MAE 340: Test I

February 11, 2003

Number 1-20 are 4 points each; Questions 21 and 22 are 10 points each

Answer questions 1–10 based on the following problem statement:

A system is modeled by the differential equations

\[
\begin{align*}
\dot{x}_1 &= 3(\dot{x}_2 - x_1) + 11(x_2 - x_1) + 2F_1(t) + 5F_2(t) \\
\dot{x}_2 &= -4(\dot{x}_2 - x_1) - 25(x_2 - x_1) + 5F_2(t) + F_1(t)
\end{align*}
\]

where \(F_1(t)\) and \(F_2(t)\) are inputs. The outputs for this system are defined as

\[
y_1 = 6(x_2 - x_1) + 2(\dot{x}_2 - \dot{x}_1), \quad y_2 = 3\dot{x}_1 - x_1 + 4F_1(t) + 2F_2(t)
\]

Write the state-space and output equations for this system in the standard form

\[
\dot{q} = Aq + Bu, \quad y = Cq + Du
\]

1. The size of the matrix is:
   (a) \(4 \times 2\) \quad (b) \(2 \times 4\) \quad (c) \(2 \times 2\) \quad (d) \(4 \times 4\) \quad (e) Other

2. The size of the matrix is:
   (a) \(4 \times 2\) \quad (b) \(4 \times 4\) \quad (c) \(2 \times 2\) \quad (d) \(2 \times 4\) \quad (e) Other

3. The size of the matrix is:
   (a) \(4 \times 2\) \quad (b) \(2 \times 4\) \quad (c) \(2 \times 2\) \quad (d) \(4 \times 4\) \quad (e) Other

4. The size of the matrix is:
   (a) \(4 \times 4\) \quad (b) \(4 \times 2\) \quad (c) \(2 \times 2\) \quad (d) \(2 \times 4\) \quad (e) Other

5. Write a state vector for this system:
   \[
   \begin{bmatrix}
   \chi_1 \\
   \chi_2
   \end{bmatrix}
   \]
   or
   \[
   \begin{bmatrix}
   \chi_1 \\
   \chi_2
   \end{bmatrix}
   \]

6. Write the input vector for this system:
   \[
   u = \{F_1, F_2\}
   \]

7. Write the matrix for this system:
   \[
   A = \begin{bmatrix}
   0 & 1 & 0 & 0 \\
   0 & 0 & 1 & 0 \\
   -11 & 11 & -3 & 3 \\
   25 & -25 & 4 & -4
   \end{bmatrix}
   \]

8. Write the matrix for this system:
   \[
   B = \begin{bmatrix}
   0 & 0 & 0 & 0 \\
   0 & 0 & 25 & 0
   \end{bmatrix}
   \]
9. Write the C matrix for this system:

\[
C = \begin{bmatrix}
-6 & 6 & -2 & 2 \\
-1 & 0 & 3 & 0 \\
\end{bmatrix}
\]

10. Write the D matrix for this system:

\[
D = \begin{bmatrix}
0 & 0 \\
4 & 2 \\
\end{bmatrix}
\]

A system's characteristic equation is given by:

\[s^2 + 2s + 5 = 0\]

\[s_{1,2} = \frac{-1 \pm \sqrt{4-20}}{2}
\]

11. Write the general expression for the homogeneous solution:

\[\chi_H(t) = C_1 e^{-t} \sin(2t + \phi)\]

12. The time constant of the homogeneous solution is:

(a) 1 sec  (b) π sec  (c) ½ sec  (d) Other  (e) Not enough information

13. The settling time of the homogeneous solution is:

a) 1 sec  (b) 2 sec  (c) 4/3 sec  (d) Other  (e) Not enough information

14. The period of the homogeneous solution is:

a) 1 sec  (b) π sec  (c) ½ sec  (d) Other  (e) Not enough information

15. The value of the solution at \(t = 1\) sec is:

a) 0.37  (b) 0.02  (c) 0.63  (d) Other  (e) Not enough information

16. Assume that a system is modeled by \(\ddot{x} + 3\dot{x} + 10x = 4\sin 3t\). The system is:

(a) Unstable  (b) Stable  (c) Marginally stable  (d) Not enough information to know

