

mae 335 Fluid Mechanics Spring 2007 Quiz 1

A vertical hinged 10 ft wide gate in a water channel open to the atmosphere has a water depth of 20 ft on one side and a water depth of 8 ft on the other side. The gate is hinged at the 20 ft water level. Determine the magnitude and direction of the force required to keep the gate closed.

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100 8

90 4

80 6

70 3

60 5

50 6

40 1

30 5

20 1

ave

$$p_{1\text{ave}} = \rho g \frac{h}{2} = 1.938 \times 32.2 \times \frac{20}{2} = 632 \text{ lb/ft}^2$$

$$F_1 = p_{1\text{ave}} \times A_1 = 624 \times (20 \times 10) = 124,807 \text{ lb}$$

$$p_{2\text{ave}} = \rho g \frac{h}{2} = 1.938 \times 32.2 \times \frac{8}{2} = 249.6 \text{ lb/ft}^2$$

$$F_2 = p_{2\text{ave}} \times A_1 = 249.6 \times (8 \times 10) = 19,969 \text{ lb}$$

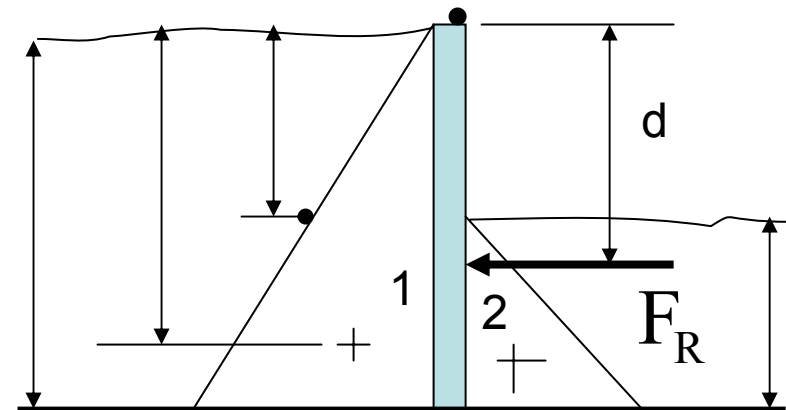
$$F_R = F_1 - F_2 = 124,807 \text{ lb} - 19,969 \text{ lb} = 104,838 \text{ lb}$$

$$\sum \text{Moments} = 0$$

$$M_{O1} - M_{O2} = F_R \times d$$

$$124,807 \times \frac{2}{3} \times 20 - 19,969 \text{ lb} \left(12 + \frac{2}{3} \times 8\right) = 104,838 \text{ lb} \times d$$

$$d = 12.57 \text{ ft below hinge point}$$



$$p = \rho g z$$

$$p = \frac{\text{slugs}}{\text{ft}^3} \frac{\text{lb}_f}{\text{slug}} \text{ft} = \frac{\text{lb}_f}{\text{ft}^2}$$

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$$p_{1\text{ave}} = \rho g \frac{h}{2}$$

$$p_{1\text{ave}} = 998.2 \times 9.807 \times \frac{6.096}{2} = 29,837.9 \text{ N/m}^2$$

$$F_1 = p_{1\text{ave}} \times A_1$$

$$F_1 = 29,837.9 \text{ N/m}^2 \times (6.096 \times 3.048)$$

$$F_1 = 554,406.9 \text{ N}$$

$$p_{2\text{ave}} = \rho g \frac{h}{2} = 998.2 \times 9.807 \times \frac{2.438}{2} = 11,932.2 \text{ N/m}^2$$

$$F_2 = p_{2\text{ave}} \times A_1 = 11,932.2 \text{ N/m}^2 \times (2.438 \times 3.048) = 88,676 \text{ N}$$

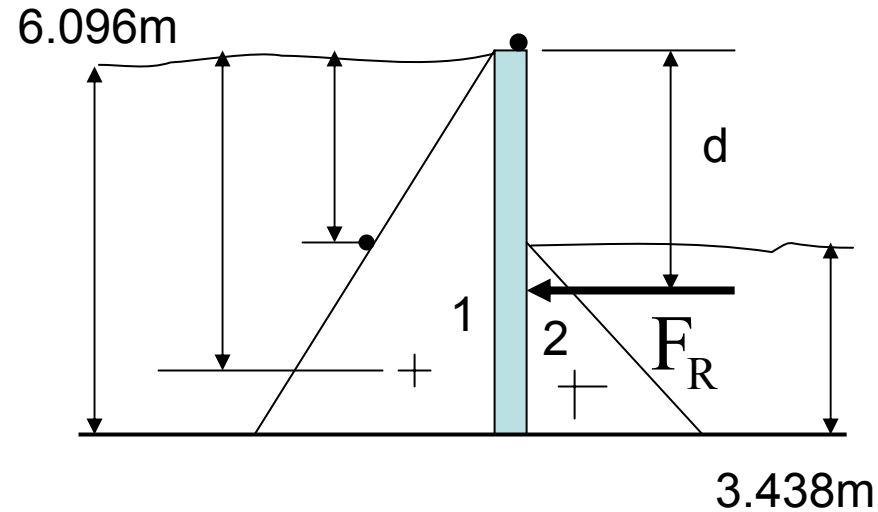
$$F_R = F_1 - F_2 = 554,406.9 \text{ N} - 88,676 \text{ N} = 465,730.9 \text{ N}$$

