The conditions in a 1-D isentropic flow of a gas with a specific heat ratio of 1.3 are 280 K, .9 atm and M=.5.

a) What is the stagnation temperature and the stagnation pressure of the flow?

Since the Isentropic Tables, A.1, are for a gas with a specific heat ratio of 1.4 equations must be used for pressure and temperature ratios rather than Table A.1.

1) $\frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} M^2 = 1 + \frac{3}{2} \times 0.5^2 = 1.0375$

$T_0 = 1.0375 \times 280 = 290.5 \text{ K}$

2) $\frac{p_0}{p_1} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} = \left(\frac{T_0}{T_1}\right)^{\frac{\gamma}{\gamma - 1}}$  

3) $\frac{p_0}{p_1} = 1.0375^{4.333} = 1.173$

$p_1 = 1.173 \times 0.9 = 1.0557 \text{ atm}$

b) $\frac{p_0}{p_2} = \frac{p_0}{p_1} \times \frac{p_1}{p_2} = 1.173 \left(\frac{0.9}{0.7}\right) = 1.508$

4) $\frac{p_0}{p_2} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$

5) $M_2 = \left(\frac{p_0}{p_2}\right)^{\frac{\gamma - 1}{\gamma}} / \left(1 + \frac{\gamma - 1}{2}\right)$

6) $M_2 = 0.814$

7) @ $M_3 = 1$

8) $\frac{T_0}{T_3} = 1 + \frac{\gamma - 1}{2} M_3^2 = 1 + \frac{3}{2} \times 1 = 1.115$

$T_3 = 290.5 / 1.115 = 252.6$

$v_3 = M_3 \times a_3 = M_3 \times \sqrt{\gamma RT}$

9) $v_3 = 1 \times \sqrt{1.3 \times 252.6 \times 1000}$

$v_3 = 573.044 \sqrt{R} \text{ m/sec}$

assuming $R = 0.287$

$v_3 = 307 \text{ m/sec}$
An oblique shock, with an angle of 28 degrees to the direction of the approaching flow, is attached to the leading edge of a wedge model mounted in a wind tunnel. The temperature of the approaching flow is 300 K. If the stagnation temperature of the wind tunnel flow is 915 K what is the velocity and direction of the flow after the oblique shock?

\[
\frac{T_o}{T} = \frac{915}{300} = 3.05
\]

Isentropic Table A.1\(\) \(\frac{T_o}{T} = 3.05; \quad M_1 = 3.2\)

\(M - \theta - \beta\) Chart \(\) \(M_1 = 3.2, \beta = 28; \quad \theta = 12.1\)

\(M_{in} = M_1 \sin \beta = 3.2 \times .4695 = 1.5023\)

Normal Shock Table A.2 \(\) \(M_{in} = 1.502; \quad M_{2n} = .700, \quad T_2/T_1 = 1.321\)

\(M_2 = \frac{M_{2n}}{\sin(\beta - \theta)} = \frac{.700}{\sin(28 - 12.1)} = 2.555\)

\(T_2 = 300 \times 1.321 = 396.3 \text{ K}\)

\(a_2 = \sqrt{\gamma RT} = \sqrt{1.4 \times .287 \times 1000 \times 396.3} = 399. \text{ m/sec}\)

\(v_2 = a_2 \times M_2 = 399 \times 2.555 = 1019.5 \text{ m/sec @ 12.1}^\circ\) to approaching flow direction
Air flows at a Mach number of 3.5 at 280 K and .9 atm through a rectangular cross section channel which has a 15 degree corner in the lower wall. An oblique shock forms at the corner and is reflected from the upper wall and lower walls. How many reflections are possible? If a normal shock occurs after the last possible reflection what is the temperature, pressure and velocity after the normal shock.

isentropic, TableA.1, \( M_1 = 3.5 \), \( \frac{p_0}{p} = 76.272 \), \( \frac{T_0}{T} = 3.45 \)

\( p_{01} = .9 \times 76.272 = 68.74 \), \( T_{01} = 3.45 \times 280 = 966.6 \).

\( M - \theta - \beta \) \( @ \) \( M_1 = 3.5 \), \( \theta = 15 \);

\( \beta = 29.192 \), \( M_{in} = M_1 \sin \beta = 1.707 \)

\( M_2 = \frac{M_{in}}{\sin(\beta - \theta)} = 2.6053 \), \( \frac{p_2}{p_1} = 3.233 \), \( \frac{T_2}{T_1} = 1.4634 \);

\( p_2 = .9 \times 3.233 = 2.91 \), \( T_2 = 280 \times 1.4634 = 409.75 \)

