Problem Set 12 (PS12) due Monday April. 23

10.10 10.13 10.32 10.34

10.10

$$\frac{u}{U_e} = \sin\frac{\pi y}{2\delta}(y \le \delta)$$

Continuity equation:

$$-U_e\delta w + \int_o^\delta uwdy + \int_0^L \bar{v}wdx = 0$$

$x-$*momentum equation*:

($-F =$ *force* by plate on fluid)

$$-F = -\rho U_e^2\delta w + \int_o^\delta \rho u^2 wdy + \int_0^L \rho U_e \bar{v}wdx$$

From the two equations, we get:

$$\therefore -F = -\rho U_e^2\delta w + \rho U_e^2\delta w + \int_o^\delta (\rho u^2 - \rho U_e u)wdy$$

$$\therefore \frac{F}{\rho U_e^2\delta w} = \int_o^\delta \left(\frac{u}{U_e} - \left(\frac{u}{U_e}\right)^2\right)dy$$

$$= \int_o^\delta \left(\sin\frac{\pi y}{2\delta} - \sin^2\frac{\pi y}{2\delta}\right)dy = \frac{4-\pi}{2\pi}$$

10.13

$(a) R_{e_x} = \dfrac{xU_e}{\nu}$

for x=1m, $U_e = 20 m/s, \nu = 1.56\times10^{-5} m^2/s$

$R_{e_x} = 1.28\times10^6$

Since $R_{e_x} > 10^5$, *we* should expect flow near trailing edge to be turbulent.

(b)Using a $\dfrac{1}{7}$ power law, assuming that the boundary layer is turbulent from the leading edgem then at x=1m:

$$\frac{\delta}{\mathrm{x}} = \frac{0.37}{R_{e_x}^{\;0.2}} = \frac{0.37}{(1.28\times10^6)^{0.2}} = 0.022$$

$\therefore \delta = 22\ mm$ (max value)

(c) *From* equation 10.21, $\mathrm{C_F} = \dfrac{0.074}{R_L^{\;0.2}} = \dfrac{0.074}{(1.28\times10^6)^{0.2}}$

(assuming boundary layer is turbulent from leading edge)

$\therefore \mathrm{C_F} = 0.00444$

$$(d)\mathrm{C_F} = \frac{\mathrm{F_V}}{\frac{1}{2}\rho U_e^2 Lw}$$

$\therefore 2\mathrm{F_V} = \rho U_e^2 Lw \times \mathrm{C_F} = total\ drag$

$\therefore total\ dragonplate = 1.184\times(20)^2\times1\times0.5\times0.00444\ N$

$=1.05$ N

10.32

$$R_e = \frac{DU_\infty}{\nu} = 25100$$

If we model the flow on the flow over an isolated cylinder of infinite length,

$$C_D = 1.5$$

$$= \frac{F}{\frac{1}{2}\rho U_\infty^2 DL}$$

$$\therefore F = \frac{1}{2}\rho U_\infty^2 DL \times C_D$$

$$= \frac{1}{2} \times 2.469 \times 10^{-3} \times (60 \times \frac{22}{15})^2 \times \frac{0.5}{12} \times 120 \times 1.5 \text{ lbf}$$

=72 lbf

10.34

$$(a)C_D = \frac{F}{\frac{1}{2}\rho U_\infty^2 A} \quad (A = frontal\ area)$$

$$\therefore F = \frac{1}{2}\rho U_\infty^2 A \times C_D$$

$$= \frac{1}{2} \times \frac{2.373}{1000} \times (75 \times \frac{22}{15})^2 \times 24 \times 0.4 \text{ lbf}$$

$$= 138 \text{ lbf}$$

(b)From definition of $C_D : U_\infty^2 = \dfrac{F}{\frac{1}{2}\rho A C_D}$

$$That \text{ is: } U_\infty^3 = \frac{F \times U_\infty}{\frac{1}{2}\rho A C_D} = \frac{P}{\frac{1}{2}\rho A C_D}$$

$$where\ P = F \times U_\infty = available\ power = 0.8 \times 120 hp$$

$$= 0.8 \times 120 \times 550 \text{ ft} \cdot \text{lbf/s}$$

If only source of drag was aerodynamic:

$$U_\infty^3 = \frac{P}{\frac{1}{2}\rho A C_D} = \frac{0.8 \times 120 \times 550}{\frac{1}{2} \times \frac{2.737}{1000} \times 30 \times 0.6} (\frac{ft}{s})^3$$

$$\therefore (U_\infty)_{\max} = 135 \frac{ft}{s} = 92 mph$$

U_∞ is a relative velocity, and if there is a 30mph headwind

$(U_\infty)_{\max} = 92 - 30$ mph=62 mph