Problem Set 6 (PS6) due
Monday March 5
6.11 6.19
6.11

From continuity equation \( \frac{D\rho}{Dt} = -\rho(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}) \)

(a) \( \frac{D\rho}{Dt} \) directly:
\[
\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z}
= \frac{1}{x} + \frac{4x}{t} \left( -\frac{t}{x^2} \right) + \frac{y^2}{t} \cdot 0 + 0
= -\frac{3}{x}
\]

(b) \( \frac{D\rho}{Dt} \) from continuity equation:
\[
\frac{D\rho}{Dt} = -\rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)
= -(3 + \frac{t}{x}) \left( \frac{4 + 2y}{t} \right)
= -\frac{12}{t} - \frac{6y}{t} - \frac{4}{x} - \frac{2y}{x}
\]

Answers to (a) and (b) do not degree: This flowfield is not possible. (mass is not conserved.)
6.19

\[ V = -2x^2 i + 4xy j + 3k \]

(a) \[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = -4x + 4x + 0 = 0 \]

\[ \therefore \text{Hence the flowfield is incompressible.} \]

(b) \[ \frac{DV}{Dt} = \frac{\partial V}{\partial t} + u \frac{\partial V}{\partial x} + v \frac{\partial V}{\partial y} + w \frac{\partial V}{\partial z} \]

\[ = 0 + (-2x^2)(-4xi + 4yj) + (4xy)(4xj) + (3)(0) \]

\[ = i(8x^3) + j(16x^2 y - 8x^2 y) = 8x^3 (i + \frac{y}{x} j) \]

at \((1,3,0)\) \[ \frac{DV}{Dt} = 8(i + 3j). \]

(c) Volume flux through area \(A = \int (n \cdot v)dA \)

..:\[ = \int j \cdot v dA = \int \int 4xy dx dz \]

\[ = \int_0^1 4xy \left[ \int_0^4 dz \right] dx \]

\[ = \int_0^1 16xy dx = 16y \int_0^1 x dx \]

\[ = 24 \]

(d) dimension of volume flux=velocity \times area = \frac{L^3}{T}