$$\sum \text{Moments} = 0$$

$$M_{O1} - M_{O2} = F_R \times d$$

$$554,406.9 \text{ N} \times \frac{2}{3} \times 6.096 - 88,676 \text{ N} \left(3.658 + \frac{2}{3} \times 2.438 \right) = 465,730.9 \text{ N} \times d$$

$$d = 3.855 \text{ m below hinge point}$$



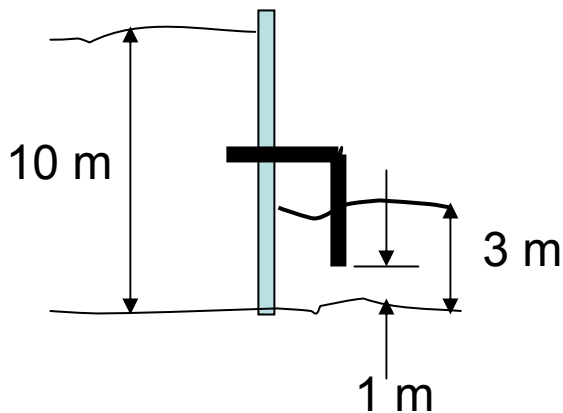
$$p = \rho g z$$

$$p = \frac{\text{kg}}{\text{m}^3} \frac{\text{m}}{\text{sec}^2} \text{m} = \frac{\text{kg}}{\text{m}^3} \frac{\text{m}}{\text{sec}^2} \text{m} \frac{\text{m}}{\text{m}}$$

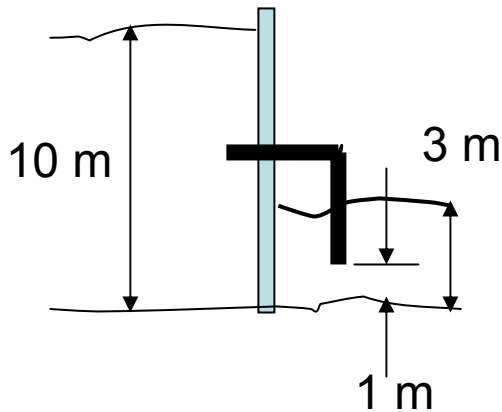
$$p = \frac{\text{kgm}}{\text{sec}^2} \frac{1}{\text{m}^2} = \text{N/m}^2$$

mae 335 Fluid Mechanics Spring 2007 Quiz 2

A fixed vertical dam in a channel has water at a depth of 10 meters on one side and 3 meters on the other side. A pipe with a 2m^2 flow area passes through the dam and discharges down. The pipe discharge is 1 m from the bottom of the channel. Ignoring viscosity what is the volume and mass flow of water through the pipe ?



A fixed vertical dam in a channel has water at a depth of 10 meters on one side and 3 meters on the other side. A pipe with a .2m² flow area passes through the dam and discharges down. The pipe discharge is 1 m from the bottom of the channel. Ignoring viscosity what is the volume and mass flow of water through the pipe ?



$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2$$

$$\frac{p_{atm}}{\rho} + 0 + 10g = \frac{p_{atm}}{\rho} + \frac{\rho g (3-1)}{\rho} + \frac{V_2^2}{2} + 1g$$

$$10g = 2g + \frac{V_2^2}{2} + 1g$$

$$\frac{V_2^2}{2} = 7g$$

$$V = \sqrt{2 \times 7 \times g} = 11.7 \text{ m/sec}$$

$$Q = V \times A = 11.7 \times .2 = 2.34 \text{ m}^3/\text{sec}$$

$$m = \rho \times V \times A = 2335.8 \text{ kg/sec}$$

Grade No

100 9

90

80 2

70 4

60 11

50 12

40 1

30

20 1

10 3

Ave 67

$$\frac{D\vec{V}}{Dt} = \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) \vec{i} + \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) \vec{j} + \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) \vec{k}$$

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} \quad \nabla \bullet \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \quad \nabla p = \frac{\partial p}{\partial x} \vec{i} + \frac{\partial p}{\partial y} \vec{j} + \frac{\partial p}{\partial z} \vec{k}$$

- 1 Reduce equation (6.8) in the Smits text to the differential continuity equation in Cartesian Coordinates for 1 D, unsteady, variable density, viscous flow.

Grade No.

