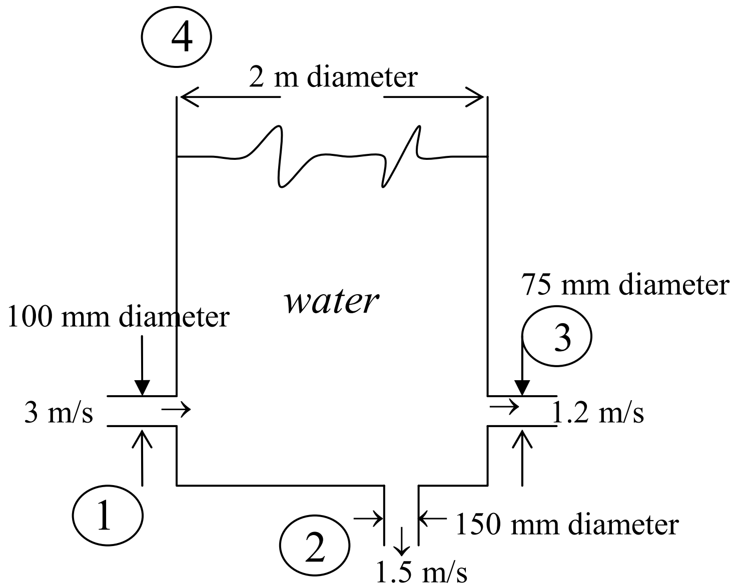
Problem Set 3 (PS3) due Monday Feb. 12

3.18 3.23 3.27 4.22

3.18



Let in = rate of accumulation of mass in tank:

By conservation of mass:

$$\dot{m} = \rho A_1 V_1 - \rho A_2 V_2 - \rho A_3 V_3$$

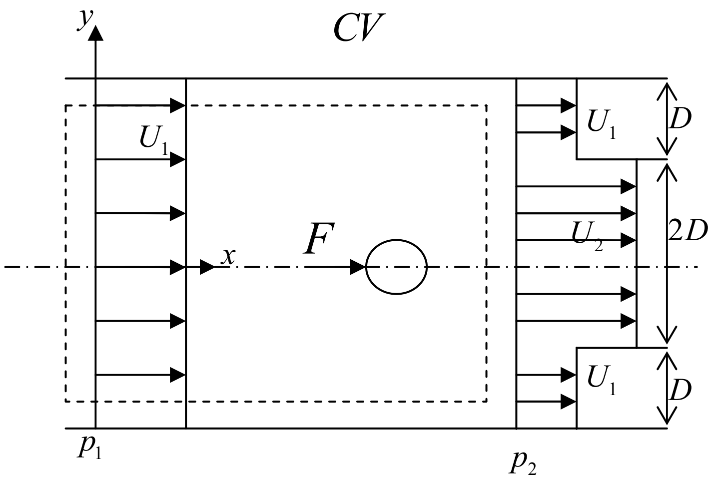
$$= \frac{\rho \pi}{4}(V_1 D_1^{\ 2} - V_2 D_2^{\ 2} - V_3 D_3^{\ 2})$$

Now $\dot{m} = \frac{\rho \pi}{4} D_4^{\ 2} \frac{dh}{dt}$

So $\frac{dh}{dt} = \frac{1}{D_4^{\ 2}}(V_1 D_1^{\ 2} - V_2 D_2^{\ 2} - V_3 D_3^{\ 2}) = -0.00263 (m/s)$

Hence, water level is falling at 0.00263 m/s.

<u>3.23</u>



For control volume CV shown;

For steady, constant density flow:

$$-U_1 \cdot 4Dw - Qw + U_2 \cdot 2Dw + U_1 \cdot 2Dw = 0$$

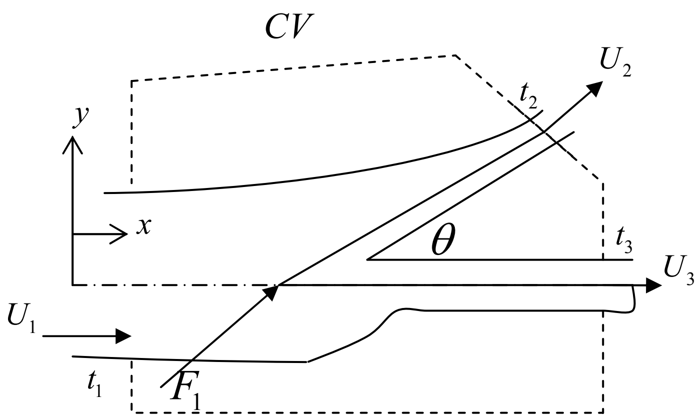
$$\therefore U_2 = U_1 + \frac{Q}{2D}$$

x-component momentumequation:

$$F + p_1 4Dw - p_2 4Dw = -\rho U_1^2 4Dw + \rho U_2^2 2Dw + \rho U_1^2 2Dw$$

$$\therefore \frac{F}{\frac{1}{2}\rho U_1^2 4Dw} = \frac{p_2 - p_1}{\frac{1}{2}\rho U_1^2} + \frac{Q}{DU_1}\left(\frac{Q}{4DU_1} + 1\right)$$