<table>
<thead>
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<th>( i )</th>
<th>( M )</th>
<th>( \theta )</th>
<th>( \beta )</th>
<th>( M_{ni} )</th>
<th>( M_{n(i+1)} )</th>
<th>( M_{(i+1)} )</th>
<th>( \frac{p_{(i+1)}}{p_i} )</th>
<th>( \frac{T_{(i+1)}}{T_i} )</th>
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<td>2.1779</td>
<td>1.2661</td>
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normal shock

\[
\begin{array}{llll}
M_4 & M_5 & p_5/p_4 & T_5/T_4 \\
1.4048 & .7377 & 2.1357 & 1.2578 \\
\end{array}
\]

\[
p_4 = p_1 \times \frac{p_2}{p_1} \times \frac{p_3}{p_2} \times \frac{p_4}{p_3} = .9 \times 3.233 \times 2.5367 \times 2.1779 \times 2.1357 = 34.33 \text{ atm}
\]

\[
T_4 = T_1 \times \frac{T_2}{T_1} \times \frac{T_3}{T_2} \times \frac{T_4}{T_3} \times \frac{T_5}{T_4} = 280 \times 1.4634 \times 1.3351 \times 1.2661 \times 1.2578 = 871.2 \text{ K}
\]

\[
a = \sqrt{\gamma RT} = \sqrt{1.4 \times .287 \times 1000 \times 871.2} = 591.6 \text{ m/sec}
\]

\[
v = M \times a = 591.6 \times .7377 = 436.4 \text{ m/sec}
\]
The conditions at the throat of a converging nozzle are: mass flow=55 kg/sec, $T=300$ K, and $p=1$ atm. The area at the throat is $0.15 \text{ m}^2$. The area at the inlet is $0.9$. What is the temperature, pressure, velocity and Mach number at the inlet?
The conditions at the throat of a converging nozzle are: mass flow=55 kg/sec, T=300 K, and p = 1 atm. The area at the throat is 0.15 m². The area at the inlet is 0.9. What is the temperature, pressure, velocity and Mach number at the inlet?

\[ v_t = \frac{RT}{p} = \frac{0.287 \times 300}{101.325} = 0.85 \text{ kg/m}^3 \]

\[ a = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 300} = 347.2 \text{ m/sec} \]

\[ V_t = \frac{m \times v_t}{A} = \frac{55 \times 0.85}{0.15} = 311.67 \text{ m/sec} \]

\[ M_t = \frac{V_t}{a_t} = \frac{311.67}{347.2} = 0.9 \]

isentropic at \( M = 0.9 \), \( \frac{A_t}{A^*} = 1.009 \)

\[ p_o = 1.691, \quad \frac{T_o}{T} = 1.172, \quad \frac{\rho_o}{\rho} = 1.456 \]

\[ \frac{A_1}{A^*} = \frac{A_1}{A_t} \frac{A_t}{A^*} = 0.9 \times 1.009 = 6.054 \]

\[ T_o = 1.172 \times 300 = 351.6 \]

\[ p_o = 1.691 \times 1 = 1.691 \text{ atm} \]

\[ p_1 = \frac{1.691}{1.0065} = 1.68 \text{ atm} \]

\[ a_1 = \sqrt{1.4 \times 287 \times 350.97} = 375.38 \text{ m/sec} \]

\[ V_1 = a_1 \times M = 375.38 \times 0.96 = 36.04 \text{ m/sec} \]
assuming exit conditions as inlet

\[ v_1 = \frac{RT}{p} = \frac{0.287 \times 300}{101.325} = 0.85 \text{ kg/m}^3 \]

\[ a_1 = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 300} = 347.2 \text{ m/sec} \]

\[ V_1 = \frac{m \times v_1}{A_1} = \frac{55 \times 0.85}{0.9} = 51.94 \text{ m/sec} \]

\[ M_1 = \frac{51.94 \text{ m/sec}}{347.2 \text{ m/sec}} = 0.15 \]

@ \( M_1 = 0.15; \frac{A_1}{A^*} = 3.91 \]

BUT \( \frac{A_1}{A_t} = \frac{A_1}{A^*} = 6. \)

need higher \( p_i \) or less mass flow

for a mass balance. \( M_i \) can't be

greater than 1 in a converging nozzle.
A converging diverging nozzle has inlet stagnation conditions of 15 atm and 399 K. The ratio of the exit area to the throat area is 5.121. Assuming 1D isentropic compressible flow determine over what range of back pressures you would expect to find an oblique shock at the nozzle exit? What would the nozzle exit velocity be at the highest and lowest pressures in this range?

Isentropic at \( \frac{A}{A^*} = 5.121 \) supersonic

\[ M_e = 3.21, \quad \frac{p_o}{p_e} = 49.44, \quad \frac{T_o}{T_e} = 3.048 \]

\[ p_e = \frac{15}{49.44} = .303 \text{ atm} \]

\[ T_e = \frac{300}{3.048} = 98.43 \text{ K} \]

\[ a_e = \sqrt{\gamma RT} \]

\[ a_e = \sqrt{1.4 \times 287 \times 98.43} = 198.8 \text{ m/sec} \]

\[ V_e = 3.2 \times 198.8 \text{ m/sec} = 636.4 \text{ m/sec} \]

Oblique shocks form at a pressure below the pressure at which a normal shock forms at the exit.

Normal shock @ \( M_1 = 3.2 \)

\[ \frac{p_2}{p_1} = 11.78, \quad \frac{T_2}{T_1} = 2.922, \quad M_2 = .4643 \]

\[ p_e = .303 \times 11.78 = 3.57 \text{ atm} \]

\[ T_e = 2.922 \times 98.43 = 287.6 \text{ K} \]

\[ a_e = \sqrt{1.4 \times 287 \times 287.6} = 339.9 \text{ m/sec} \]

\[ V = M \times a = .4643 \times 339.9 = 157.8 \text{ m/sec} \]