100

90 9

80 9

70 11

ave 80

$$\nabla \bullet \vec{V} = -\frac{1}{\rho} \frac{D\rho}{Dt}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} \right)$$

$$\rho \frac{\partial u}{\partial x} = -\frac{\partial \rho}{\partial t} - u \frac{\partial \rho}{\partial x}$$

$$\rho \frac{\partial u}{\partial x} + u \frac{\partial \rho}{\partial x} = -\frac{\partial \rho}{\partial t}, \quad \text{or} \quad \frac{\partial(\rho u)}{\partial x} = -\frac{\partial \rho}{\partial t}$$

2. Reduce equation (6.18) in the Smits text to the differential momentum equation in Cartesian Coordinates for 2D, unsteady, viscous, incompressible flow ignoring body forces

$$\rho \frac{D\vec{V}}{Dt} = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \vec{V}$$

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad \text{x direction}$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \quad \text{y direction}$$

A potential flow is comprised of a flow with a stream function $\Psi = xy$ and a flow with a stream function $\Psi = -y$. What is the stream function for the resulting flow? What is the location in the flow of the stagnation point? What is the velocity at the point in the flow $x=3, y=2$? Define the locations on the streamlines where the pressure is a minimum? Sketch the streamlines.

$$\Psi_1 = xy, \quad \Psi_2 = -y$$

a) $\Psi = xy - y$

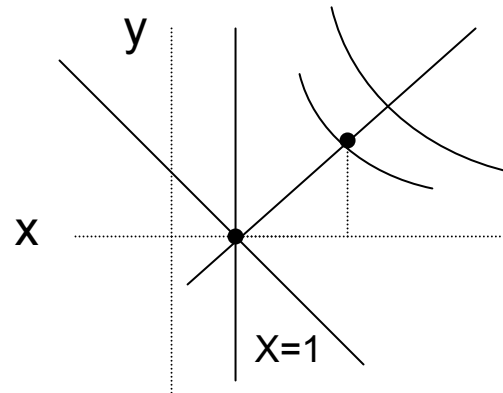
b) $u = \frac{\partial \Psi}{\partial y} = x - 1$

$$v = -\frac{\partial \Psi}{\partial x} = -y$$

stagnation point at $u = 0, v = 0$

$$x = 1, y = 0$$

$$V = (u^2 + v^2)^{1/2} = ((x - 1)^2 + (-y)^2)^{1/2}$$



c) at $x = 3, y = 2, \vec{V} = 2\vec{i} + 2\vec{j} \quad V = 2\sqrt{2}$

angle to x axis = $-\frac{\pi}{4}$

d) $V = (u^2 + v^2)^{1/2} = ((x - 1)^2 + (-y)^2)^{1/2}$

pressure is maximum where velocity is minimum

$$dV = 2(x - 1) - 2y = 0$$

$$y = x - 1$$

max far from origin along x and y axis of the flow.

Grade No

100 1

90 7

80 8

70 8

60 2

50 1

40 5

30 3

20 2

Ave 68

Water at 60 F flows through a square edged entrance from a tank into a 2 in ID hydraulically smooth copper pipe 200 ft long. The pipe run has two standard 90 degree elbows and discharges abruptly into the atmosphere. Calculate the head loss for this pipe run if the flow rate is 60 gpm, (8.ft³/min). The kinematic viscosity of water at 60 F is 1.2 x 10⁻⁵ ft²/sec.

90 5

80 3

70 10

60 4

50 1

<50 1

ave 73

$$D = \frac{2 \text{ in}}{12 \text{ in/ft}} = .1667$$

$$A = \frac{\pi D^2}{4} = \frac{\pi \times .1667^2}{4} = .0218 \text{ ft}^2$$

$$V = \frac{Q}{A} = \frac{8 \text{ ft}^3/\text{min}}{.0218 \text{ ft}^2 \times 60 \text{ sec/min}} = 6.11 \text{ ft/sec}$$

$$N_{RE} = \frac{VD}{\nu} = \frac{6.11 \text{ ft/sec} \times .1667 \text{ ft}}{1.2 \times 10^{-5} \text{ ft}^2/\text{sec}} = 84,279$$

Figure 9 - 7 @ (N_{RE} = 84,279) f = .019

Pipe Loss

$$h_1 = f \frac{L}{D} \frac{V^2}{2g} = .019 \frac{200}{.1667} \frac{6.11^2}{2 \times 32.2} = 13.21 \text{ ft}$$

Minor Losses – loss coefficients

Figure 10 – 22a, Inlet K = .5, Outlet K = 1

Figure 9.3, Elbow K = .019 x 30 = .57

$$h_1 = \sum K \frac{V^2}{2g} = (.5 + 2 \times .57 + 1) \frac{6.11^2}{2 \times 32.2} = 1.55 \text{ ft}$$

$$h_1 = 13.21 + 1.55 = 14.7 \text{ ft}$$

Minor Losses – equivalent length

Figure 9.3 Elbows L_{eq} / D = 30

$$h_1 = f \frac{L}{D} \frac{V^2}{2g} = .019 \times 30 \times \frac{6.11^2}{2 \times 32.2} = .66 \text{ ft}$$