Problem Set 7 (PS7) due Monday March 19

6.16  6.37
The divergence of the velocity is important because when it equals to 0, the flowfield is incompressible. For the case given here, $\nabla \cdot V \neq 0$ (for any value of $z$)

\[ V = 2xi + 5yz^2 j - t^3 k \]

\[ \nabla \cdot V = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 2 + 5z^2 \]
Incompressible N-S equation:

\[
\rho \frac{DV}{Dt} = -\nabla p + \rho g + \mu \nabla^2 V
\]

Pressure is constant everywhere; ignore gravity:

\[
\rho \frac{DV}{Dt} = \mu \nabla^2 V
\]

Now \(V = u_i\) where \(u = u(y)\)

\[
\frac{DV}{Dt} = i \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right]
\]

\[
= i [0 + u(0) + 0 \frac{\partial u}{\partial y} + 0(0)]
\]

By continuity, \(\nabla \cdot V = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\)

Since \(\frac{\partial u}{\partial x} = 0, v = \text{cons tan } t\).

At \(y = 0, v = 0 \rightarrow v = 0\) everywhere.

Hence \(\frac{DV}{Dt} = 0\).

\[
\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = i [0 + \frac{\partial^2 u}{\partial y^2} + 0]
\]

Since \(u = U \frac{y}{h}, \frac{\partial^2 u}{\partial y^2} = 0\).

Hence \(\nabla^2 V = 0\).

Therefore, linear couette flow is an exact solution to the N-S equation.