Problem Set 1 (PS1) due Monday January 29

1.14 Estimate the change in pressure that occurs when still air at 100°*F* and atmospheric pressure is accelerated to a speed of 100 *mph* at constant temperature. Neglect compressibility effects.

2.14 The gate shown in Figure P2.14 has a width w and a height H and it is pivoted on a frictionless hinge at a point z^* below the surface of the water. The top of the gate is level with the surface of the water. The water is of density ρ, and outside the tank the pressure is uniform everywhere and equal to the atmospheric pressure.

(a) Find the magnitude of the resultant force F on the gate, in terms of ρ, g, w, and H.

(b) Find the location of the pivot line z^* below the top of the gate, so that there is no resultant moment about the hinge tending to open the gate.

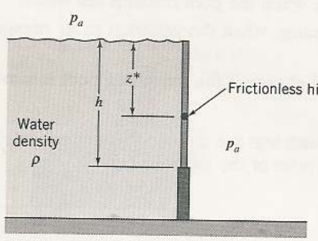


FIGURE P2.14

2.10 For the manometer shown in Figure P2.10, both legs are open to the atmosphere. It is filled with liquids A and B as indicated. Find the ratio of the liquid densities.

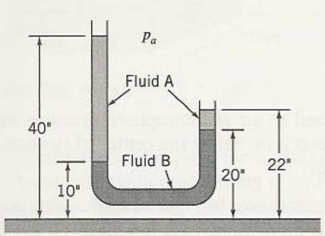


FIGURE P2.10

2.28 A rigid, weightless, two-dimensional gate of width w separates two liquids of density ρ_1 and ρ_2, respectively, as shown in Figure P2.28. The gate pivots on a frictionless hinge and it is in static equilibrium. Find the ratio ρ_2/ρ_1 when $h = b$.

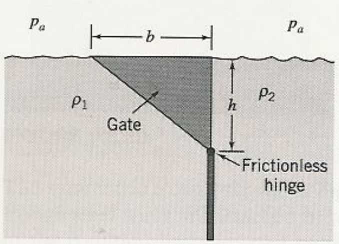


FIGURE P2.28

1.14

$$1\,\text{lb}_f = 1\,\text{lb}_m \times 32.1739\,\frac{\text{ft}}{\text{sec}^2}$$

$$1\,\text{lb}_f = 1\,\text{slug} \times 1\,\frac{\text{ft}}{\text{sec}^2}$$

$$1\,\text{slug} = \frac{\text{lb}_m}{g_c} = \frac{\text{lb}_f}{\text{ft/sec}^2}$$

$$\rho\ @\ 100\ \text{F} = .002204\,\frac{\text{slugs}}{\text{ft}^3}$$

$$V_1 = 0$$

$$V_2 = 100\,\frac{\text{miles}}{\text{hr}} \times \frac{5280\,\frac{\text{ft}}{\text{mile}}}{3600\,\frac{\text{sec}}{\text{hr}}} = 146.67\ \text{ft/sec}$$

$$\Delta p = -\frac{1}{2}\rho\left(V_2^2 - V_1^1\right) \qquad (\text{eq. } 1.6)$$

$$\Delta p = -\frac{1}{2} \times .002204 \times (146.67^2 - 0)$$

$$\Delta p = -23.7\,\frac{\text{lb}_f}{\text{ft}^3}$$

$$\frac{\text{lb}_f}{\text{ft}^3} = \frac{1}{\text{ft}^3}\,\frac{\text{lb}_f}{\text{ft/sec}^2} \times \frac{\text{ft}^2}{\text{sec}^2}$$

