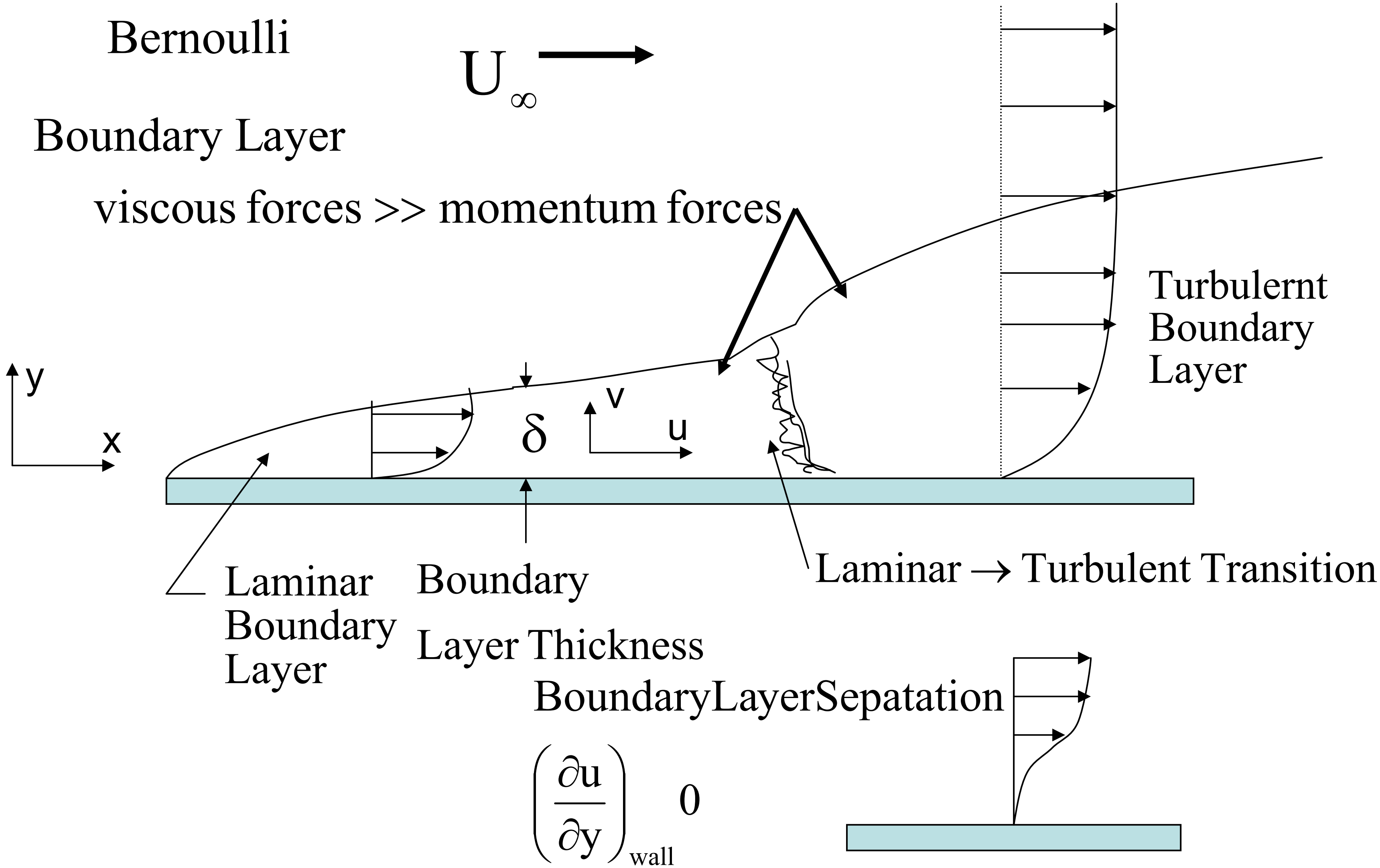
Free Stream

potential flow momentum forces >> friction forces

Bernoulli

Boundary Layer

viscous forces >> momentum forces



BOUNDARY LAYER EQUATIONS

steady, 2 Dimensional, viscous, constant density, constant body forces

CONTINUITY

$$\frac{\partial\left(\rho\,\vec{V}\right)}{\partial t} = -\nabla p - \nabla\left(\rho\,\vec{V}\right)\vec{V} - \nabla\tau + \rho\, f$$