3.27



For steady, constant density flow, for CV shown, mass conservation gives:

$$-U_1 t_1 + U_2 t_2 + U_3 t_3 = 0$$

$$\therefore U_1 t_1 = 2U_1 t_2 + 2U_1 t_3$$

$$\therefore t_2 = t_3 = \frac{t_1}{4}$$

x-component momentumequation:

$$F_x = -\rho U_1^{\,2} t_1 w + \rho U_2^{\,2} t_2 w \cos\theta + \rho U_3^{\,2} t_3 w$$

$$= -\rho U_1^{\,2} t_1 w + \rho 4 U_1^{\,2} \frac{t_1}{4} w \cos\theta + \rho 4 U_1^{\,2} \frac{t_1}{4} w$$

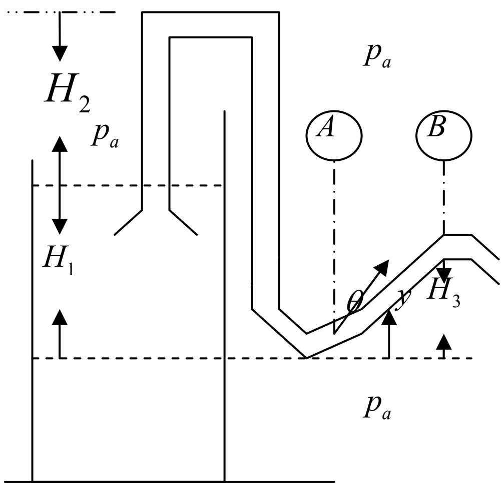
$$= \rho U_1^{\,2} t_1 \cos\theta$$

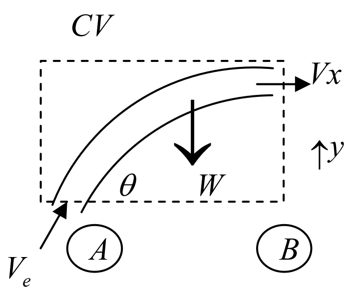
y-component momentumequation:

$$F_y = \rho U_2^{\,2} t_2 w \sin\theta = \rho 4 U_1^{\,2} \frac{t_1}{4} w \sin\theta = \rho U_1^{\,2} t_1 \sin\theta$$

Now $F_x = F_y$ when $\dfrac{\sin\theta}{\cos\theta} = 1$, Hence $\theta = 45^\circ$

4.22





(a) Cavitation may occur at the point of lowest pressure In the tube. That is, when H_2 is such that p_3<vapor Pressure of water, Cavitation can occur, and the siphon may not continue to operate.

(b) Bernoulli equation (between free surface and exit plane)

$$gH_1 = \frac{1}{2}V_e^2$$

Bernoulli equation (between exit plane and height y)

$$\frac{1}{2}V_e^2 = \frac{1}{2}V_g^2 + gy$$

$$\therefore gy = \frac{1}{2}(V_e^2 - V_g^2)$$

By continuity: $V_e A_t = V_y A$

$$\therefore \frac{A}{A_t} = \frac{V_e}{V_y} = \frac{\sqrt{2gH_1}}{\sqrt{V_e^2 - 2gy}} = \sqrt{\frac{H_1}{H_1 - y}}$$

(c) At apogee: $V_x = V_e \cos\theta,\ V_y = 0$

Bernoulli equation (between exit plane and apogee)

$$\frac{1}{2}V_e^2 = \frac{1}{2}(V_e^2\cos^2\theta + V_e^2\sin^2\theta) = \frac{1}{2}V_x^2 + gH_3$$

$$= \frac{1}{2}V_e^2\cos^2\theta + gH_3$$

$$\therefore \frac{1}{2}V_e^2\sin^2\theta = gH_3$$

$$\therefore H_3 = H_1\sin^2\theta$$

(d) Use vertical component momentum equation applied to CV shown :

-Weight=$-\rho V_e^2 A_t \sin\theta + 0$

That is:

ρg(Volume)=$\rho(2gH_1)A_t\sin\theta$

$\therefore$ Volume=$2H_1A_t\sin\theta$