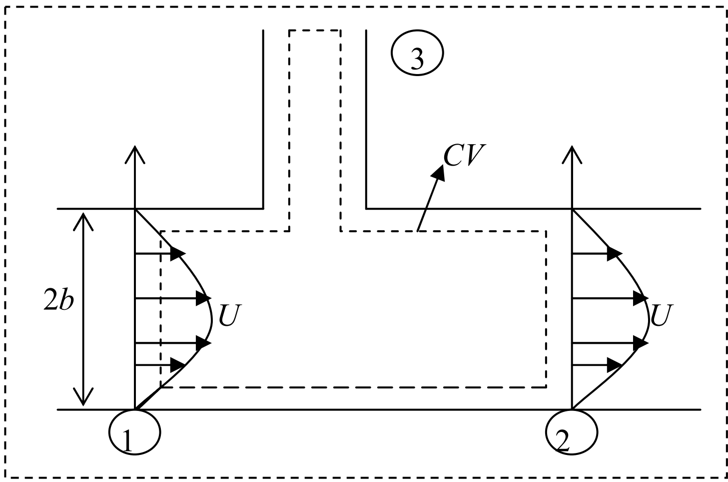
Problem Set 5 (PS5) due Monday Feb. 19

5.4 5.21 5.28

5.4



Steady continuity equation:

$$\int n\cdot \rho V dA = 0$$

$$\therefore \int n\cdot V dA = 0$$

That is:

$$\int n\cdot V dA_1 + \int n\cdot V dA_2 + \dot{q} = 0$$

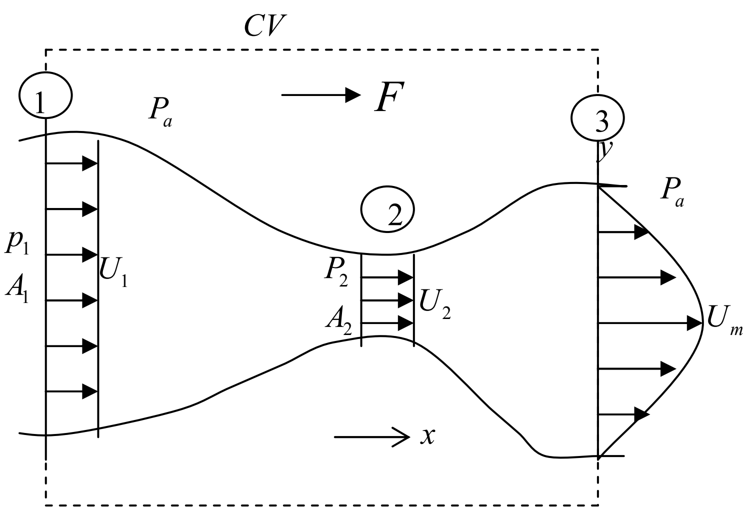
$$\therefore -\int_{-b}^{b} Q(1-\frac{y^2}{b^2})w dy + \int_{-b}^{b} \frac{2Q}{3}(1-\frac{y^2}{b^2})w dy + \dot{q} = 0$$

$$\therefore \dot{q} = \frac{Q}{3} w\int_{-b}^{b}(1-\frac{y^2}{b^2})dy = \frac{2Q}{3} w\int_{0}^{b}(1-\frac{y^2}{b^2})dy$$

$$= \frac{4}{9}\rho Q w b$$

$$\therefore \dot{q} = \frac{4}{9}\rho Q w b (outflow)$$

5.21



(a)Steady, constant density flow:

$$\int n \cdot V dA = 0$$

Between 1 and 2:

$$-U_1 3hw + U_2 hw = 0$$

$$\therefore \frac{U_2}{U_1} = 3$$

Between 2 and 3:

$$-U_2 hw + 2\int_0^h U_m (1 - \frac{y^2}{h^2}) w dy = 0$$

$$\therefore U_2 = \frac{2}{h} U_m [y - \frac{y^3}{3h^2}]_0^h$$

$$3U_1 = \frac{2}{h} \times \frac{2h}{3} U_m$$

$$\therefore \frac{U_m}{U_1} = \frac{9}{4}$$

(b)Assume no loss. From 1 to 2:

$$p_1 + \frac{1}{2}\rho U_1^2 = p_2 + \frac{1}{2}\rho U_2^2$$

$$\therefore p_1 - p_2 = \frac{1}{2}\rho (U_2^2 - U_1^2) = \frac{1}{2}\rho (9U_1^2 - U_1^2)$$

$$\therefore p_1 - p_2 = 4\rho U_1^2$$

<u>5.21</u>

(*c*) $x-component\ momentum$ equation:

+F force acting on fluid

$$+\mathrm{F}+\mathrm{P}_1 3hw - P_a 2hw = -\rho U_1^2 3hw + \int_{-h}^{h} \rho U_m^2 (1-\frac{y^2}{b^2}) w dy$$

$$\therefore \mathrm{F}=-\mathrm{P}_1 3hw + P_a 2hw + \frac{12}{5}\rho \mathrm{U}_1^2 hw$$

Here P_1 is the gauge pression.

$$\therefore \mathrm{F}=-\mathrm{P}_1 3hw + \frac{12}{5}\rho \mathrm{U}_1^2 hw$$

$$\therefore \frac{\mathrm{F}}{\rho \mathrm{U}_1^2 hw} = \frac{-3\mathrm{P}_1}{\rho \mathrm{U}_1^2} + \frac{12}{5}$$

5.28

F=Force exerted by fluid on cane to keep ot moving at constant

-F= Force exerted by cane on fluid.

Steady, constant density flow:

$$-\mathrm{F} = \int (\mathrm{n} \cdot \rho V) V dA$$

$$\therefore -\mathrm{F} = (-\rho(V-U)) \cdot (V-U) \cdot \mathrm{i}A + \rho(V-U)$$

$$[(V-U)\cos\theta \cdot \mathrm{i} + (V-U)\sin\theta \cdot \mathrm{j}]$$

$$\therefore \mathrm{F} = \rho(V-U)^2 \cdot \mathrm{i}A - \rho(V-U)^2 A(\cos\theta \cdot \mathrm{i} + \sin\theta \cdot \mathrm{j})$$

$$= \rho(V-U)^2 A[(1-\cos\theta)\mathrm{i} - \sin\theta \cdot \mathrm{j}] \text{ N}$$

$$\therefore \mathrm{F} = (998\mathrm{i} - 1729\mathrm{j}) \text{ N}$$