17. Assume that a system is modeled by \(\ddot{x} + 18x = 0\). The system is:

(a) Unstable  (b) Stable  (c) Marginally stable  (d) Not enough information to know

18. Assume that a system is modeled by \(\dot{x} - 2\dot{x} - 12x = e^{-2t}\). The system is:

(a) Unstable  (b) Stable  (c) Marginally stable  (d) Not enough information to know
19. A system is modeled by \( \ddot{x} + 7\dot{x} + 12x = 0 \). The settling time of the system is:
   (a) 3 secs    (b) 1 sec    (c) 1/3 sec    (d) 4 sec    (e) Other

   \( \zeta = \frac{-7 + \sqrt{49 - 48}}{2} = -2.5 \) \( \zeta = -\frac{3}{2} + \frac{1}{2} = -1.5 \) \( \zeta = -3 + \frac{1}{2} = -3.5 \)

20. The particular solution for \( \ddot{x} + 2\dot{x} + x = 2t + 5 \) is:
   (a) \( x_p = 6t + 7 \)    (b) \( x_p = 2t + 1 \)    (c) \( x_p = 2t + 5 \)    (d) Not enough info
   (e) None of the above

   \[ \begin{align*}
   x_p &= c_1 + c_2 \\
   x_q &= c_1 \\
   \ddot{x} &= 2t + 5 \\
   c_1 &= 2 \\
   c_2 &= 5 - 4 = 1
   \end{align*} \]

21. For the system shown, find the ordinary differential equation model.

\[ \begin{align*}
\dot{z} &= \text{position of base (input)} \\
\dot{x}_1, \dot{x}_2 &= \text{position of masses } m_1, m_2
\end{align*} \]

\[ \begin{align*}
m_1 \ddot{x}_1 &= m_1 g + k_2 (x_2 - x_1) + b (\dot{x}_2 - \dot{x}_1) - k_1 (x_1 - z) \\
m_2 \ddot{x}_2 &= m_2 g - k_2 (x_2 - x_1) - b (\dot{x}_2 - \dot{x}_1)
\end{align*} \]

22. Sketch the solution of \( 4\ddot{x} + 8\dot{x} + 104x = 8 \) for the time period \( t = 0 \) to the settling time, with \( x(0) = 10 \) and \( \dot{x}(0) = 0 \).

\[ \begin{align*}
\ddot{x} + 2\dot{x} + 26x &= 2 \\
\dot{x} &= \frac{2}{26} x \\
\ddot{x} &= -2 - \frac{\sqrt{4 - 104}}{2} = -1 \pm 5i \\
\chi(0) &= 0 \Rightarrow C + \frac{2}{26} = 0
\end{align*} \]

\[ \begin{align*}
\chi(t) &= C e^{-\frac{t}{13}} \sin \left( 5t + \phi \right) + \frac{2}{26} \\
\chi(0) &= 0 \Rightarrow C_1 + \frac{2}{26} ; \dot{x}(0) = 0 \Rightarrow x(0) = 0
\end{align*} \]
\[ x(t) = Ce^{\frac{-t}{\tau}} \sin \left( \omega t + \phi \right) + \frac{\omega}{\omega^2} \]

\[ x(0) = 10 \]
\[ \dot{x}(0) = 0 \]

Settling time = 4 secs; period = \[ \frac{2\pi}{\omega} \] secs = 1.2566 secs

Approx 3.2 cycles to settle.
MAE 340: Test I
February 11, 2003

Number 1-20 are 4 points each; Questions 21 and 22 are 10 points each

Answer questions 1–10 based on the following problem statement:

