Problem Set 4 (PS4) due Monday Feb. 19

4.33  5.6   5.22
If there are no losses in the system,

\[ p_2 = p_3 = p_a \]

Applying Bernoulli:

\[ \frac{p_1}{\rho_a} + \frac{1}{2}V_1^2 = \frac{p_2}{\rho_a} + \frac{1}{2}V_2^2 \]

\[ \therefore \frac{p_1 - p_a}{\rho_a} = \frac{1}{2}(V^2 - V_1^2) \]

Continuity equation:

\[ V_1^2 \frac{\pi D_1^2}{4} = V_2^2 \frac{\pi D_2^2}{4} \]

And also,

\[ p_2 - p_1 = \rho_m gh \]

\[ \therefore p_1 - p_a = -\rho_m gh \]

Hence

\[ -\frac{\rho_m}{\rho_a} gh = \frac{1}{2}(V^2 - V_1^2) \left( \frac{D}{d} \right)^4 \]

\[ \therefore V = \left[ \frac{\rho_a}{\rho_m} gh \right]^{\frac{1}{2}} \left( \frac{D}{d} \right)^{4 - 1} \]
Steady, constant density flow.

Continuity equation:

\[ \int n \cdot VdA = 0 \]
\[ \int n \cdot VdA_1 + \int n \cdot VdA_2 - q = 0 \]

\[ \therefore q = \int_{h_1}^{h_2} U_2 \left(1 - \frac{y}{h_2}\right)wdy \]
\[ = 2U_1h_1w + U_2h_2w = -2U_1h_1w + 2U_1h_2w \]
\[ \therefore q = 2U_1w(h_2 - h_1) \]
5.22

(a) Constant density, steady flow:
\[ \int n \cdot V dA = 0 \]

\[-U_1 Dw + 2(U_2 \times \frac{\delta}{2})w + U_2(D - 2\delta)w = 0\]

\[ \therefore U_1 = \frac{\delta}{D} U_2 + U_2 \left(1 - \frac{2\delta}{D}\right) \]

\[ \therefore \frac{U_1}{U_2} = 1 - \frac{\delta}{D} \]

(b) Outside the boundary layers, the flow is inviscid, and Bernoulli equation can be used:

\[ \therefore p_1 + \frac{1}{2} \rho U_1^2 = p_2 + \frac{1}{2} \rho U_2^2 \]

\[ \therefore p_1 - p_2 = \frac{1}{2} \rho (U_2^2 - U_1^2) \]

\[ \therefore \frac{p_1 - p_2}{\frac{1}{2} \rho U_1^2} = \frac{U_2^2}{U_1^2} - 1 \]
(c) x-component momentum equation:

\[ F_v = \text{viscous force acting on walls}, \]
\[ \therefore -F_v = \text{viscous force acting on fluid}. \]
\[ \therefore -F_v + (p_1 - p_2)Dw = -\rho U_1^2 Dw + \]
\[ \rho U_2^2 (D - 2\delta)w + 2 \rho w \int_0^\delta U^2 dy \]
\[ \therefore \frac{F_v}{\frac{1}{2} \rho U_1^2 Dw} = \frac{p_1 - p_2}{\frac{1}{2} \rho U_1^2} + 2 - \]
\[ \frac{2U_2^2}{U_1^2} (1 - \frac{2\delta}{D}) - \frac{4}{D} \frac{U_2^2}{U_1^2} \int_0^\delta \frac{y^2}{\delta^2} dy \]
\[ = 1 - \frac{U_2^2}{U_1^2} (1 - \frac{8\delta}{3D}) \]
\[ \therefore \frac{F_v}{\frac{1}{2} \rho U_1^2 Dw} = 1 - \frac{U_2^2}{U_1^2} (1 - \frac{8\delta}{3D}) \]