_2}{M_1 a_1} = \frac{M_2}{M_1} \left( \frac{\gamma RT_2}{\gamma RT_1} \right)^{1/2} \]
substituting, \[ \frac{T_2}{T_1} = \frac{p_2}{p_1} \frac{M_2}{M_1} \left( \frac{\gamma RT_2}{\gamma RT_1} \right)^{1/2} \]
\[ \left( \frac{T_2}{T_1} \right)^{1/3} = \frac{p_2}{p_1} \frac{M_2}{M_1} \]
\[ \frac{T_2}{T_1} = \left( \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \right)^2 \frac{M_2}{M_1} \] \( (3.81) \)
PS 3.2

\[ P = 100 \text{ kPa} \]
\[ T = 280 \text{ K} \]
\[ V = 200 \text{ m/sec} \]

\[ M = 1 \]

\[ a_1 = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 1000 \times 280} = 335.4 \text{ m/sec} \]

\[ M_1 = \frac{100}{335.4} = .3 \]

@ \( M = .6 \), \[ \frac{p_o}{p_1} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} = 1.276 \text{ or Table A.1} \]

\[ p_o = 1.276 \times 100 = 127.6 \text{ kPa} \]

\[ \frac{T_o}{T} = 1.072 \]

\[ T_o = 1.072 \times 280 = 300.16 \text{ K} \]

@ \( M = 1 \), \[ \frac{p_o}{p_2} = 1.893 \]

\[ \left(\frac{p_o}{p_2}\right) = \left(\frac{p_o}{p_1}\right) \frac{p_1}{p_2} \]

\[ 1.893 = 1.276 \frac{p_1}{p_2} \]

\[ p_2 = \frac{1.276 p_1}{1.893} = .674 \times 100 = 67.4 \text{ kPa} \]
PS 3.3

| 300 K | 1 |
| 100 kPa | 2 |
| 6.94 m/sec | M=1 |

\[ a_1 = \sqrt{\gamma RT_1} = \sqrt{1.4 \times 287 \times 1000 \times 300} = 347.2 \]

\[ M_1 = \frac{6.94}{347.2} = 0.02 \]

\[ @ M_1 = 0.02, \quad \frac{4fL^*}{D} = 1778 \quad (3.107) \text{ or Table A.4} \]

\[ f = \frac{1778 \times 0.5}{4 \times 6000} = 0.037 \]
\[
q = \frac{3.9}{\frac{4.5 \times 10^8 \text{ ft lbf}}{\text{slug}}} \frac{1 \text{ slug}}{32.2 \text{ lb fuel}} \frac{778 \text{ ft lbf}}{1 \text{ Btu}} \frac{1 \text{ lbm fuel}}{0.06 \text{ lbm air}} \frac{1 \text{ lb products}}{1.06 \text{ lb products}}
\]

\[
q = 1016.8 \frac{\text{Btu}}{\text{lb products}}
\]

\[
q = c_p \left( T_{O2} - T_{O1} \right)
\]

10 atm

\[
1000 \text{ R}
\]

M = 0.2

\[
1 + \frac{q}{c_p T_1} \frac{T_{O1}}{T_1} = \frac{T_{O2}}{T_{O1}}
\]

\[
1 + \frac{q}{c_p T_1} \frac{T_{O1}}{T_1} = \frac{T_{O2}}{T_{O}^*}
\]

(a)

\[
q = \frac{4.5 \times 10^8 \text{ ft lbf}}{\text{slug}}
\]

substituting into (a),

\[
1 + \frac{q}{c_p T_1} \frac{T_{O1}}{T_1} = \frac{T_{O2}}{T_{O}^*}
\]

\[
1 + \frac{1016.8}{0.24 \times 1000 \times 1.008} = \frac{T_{O2}}{T_{O}^*} \frac{1}{1.736}
\]

\[
\frac{T_{O2}}{T_{O}^*} = 0.903
\]

Table A.3 @ \( \frac{T_{O2}}{T_{O}^*} = 0.903 \),

M = 0.96, \( \frac{p_2}{p}^* = 1.433, \frac{T_2}{T^*} = 0.9895 \)

\[
p_2 = \frac{p_2}{p}^* \frac{p_1}{p} = 1.433 \frac{1}{2.273} \times 10 = 6.30 \text{ atm}
\]

\[
T_2 = \frac{T_2}{T^*} \frac{T^*}{T_1} \times T_1 = 0.9895 \frac{1}{0.2066} \times 1000 = 4789 \text{ R}
\]
3.10

Table A.3 @ $M_2 = 1, \frac{T_{O_2}}{T_{O}} = 1$

From 3.9,

\[
1 + \frac{q}{c_p T_1 \frac{T_{O_1}}{T_1}} = \frac{T_{O_2}}{T_{O}} \frac{T_{O}}{T_{O_1}} \quad \text{(a)}
\]

\[
1 + \frac{q}{.24 \times 1000 \times 1.008} = 1 \times \frac{1}{.1736}
\]

\[
q = 1151.6 \text{ Btu/lb products}
\]

\[
q = \frac{4.5 \times 10^8 \times AF}{(1 + AF) \times 778 \times 32.2} = 1151.5 \text{ Btu/lb products}
\]

\[
AF = .0685
\]