A system is modeled by the differential equations

\[
\begin{align*}
\dot{x}_1 &= 11(\dot{x}_2 - \dot{x}_1) + 3(x_2 - x_1) + 5F_1(t) + 2F_2(t) \\
\dot{x}_2 &= -25(\dot{x}_2 - \dot{x}_1) - 4(x_2 - x_1) + F_2(t) + 5F_1(t)
\end{align*}
\]

where \(F_1(t)\) and \(F_2(t)\) are inputs. The outputs for this system are defined as

\[
y_1 = 2(x_2 - x_1) + 6(\dot{x}_2 - \dot{x}_1), \quad y_2 = 4\dot{x}_1 - x_1 + 2F_1(t) + 3F_2(t)
\]

Write the state–space and output equations for this system in the standard form

\[
\dot{q} = Aq + Bu, \quad y = Cq + Du
\]

1. The size of the \(A\) matrix is:
   (a) 2 x 4  (b) 4 x 4  (c) 2 x 2  (d) 4 x 2  (e) Other

2. The size of the \(B\) matrix is:
   (a) 2 x 4  (b) 4 x 2  (c) 2 x 2  (d) 4 x 4  (e) Other

3. The size of the \(C\) matrix is:
   (a) 4 x 2  (b) 2 x 2  (c) 2 x 4  (d) 4 x 4  (e) Other

4. The size of the \(D\) matrix is:
   (a) 2 x 2  (b) 2 x 4  (c) 4 x 2  (d) 4 x 4  (e) Other

5. Write a state vector for this system:
   \(\mathbf{q} = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix}\)

6. Write the input vector for this system:
   \(u = \{F_1, F_2\}\)

7. Write the \(A\) matrix for this system:
   \[
   A = \begin{bmatrix}
   0 & 0 & 0 & 0 \\
   -3 & -11 & 3 & 11 \\
   0 & 0 & 0 & 1 \\
   4 & 25 & -4 & -25
   \end{bmatrix}
   \]

8. Write the \(B\) matrix for this system:
   \[
   B = \begin{bmatrix}
   0 & 6 \\
   5 & 2 \\
   0 & 0 \\
   5 & 1
   \end{bmatrix}
   \]
9. Write the C matrix for this system:
\[
C = \begin{bmatrix}
-2 & -6 & 2 & 6 \\
0 & 2 & 0 & 0 \\
2 & 3 & \end{bmatrix}
\]

10. Write the D matrix for this system:
\[
D = \begin{bmatrix}
0 & 0 \\
2 & 3 \\
\end{bmatrix}
\]

A system's characteristic equation is given by: \( s^2 + 4s + 13 = 0 \)

\[ s_{1,2} = \frac{-4 \pm \sqrt{16 - 52}}{2} \]

\[ s_{1,2} = -2 \pm 3i \]

11. Write the general expression for the homogeneous solution:
\[
\chi_h(t) = C_1 e^{-2t} \sin (3t + \phi)
\]

12. The time constant of the homogeneous solution is:
(a) 1 sec (b) \( \pi \) sec (c) \( \frac{1}{2} \) sec (d) Other (e) Not enough information

13. The settling time of the homogeneous solution is:
   a) 1 sec (b) 2 sec (c) \( \frac{4}{3} \) sec (d) Other (e) Not enough information

14. The period of the homogeneous solution is:
   a) 3 sec (b) \( \pi \) sec (c) \( \frac{1}{2} \) sec (d) Other (e) Not enough information

15. The value of the solution at \( t = 1 \) sec is:
   a) 0.37 (b) 0.02 (c) 0.63 (d) Other (e) Not enough information

16. Assume that a system is modeled by \( \ddot{x} + 3\dot{x} - 10x = e^{-2t} \). The system is:
   (a) Unstable (b) Stable (c) Marginally stable (d) Not enough information to know

17. Assume that a system is modeled by \( \ddot{x} + 6\dot{x} + 18x = 4\sin 3t \). The system is:
   (a) Unstable (b) Stable (c) Marginally stable (d) Not enough information to know

