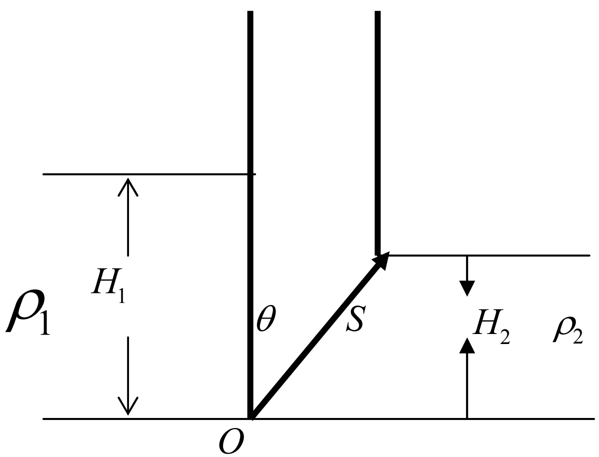
Problem Set 2 (PS2) due Monday Feb. 5

2.25 2.33 2.34 2.54

2.25



$$P_1 = \rho_1 g(H_1 - Z)$$

$$dA_1 = w \cdot dz$$

$$S_1 = z$$

$$F_1 = \int_0^{H_1} \rho_1 g(H_1 - z) \cdot w\,dz$$

$$M_1 = \int_0^{H_1} \rho_1 g(H_1 - z) \cdot w \cdot z \cdot dz = \frac{H_1^{\ 3}}{6} \rho_1 g w$$

$$P_2 = \rho_2 g(H_2 - Z)$$

$$dA_2 = w \cdot \frac{dz}{\cos\theta}$$

$$S_2 = \frac{z}{\cos\theta}$$

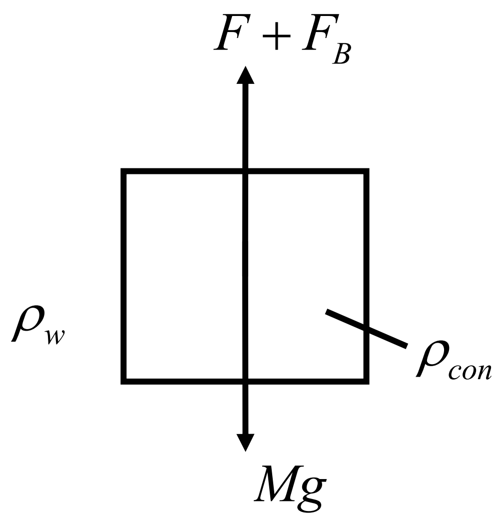
$$F_2 = \int_0^{H_2} \rho_2 g(H_2 - z) \cdot w \frac{dz}{\cos\theta}$$

$$M_2 = \int_0^{H_2} \rho_2 g(H_2 - z) \cdot w \cdot \frac{z}{\cos\theta} \cdot \frac{dz}{\cos\theta} = \frac{H_2^{\ 3}}{6 \cdot \cos^2\theta} \rho_2 g w$$

For moment equilibrium: $M_1 = M_2$

We can get: $\dfrac{\rho_2}{\rho_1} = \dfrac{H_1^3 \cdot \cos^2\theta}{H_2^3}$

2.33



Let F be the force required to hold the block at a constant depth, and the force due to buoyancy. Then,

$$F + F_B = Mg$$

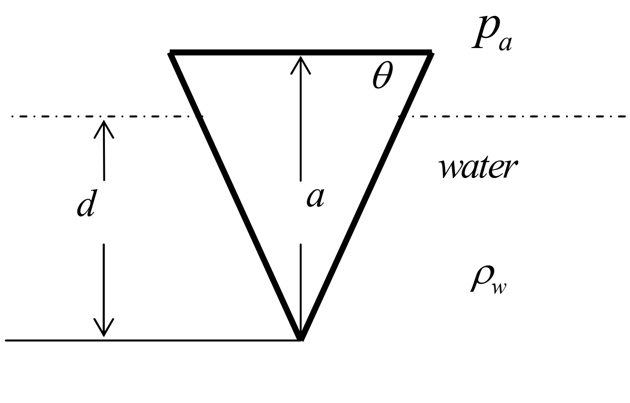
$$F = Mg - F_B = \rho_{con} gV - \rho_W gV$$

$$= (\rho_{con} - \rho_W) gV$$

$$= (2400 - 1000) \times 9.8 \times (\frac{12 \times 2.54}{100})^3 N$$

$$= 389 N$$

2.34



For equilibrium,

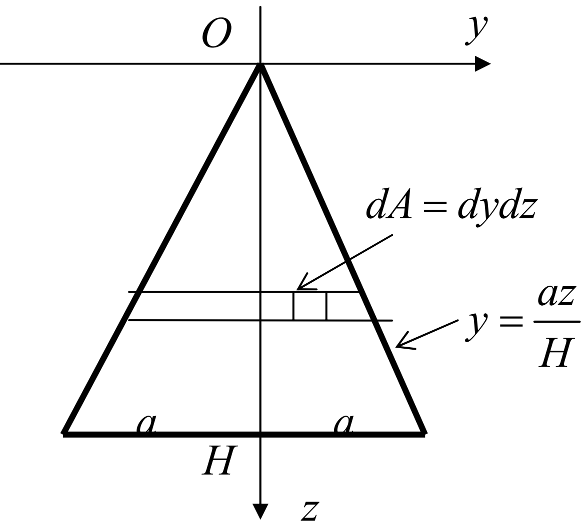
Weight of prism = buoyancy force,

$$\rho_P g \times a \times \frac{a}{\tan\theta} \times w = \rho_w g \times d \times \frac{d}{\tan\theta} \times w$$

$$\therefore \rho_P \times a^2 = \rho_w \times d^2$$

$$\therefore \frac{a}{d} = \sqrt{\frac{\rho_w}{\rho_P}} = 2$$

<u>2.54</u>



$$F = \int \rho_g \, dA$$

$$\therefore F = \iint \rho g z \, dy \, dz$$

$$\therefore F = \int_0^H \rho g z \left[2 \int_0^{\frac{az}{H}} dy \right] dz$$

$$= \int_0^H 2 \rho g z \left(\frac{az}{H} \right) dz = \frac{2 \rho g H^2 a}{3}$$

The force F will act on the line y=0, at a distance from the water surface, where, if we take moment of y-axis.

$$F \times \bar{z} = \int dM = \int z \, dF = \iint \rho g z^2 \, dy \, dz$$

$$= \int_0^H \rho g z^2 \left[2 \int_0^{\frac{az}{H}} dy \right] dz$$

$$= \int_0^H 2 \rho g z^2 \left(\frac{az}{H} \right) dz = \frac{\rho g H^3 a}{2}$$

$$\therefore \bar{z} = \frac{3}{4} H$$