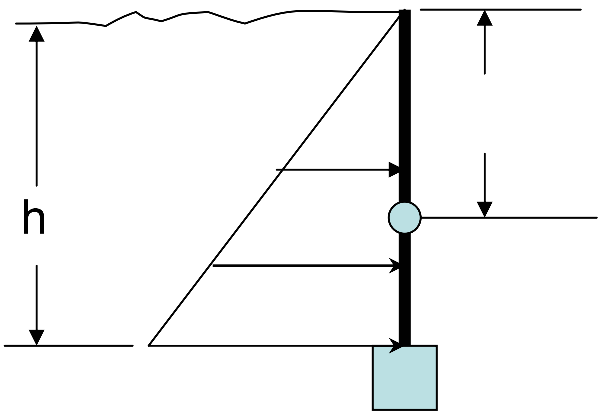
2.10

$$\rho_A h_A = \rho_B h_B$$

$$\rho_A\left(\left(40-10\right)-\left(22-20\right)\right) = \rho_B\left(20-10\right)$$

$$\frac{\rho_B}{\rho_A} = \frac{28}{10} = 2.8$$

<u>2.14</u>



$$p = g\,\rho\, z$$

$$F = \int p\, dA = w \int p\, dz$$

$$F = w\, g\, \rho \int z\, dz$$

$$\text{moment arm} = \left(z^* - z\right)$$

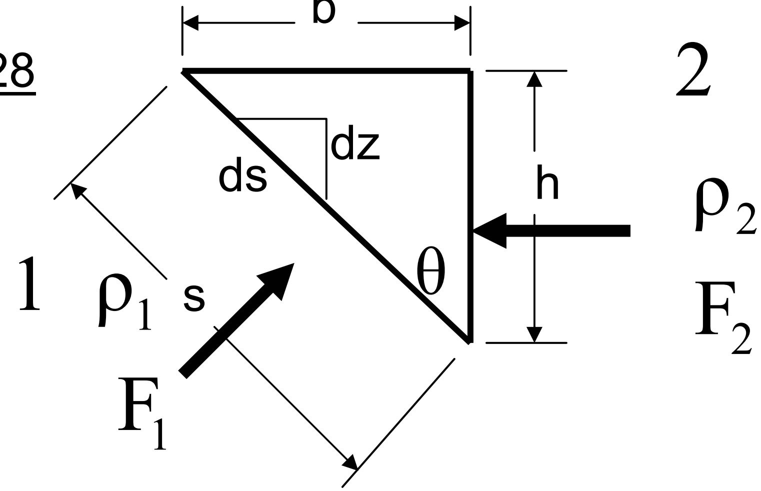
$$\text{Moment} = w\, g\, \rho \int_0^{-h} \left(z^* - z\right) z\, dz = 0$$

$$z^* \frac{z^2}{2} - \frac{z^3}{3} = 0$$

$$z^* \frac{h^2}{2} + \frac{h^3}{3} = 0$$

$$z^* = -\frac{2 \times h}{3} \quad \text{(below surface)}$$

2.28



$$\cos\theta = \frac{h}{s} = \frac{dz}{ds} = \frac{h}{\sqrt{h^2+b^2}}$$

$$dA_1 = w \times ds = \frac{w \times ds}{\cos\theta}$$

$$p_2 = g\,\rho_1\,(h-z)$$

$$F_2 = \int p\,dA_2 = w\,g\,\rho_2 \int (h-z)\,dy$$

$$\text{momment arm} = z$$

$$M_2 = F_2 \times s = w\,g\,\rho_2 \int (h-z)\,z\,dy$$

$$p_1 = g\,\rho_1\,(h-z)$$

$$F_1 = \int p\,dA_1$$

$$\text{mommentarm} = s = \frac{z}{\cos\theta}$$

$$M_1 = F_1 \times s = \frac{w\,g\,\rho_1}{\cos\theta} \int (h-z) \frac{z}{\cos\theta}\,dy$$

$$M_1 = F_1 \times s = \frac{w\,g\,\rho_1}{\cos\theta^2} \int (h-z)\,z\,dy$$

$$M_1 = M_2$$

$$\frac{w\,g\,\rho_1}{\cos\theta^2} \int (h-z)\,z\,dy = w\,g\,\rho_2 \int (h-z)\,z\,dy$$

$$\frac{\rho_2}{\rho_1} = \frac{1}{\cos\theta^2}$$

$$\text{forh} = b$$

$$\frac{\rho_2}{\rho_1} = \frac{1}{\cos\theta^2} = \frac{1}{\left(\frac{b}{\sqrt{b^2+b^2}}\right)^2} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$\frac{\rho_2}{\rho_1} = 2$$