18. Assume that a system is modeled by \( \ddot{x} + 12x = 0 \). The system is:
   (a) Unstable (b) Stable (c) Marginally stable (d) Not enough information to know
19. A system is modeled by $\ddot{x} + 5\dot{x} + 6x = 0$. The settling time of the system is:
(a) 2 secs  (b) 1 sec  (c) 1/2 sec  (d) 4 sec  (e) Other  
\[ S_{1/2} = \frac{-5 \pm \sqrt{25 - 24}}{2} \]

20. The particular solution for $\ddot{x} + \dot{x} + x = 6t + 13$ is:
(a) $x_p = 6t + 7$  (b) $x_p = 2t + 1$  (c) $x_p = 6t + 13$  (d) Not enough info  
(e) None of the above  
\[ x_p = c_1e^t + c_2 \]
\[ x_2 = c_1 \]
\[ c_1 = 6 \]
\[ c_2 = 13 - 6 = 7 \]

21. For the system shown, find the ordinary differential equation model:
\[ x_1 = 0 \text{ at } z = 0 \]
\[ x_2 = 0 \text{ at } z = 0 \]
\[ k_1, k_2 \text{ unstretched} \]
\[ z = \text{position of base (input)} \]
\[ x_1, x_2 = \text{position of masses } m_1, m_2 \]
\[ m_1 \ddot{x}_1 = m_1 g + k_2(x_2 - x_1) - k_1(x_1 - z) = b(\dot{x}_1 - \dot{z}) \]
\[ m_2 \ddot{x}_2 = m_2 g - k_2(x_2 - x_1) \]

22. Sketch the solution of $4\ddot{x} + 16\dot{x} + 80x = 8$ for the time period $t = 0$ to the settling time, with $x(0) = 5$ and $\dot{x}(0) = 0$.
\[ \ddot{x} + 4\dot{x} + 20x = 2 \implies x_p = \frac{1}{10} \]
\[ S_{1/2} = \frac{-4 \pm \sqrt{16 - 80}}{2} = -2 \pm 4i \]
\[ \text{Settling time } t_w = 2 \text{ secs} \]
\[ \text{Period } = \frac{2\pi}{4} \approx 1.57 \text{ secs} \]
Approx 1\(\frac{1}{3}\) cycles to settle
MAE 340: Test I

February 11, 2003

Number 1-20 are 4 points each; Questions 21 and 22 are 10 points each

Answer questions 1–10 based on the following problem statement:

A system is modeled by the differential equations

\[ \dot{x}_1 = -25(x_2 - x_1) - 4(x_2 - x_1) + F_1(t) + 5F_2(t) \]
\[ \dot{x}_2 = 11(x_2 - x_1) + 3(x_2 - x_1) + 5F_2(t) + 2F_1(t) \]

where \( F_1(t) \) and \( F_2(t) \) are inputs. The outputs for this system are defined as
\[ y_1 = 4(x_2 - x_1) - (x_2 - x_1) + 2F_1(t) + 3F_2(t) \]
\[ y_2 = 2x_2 + 6x_1 \]

Write the state-space and output equations for this system in the standard form

\[ \dot{q} = Aq + Bu \quad y = Cq + Du \]

1. The size of the A matrix is:
   (a) 4 x 4  (b) 2 x 4  (c) 2 x 2  (d) 4 x 2  (e) Other

2. The size of the B matrix is:
   (a) 2 x 4  (b) 2 x 2  (c) 4 x 2  (d) 4 x 4  (e) Other

3. The size of the C matrix is:
   (a) 2 x 4  (b) 4 x 4  (c) 4 x 2  (d) 2 x 2  (e) Other

4. The size of the D matrix is:
   (a) 4 x 4  (b) 2 x 4  (c) 4 x 2  (d) 2 x 2  (e) Other

5. Write a state vector for this system:
   \[ q = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \]
   \[ q = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} \]

