Problem Set 1 (PS1) Due Tuesday January 24

1. Air at 80 kPa, 27 C and 220 m/sec enters a diffuser at a rate of 3 kg/sec and leaves at 42 C. The exit area of the diffuser is 400 square cm. The air looses heat at a rate of 18 kJ/sec. Determine the exit velocity and the exit air pressure.

2. In an insulated heat exchanger air is heated at atmospheric pressure from 25 C to 100 C and 2 kg/sec of water is cooled at atmospheric pressure from 150 C to 100 C. What is the entropy change for the air stream, the water stream and for the overall heat transfer process?

3. 3 lbs of air is expanded in an adiabatic isentropic process from 500 R and a gage pressure 200 psi to a gage pressure off 100 psi. Atmospheric pressure is 14.6 psi. What are the temperature, pressure, specific density and total volume at the end of the process?
steady flow energy equation

\[ m \left( c_p T_1 + \frac{u_1^2}{2 \times 1000} \right) + Q = m \left( c_p T_2 + \frac{u_2^2}{2 \times 1000} \right) \]

\[ 3 \text{ kg} \left( 1.005 \times 300.15 \times \frac{220^2}{2000} \right) + 18 \text{ kJ/sec} = 3 \text{ kg} \left( 1.005 \times 315.15 \times \frac{u_2^2}{2000} \right) \]

\[ 941 + 18 = 950.18 + .0015 u_2^2 \]

\[ u_2 = 76.7 \text{ m/sec} \]

continuity equation

\[ m = \rho_2 A_2 u_2 \]

ideal gas law

\[ m = \frac{p_2}{R T_2} A_2 u_2 \]

\[ p_2 = \frac{m R T_2}{A_2 u_2} = \frac{3 \times 8.324 \text{ kPa/kgmole K} \times 315.5 \text{ K}}{28.97 \times .04 \text{ m}^2 \times 76.68 \text{ m/sec}} = 88.46 \text{ kPa} \]
energy balance

\[ Q_{\text{water}} = Q_{\text{air}} \]
\[ m_{\text{water}} \times c_p \times (T_1 - T_2) = m_{\text{air}} \times c_p \times (T_1 - T_2) \]
\[ m_{\text{air}} = \frac{2 \text{ kg} \times 4.18 \text{ kJ/kg K} \times (150 - 100)}{1.005 \text{ kJ/kg K} \times (100 - 25)} \]
\[ m_{\text{air}} = 5.5 \text{ kg/sec} \]

entropy change

\[ \Delta S_{\text{air}} = m_{\text{air}} \times c_p \times \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{p_2}{p_1} \right) \]
\[ \Delta S_{\text{air}} = 5.5 \times 1.005 \times \ln \left( \frac{373.15}{298.15} \right) = +1.228 \text{ kJ/K} \]
\[ \Delta S_{\text{water}} = 2 \times 4.18 \times \ln \left( \frac{373.15}{423.15} \right) = -1.0512 \text{ kJ/K} \]
\[ \Delta S_{\text{process}} = \Delta S_{\text{air}} + \Delta S_{\text{water}} = +1.228 - 1.0512 \]
\[ \Delta S_{\text{process}} = +.177 \text{ kJ/K} \]
\[ \Delta S_{\text{isolated}} \geq 0 \]
ideal gas law

\[ \rho_1 = \frac{p}{RT} = \frac{214.6 \text{ lbf/in}^2 \times 144 \text{ in}^2/\text{ft}^2}{1545.15 \text{ ft lbf/lbmole} \times 28.97} \]

\[ \rho_1 = 1.158 \text{ lbm/ft}^3 \]

isentropic process

\[ \frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\gamma-1} = \left( \frac{114.6}{214.6} \right)^{2857} = .8359 \]

\[ T_2 = 500 \times .8359 = 417.96 \text{ R} \]

\[ \frac{\rho_2}{\rho_1} = \left( \frac{p_2}{p_1} \right)^{\frac{1}{\gamma}} = \left( \frac{114.6}{214.6} \right)^{.7143} = .6388 \]

\[ \rho_2 = .6388 \times 1.158 = .7398 \text{ lbm/ft}^3 \]

\[ V_2 = \frac{m}{\rho} = \frac{3 \text{ lbm}}{.7398} = 4.055 \text{ ft}^3 \]