Problem Set 3 (PS3) due Monday  Feb. 12
3.18  3.23  3.27  4.22
Let $\dot{m}$ = rate of accumulation of mass in tank:

By conservation of mass:

$$\dot{m} = \rho A_1 V_1 - \rho A_2 V_2 - \rho A_3 V_3$$

$$= \frac{\rho \pi}{4} (V_1 D_1^2 - V_2 D_2^2 - V_3 D_3^2)$$

Now $\dot{m} = \frac{\rho \pi}{4} D_4^2 \frac{dh}{dt}$

So

$$\frac{dh}{dt} = \frac{1}{D_4^2} (V_1 D_1^2 - V_2 D_2^2 - V_3 D_3^2) = -0.00263(m/s)$$

Hence, water level is falling at 0.00263 m/s.
For control volume CV shown;
For steady, constant density flow:

\[-U_1 \overrightarrow{4Dw} - Q_w + U_2 \overrightarrow{2Dw} + U_1 \overrightarrow{2Dw} = 0\]

\[\therefore \ U_2 = U_1 + \frac{Q}{2D}\]

x-component momentum equation:

\[F + p_1 \overrightarrow{4Dw} - p_2 \overrightarrow{4Dw} = -\rho U_1^2 4Dw + \rho U_2^2 2Dw + \rho U_1^2 2Dw\]

\[\therefore \ \frac{F}{\frac{1}{2} \rho U_1^2 4Dw} = \frac{p_2 - p_1}{\frac{1}{2} \rho U_1^2} + \frac{Q}{DU_1} \left(\frac{Q}{4DU_1}\right) + 1\]
For steady, constant density flow, for CV shown, mass conservation gives:

\[-U_1 t_1 + U_2 t_2 + U_3 t_3 = 0\]

\[\therefore U_1 t_1 = 2U_1 t_2 + 2U_1 t_3\]

\[\therefore t_2 = t_3 = \frac{t_1}{4}\]

x-component momentum equation:

\[F_x = -\rho U_1^2 t_1 w + \rho U_2^2 t_2 w \cos \theta + \rho U_3^2 t_3 w\]

\[= -\rho U_1^2 t_1 w + \rho 4U_1^2 \frac{t_1}{4} w \cos \theta + \rho 4U_1^2 \frac{t_1}{4} w\]

\[= \rho U_1^2 t_1 \cos \theta\]

y-component momentum equation:

\[F_y = \rho U_2^2 t_2 w \sin \theta = \rho 4U_1^2 \frac{t_1}{4} w \sin \theta = \rho U_1^2 t_1 \sin \theta\]

Now \(F_x = F_y\) when \(\frac{\sin \theta}{\cos \theta} = 1\), Hence \(\theta = 45^\circ\)
(a) Cavitation may occur at the point of lowest pressure in the tube. That is, when $H_2$ is such that $p < \text{vapor Pressure of water}$, cavitation can occur, and the siphon may not continue to operate.

(b) Bernoulli equation (between free surface and exit plane)

$$gH_1 = \frac{1}{2}V_e^2$$

Bernoulli equation (between exit plane and height $y$)

$$\frac{1}{2}V_e^2 = \frac{1}{2}V_g^2 + gy$$

$$\therefore gy = \frac{1}{2}(V_e^2 - V_g^2)$$

By continuity: $V_e A_e = V_y A_y$

$$\therefore \frac{A_e}{A_y} = \frac{V_e}{V_y} = \frac{\sqrt{2gH_1}}{\sqrt{V_e^2 - 2gy}} = \frac{H_1}{\sqrt{H_1 - y}}$$

(c) At apogee: $V_x = V_e \cos \theta$, $V_y = 0$

Bernoulli equation (between exit plane and apogee)

$$\frac{1}{2}V_e^2 = \frac{1}{2}(V_e^2 \cos^2 \theta + V_e^2 \sin^2 \theta) = \frac{1}{2}V_x^2 + gH_3$$

$$= \frac{1}{2}V_e^2 \cos^2 \theta + gH_3$$

$$\therefore \frac{1}{2}V_e^2 \sin^2 \theta = gH_3$$

$$\therefore H_3 = H_1 \sin^2 \theta$$

(d) Use vertical component momentum equation applied to CV shown:

- Weight = $-\rho V_e^2 A \sin \theta + 0$

That is:

$$\rho g \text{(Volume)} = \rho(2gH_1)A \sin \theta$$

$$\therefore \text{Volume} = 2H_1 A \sin \theta$$