6. Write the input vector for this system:

7. Write the A matrix for this system:
   \[ A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 4 & -4 & 25 & -25 \\ 3 & 3 & -11 & 11 \end{bmatrix} \]

8. Write the B matrix for this system:
   \[ B = \begin{bmatrix} 0 & 2 \\ 0 & 5 \end{bmatrix} \]
9. Write the C matrix for this system:

\[ C = \begin{bmatrix} -4 & 4 & 1 & -1 \\ 6 & 0 & 0 & 2 \end{bmatrix} \]

10. Write the D matrix for this system:

\[ D = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} \]

\[ s_{1,2} = \frac{-6 \pm \sqrt{36 - 40}}{2} = -3 \pm 1 i \]

11. Write the general expression for the homogeneous solution:

\[ \chi_h(t) = C_1 e^{-3t} \sin(t + \phi) \]

12. The time constant of the homogeneous solution is:
(a) 1 sec  (b) π sec  (c) ½ sec  (d) Other  (e) Not enough information

13. The settling time of the homogeneous solution is:
(a) 1 sec  (b) 2 sec  (c) 4/3 sec  (d) Other  (e) Not enough information

14. The period of the homogeneous solution is:
(a) 3 sec  (b) π sec  (c) 2π sec  (d) Other  (e) Not enough information

15. The value of the solution at \( t = 1 \) sec is:
(a) 0.37  (b) 0.02  (c) 0.63  (d) Other  (e) Not enough information

16. Assume that a system is modeled by \( \ddot{x} + 4x = 0 \). The system is:
(a) Unstable  (b) Stable  (c) Marginally stable  (d) Not enough information to know

17. Assume that a system is modeled by \( \ddot{x} - \dot{x} + 12x = e^{-2t} \). The system is:
(a) Unstable  (b) Stable  (c) Marginally stable  (d) Not enough information to know

18. Assume that a system is modeled by \( \ddot{x} + 2\dot{x} + 12x = 6\sin 2t \). The system is:
(a) Unstable  (b) Stable  (c) Marginally stable  (d) Not enough information to know
19. A system is modeled by \( \ddot{x} + 4 \dot{x} + 3x = 0 \). The settling time of the system is:
(a) 2 secs  
(b) 1 sec  
(c) 3 sec  
(d) 4 sec  
(e) Other  
\( s_{1/2} = \frac{-4 \pm \sqrt{16 - 12}}{2} \)

20. The particular solution for \( \ddot{x} + 3 \dot{x} + x = 2t + 13 \) is:
(a) \( x_p = 6t + 7 \)  
(b) \( x_p = 2t + 1 \)  
(c) \( x_p = 2t + 13 \)  
(d) Not enough info  
(e) None of the above  
\[ \begin{align*}
\dot{x}_2 &= c_1 + c_2 \\
\ddot{x}_2 &= c_1 \\
c_1 &= 7 \\
c_2 &= 13 - 6 = 7 \\
\end{align*} \]

21. For the system shown, find the ordinary differential equation model.

\[ m_2 \dddot{x}_2 = k_2 (x_1 - x_2) - m_2 g \]

\[ m_1 \dddot{x}_1 = k_1 (z - x_1) + b (\ddot{z} - \dot{x}_1) - m_1 g - k_2 (x_1 - x_2) \]

22. Sketch the solution of \( 3 \dddot{x} + 12 \dot{x} + 120x = 6 \) for the time period \( t = 0 \) to the settling time, with \( x(0) = 20 \) and \( \dot{x}(0) = 0 \).

\[ \dddot{x} + 4 \dot{x} + 40x = 2 \implies x_p = \frac{1}{20} \]

\[ s_{1/2} = -4 \pm \frac{\sqrt{16 - 160}}{2} = -2 \pm 6i \]

Settling time 2 secs  
Period \( \frac{\pi}{6} \) 1.05 secs