Problem Set 11 (PS11) due Monday April 16
9.30 9.36 9.32
Write energy equation between surface of water in funnel to exit from pipe:

\[ gz_0 = \frac{1}{2} V^2 - g z_2 + g h_e (\alpha = 1) \]

\[ \therefore g(z_0 - z_2) = \frac{1}{2} V^2 + \frac{1}{2} V^2 (k_1 + k_2 + f \frac{L}{d}) \]

\[ = \frac{1}{2} V^2 (1 + k_1 + k_2 + f \frac{L}{d}) \]

\[ \therefore V^2 = \frac{2 g(z_0 - z_2)}{1 + k_1 + k_2 + f \frac{L}{d}} \]

\[ \therefore V^2 = 2 g \times 140d \]

\[ = 80gd \]

\[ \therefore V = \sqrt{80gd} \]
\[
\frac{u}{U_{CL}} = 1 - \left(\frac{r}{R}\right)^2
\]

\[(a)\bar{V} = \frac{1}{A} \int u dA = \frac{1}{\pi R^2} \int_0^R u(2\pi r) dr
\]

\[= \frac{2U_{CL}}{R^2} \int_0^R r(1 - \left(\frac{r}{R}\right)^2) dr
\]

\[= \frac{2U_{CL}}{R^2} \left[ \frac{r^2}{2} - \frac{r^4}{4R^4} \right]_0^R
\]

\[= \frac{U_{CL}}{2}
\]

\[(b)\text{Re} = \frac{2\bar{V}R}{\nu} = \frac{\bar{V}D}{\nu}
\]

When \(\text{Re} > 2300\), the flow can be turbulent, an exact value for turbulent \(\text{Re}\) does not exist because the transition to turbulence depends on many things, including the surface roughness, the pressure of flow disturbances, vibrations, noise and thermal disturbances, as well as the \(\text{Re}\) number.
(c) For a Newtonian flow, \( \tau_w = \mu \frac{\partial u}{\partial y} \)

In this case: \( \tau_w = \mu U_{cl} \left( -\frac{2r}{R^2} \right)_{r=R} = -\frac{2\mu U_{cl}}{R} \)

\[ \therefore \tau_w = -\frac{2\mu U_{cl}}{R} \]

The negative sign is caused by the choice of coordinate system.

Now \( c_f = \frac{\tau_w}{\frac{1}{2} \rho V_r^2} = \frac{4 \mu \bar{V}}{\frac{1}{2} \rho V_r^2} = \frac{16 \mu}{\rho V D \text{ Re}} = 16 \)
Energy equation from free surface of reservoir to the exit:

\[ (p_0 = p_a, V_1 = \dot{V}) \]

\[ g z_0 = \frac{\alpha V}{2} + g h_e - \frac{w_{\text{shaft}}}{m} + g z_1 \]

That is \( \frac{w_{\text{shaft}}}{m} = \frac{\alpha V}{2} + f \frac{L V}{d} + g(z_0 - z_1), \, (\alpha = 1) \)

Hence \( \frac{w_{\text{shaft}}}{m} = \frac{V}{2}(1 + f \frac{L}{d}) - g(z_0 - z_1) \)

Now: \( \dot{V} = \frac{q}{\pi D^2} = 12.7 \, m/s \)

\[ \therefore \frac{w_{\text{shaft}}}{m} = \frac{(12.7)^2}{2}(1 + 0.025 \times \frac{100}{0.1}) - 9.8(10) \, m^2/s^2 \]

= 1999 \, m^2/s^2
\[ \therefore w_{\text{shaft}} = 1999 \cdot m = 1999 \rho g \]

\[ = 1999 \times 999 \times \frac{100}{1000} \text{ watt (15 } ^\circ \text{C)} \]

\[ \therefore w_{\text{shaft}} = +200 \text{kw} \]

\[ w_{\text{shaft}} \text{ is positive, so that work is done on the fluid, so machine is a pump.} \]