

**Problem Set 13 (PS13) due**  
**Monday April. 30**  
**129 12.12 12.19 12.20**

## 12.9

$$(a) \text{ Model : } \alpha_M = 28^\circ \rightarrow M = \frac{1}{\sin \alpha_M} = 2.13$$

$$\text{Re} = \frac{VL}{\nu} = \frac{LM\sqrt{rRT}}{\nu} = 93 \times 10^6$$

(b) Full – scale : At 20,000m,  $T = 273.15 - 56.5K = 216.65K$

$$\rho = 8.891 \times 10^{-2} \frac{\text{Kg}}{\text{m}^{-3}}, \mu = 1.422 \times 10^{-5} \text{ N} \cdot \text{s} / \text{m}^2$$

$$\therefore \text{ fullscale } M = \frac{V}{\sqrt{rRT}} = 2.03$$

$$\text{fullscale } \text{Re} = \frac{\rho VL}{\mu} = 113 \times 10^6$$

Hence, model conditions close to full – scale performance.

12.12

$$(a) \frac{T_2}{T_1} = \frac{540}{290} = 1.862$$

*From the normal shock table, get :*

$$M_1 = 2.206, M_2 = 0.546, \frac{P_2}{P_1} = 5.509$$

$$\therefore V_2 = M_2 \sqrt{rRT_2} = 255 \text{ m/s}$$

$$(b) \frac{P_2}{P_1} = 5.509 \rightarrow P_2 = 5.51 \times 10^5 \text{ pa} = (P_2)_{shock}$$

$$\text{Isentropic flow: } \frac{P_2}{P_1} = \left( \frac{1 + \frac{r-1}{2} M_1^2}{1 + \frac{r-1}{2} M_2^2} \right)^{\frac{r}{r-1}} = 8.81$$

$$\therefore (P_2)_{isentropic} = 8.81 \times 10^5 \text{ pa} = 1.6 \times (P_2)_{shock}$$

## 12.19

*Assume steady, one dimensional flow,  
isentropic conditions.*

$$(a) \dot{m} = \rho_1 A_1 V_1 = \frac{P_1}{RT_1} A_1 M_1 \sqrt{rRT_1}$$

$$\frac{A_1}{A^*} = 3.5$$

$$\therefore M_1 = 2.80$$

$$\therefore P_1 = \frac{P_0}{\left(1 + \frac{r-1}{2} M_1^2\right)^{\frac{r}{r-1}}} = 3734 Pa$$

$$\text{and } T_1 = \frac{T_0}{1 + \frac{r-1}{2} M_1^2} = 114 K$$

$$\therefore \dot{m} = \frac{P_1}{RT_1} A_1 M_1 \sqrt{rRT_1} = 0.120 \text{ kg / s}$$

(b) *Range of back pressures :*

(1) *No shock will occur in the nozzle when*

$$p_b \leq p_{isentropic} = p_1 \text{ of part (a)}$$

(2) *Strongest shock will occur when shock sits at exit from nozzle. That is  $M_1 = 2.80$*

$$\text{From table, get } \frac{P_2}{P_1} = 8.980,$$

$$P_b = P_2 = 8.98P_1$$

*∴ Range of back pressures :*

$$8.98P_1 \geq P_b \geq P_1$$

12.20

*Assume steady, one dimensional flow, isentropic conditions.*

*Stagnation conditions :*

$$P_0 = 6.8 \times 10^5 \text{ pa}, T_0 = 15^\circ \text{ C} = 288.15 \text{ K}$$

*(a) Maximum mass flow rate, when Mach number at throat is one ( $M^* = 1$ ):*

$$\square \quad m = \rho^* A^* V^* = \frac{P^*}{RT^*} A^* \sqrt{rRT^*}$$

$$P^* = \frac{P_0}{\left(1 + \frac{r-1}{2}\right)^{\frac{r}{r-1}}} = 3.59 \times 10^5$$

$$T^* = \frac{T_0}{1 + \frac{r-1}{2}} = 240.1 \text{ K}$$

$$\therefore \square \quad m = \frac{P^*}{RT^*} A^* \sqrt{rRT^*} = 4.05 \text{ kg / s}$$

*(b) From isentropic flow table, two solutions*

*exist for  $\frac{A_1}{A^*} = 4 : M_1 = 0.147$  &  $M_2 = 2.94$*

*(c) Back pressure: from table,*

*For  $M_1 = 0.147$ ,  $\frac{P_1}{P_0} = 0.985$*

*$M_2 = 2.94$ ,  $\frac{P_1}{P_0} = 0.0298$*

*with  $P_0 = 6.8 \times 10^5$  pa*

*$P_b = 6.70 \times 10^5$  pa (subsonic case)*

*$P_b = 0.203 \times 10^5$  pa (supersonic case)*