Problem Set 5 (PS5) due Monday Feb. 19
5.4 5.21 5.28
Steady continuity equation:
\[ \int n \cdot \rho V dA = 0 \]
\[ \therefore \int n \cdot V dA = 0 \]

That is:
\[ \int n \cdot V dA_1 + \int n \cdot V dA_2 + q = 0 \]
\[ \therefore -\int_{-b}^{b} Q(1 - \frac{y^2}{b^2})wdy + \int_{-b}^{b} \frac{2Q}{3} (1 - \frac{y^2}{b^2})wdy + q = 0 \]
\[ \therefore q = \frac{Q}{3} w\int_{-b}^{b} (1 - \frac{y^2}{b^2})dy = \frac{2Q}{3} w\int_{0}^{b} (1 - \frac{y^2}{b^2})dy \]
\[ = \frac{4}{9} \rho Q wb \]
\[ \therefore q = \frac{4}{9} \rho Q wb (\text{outflow}) \]
5.21

(a) Steady, constant density flow:
\[ \int n \cdot V dA = 0 \]

Between 1 and 2:
\[-U_1 3hw + U_2 hw = 0\]
\[ \therefore \frac{U_2}{U_1} = 3 \]

Between 2 and 3:
\[-U_2 hw + 2 \int_0^h U_m (1 - \frac{y^2}{h^2}) wdy = 0\]
\[ \therefore \ U_2 = \frac{2}{h} U_m \left[ y - \frac{y^3}{3h^2} \right]_0^h \]
3 \[ U_1 = \frac{2}{h} \times \frac{2h}{3} U_m \]
\[ \therefore \ U_m = \frac{9}{4} U_1 \]

(b) Assume no loss. From 1 to 2:
\[ p_1 + \frac{1}{2} \rho U_1^2 = p_2 + \frac{1}{2} \rho U_2^2 \]
\[ \therefore \ p_1 - p_2 = \frac{1}{2} \rho (U_2^2 - U_1^2) = \frac{1}{2} \rho (9U_1^2 - U_1^2) \]
\[ \therefore \ p_1 - p_2 = 4 \rho U_1^2 \]
(c) $x$-component momentum equation:

$+F$ force acting on fluid

$+F + P_1 3hw - P_a 2hw = -\rho U_1^2 3hw + \int_{-h}^{h} \rho U_m^2 (1 - \frac{y^2}{b^2}) wdy$

∴ $F = -P_1 3hw + P_a 2hw + \frac{12}{5} \rho U_1^2 hw$

*Here* $P_1$ is the gauge pression.

∴ $F = -P_1 3hw + \frac{12}{5} \rho U_1^2 hw$

∴ $\frac{F}{\rho U_1^2 hw} = \frac{-3P_1}{\rho U_1^2} + \frac{12}{5}$
$F =$ Force exerted by fluid on cane to keep it moving at constant speed.

$\mathbf{-F} =$ Force exerted by cane on fluid.

Steady, constant density flow:

$\mathbf{-F} = \int (\mathbf{n} \cdot \rho \mathbf{V}) \mathbf{V} dA$

$\therefore \mathbf{-F} = (\rho (\mathbf{V} - \mathbf{U})) \cdot (\mathbf{V} - \mathbf{U}) \cdot \mathbf{i} + \rho (\mathbf{V} - \mathbf{U}) [(\mathbf{V} - \mathbf{U}) \cos \theta \cdot \mathbf{i} + (\mathbf{V} - \mathbf{U}) \sin \theta \cdot \mathbf{j}]

\therefore \mathbf{F} = \rho (\mathbf{V} - \mathbf{U})^2 \cdot \mathbf{i} - \rho (\mathbf{V} - \mathbf{U})^2 A(\cos \theta \cdot \mathbf{i} + \sin \theta \cdot \mathbf{j})

= \rho (\mathbf{V} - \mathbf{U})^2 A [(1 - \cos \theta) \mathbf{i} - \sin \theta \cdot \mathbf{j}] \text{ N}

\therefore \mathbf{F} = (998 \mathbf{i} - 1729 \mathbf{j}) \text{ N}