$$\frac{\partial\rho}{\partial t} + \left(u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y} + w\frac{\partial\rho}{\partial z}\right) + \left(\rho\frac{\partial u}{\partial x} + \rho\frac{\partial v}{\partial y} + \rho\frac{\partial w}{\partial z}\right) = 0$$

$$\text{steady} \Rightarrow \frac{\partial}{\partial t} = 0, \quad \text{constant density} \Rightarrow \partial\rho = 0, \qquad 2D \Rightarrow w = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

MOMENTUM $$\frac{\partial\left(\rho \bullet \vec{V}\right)}{\partial t} = -\nabla p - \nabla\left(\rho \vec{V}\right)\vec{V} + \nabla\tau + \rho f$$

$$\frac{\partial}{\partial t}\rho u + \rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} + \rho w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \left(\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}\right) + \rho f_x$$

$$\frac{\partial}{\partial t}\rho v + \rho u\frac{\partial v}{\partial x} + \rho v\frac{\partial v}{\partial y} + \rho w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \left(\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy}\right) + \rho f_y$$

$$\frac{\partial}{\partial t}\rho w + \rho u\frac{\partial w}{\partial x} + \rho v\frac{\partial v}{\partial y} + \rho w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \left(\frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{xz} + \frac{\partial}{\partial z}\tau_{zz}\right) + \rho f_z$$

steady $\Rightarrow \frac{\partial}{\partial t} = 0$, constant density $\Rightarrow \partial\rho = 0$, 2D $\Rightarrow w = 0$, $\frac{\partial}{\partial z} = 0$, ignoring body force chnges ρf_x and ρf_y,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\left(\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{1}{\rho}\left(\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy}\right)$$

EXACT 2D MOMENTUM BOUNDARY LAYER EQUATIONS

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

NonleinarizingParameters

$$x'=\frac{x}{L}, x=x'L, y'=\frac{y}{L}, y=y'L$$

$$u'=\frac{u}{U_\infty}, u=u'U_\infty, \frac{\partial u}{\partial x}=U_\infty L\frac{\partial u'}{\partial x}$$

$$v'=\frac{v}{U_\infty}, n=v'U_\infty, \frac{\partial v}{\partial y}=U_\infty L\frac{\partial v'}{\partial y}$$

$$p'=\frac{p}{\rho U_\infty}, p=\rho U_\infty p'$$

substituting,

$$\frac{U_\infty{}^2}{L}u'\frac{\partial u'}{\partial x'}+\frac{U_\infty{}^2}{L}v'\frac{\partial u'}{\partial y'}=-\frac{\rho U_\infty{}^2}{\rho L}\frac{\partial p'}{\partial x'}+\frac{\mu}{\rho}\frac{U_\infty{}^2}{L}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{U_\infty{}^2}{L}u'\frac{\partial v'}{\partial x'}+\frac{U_\infty{}^2}{L}v'\frac{\partial v'}{\partial y'}=-\frac{\rho U_\infty{}^2}{\rho L}\frac{\partial p'}{\partial y'}+\frac{\mu}{\rho}\left(\frac{\partial^2 v'}{\partial x'^2}+\frac{\partial^2 v'}{\partial y'^2}\right)$$

dividing by $L/U_\infty{}^2$

$$u'\frac{\partial u'}{\partial x'}+v'\frac{\partial u'}{\partial y'}=-\frac{\partial p'}{\partial x'}+\frac{\mu}{\rho}\frac{}{LU_\infty}\left(\frac{\partial^2 u'}{\partial x^2}+\frac{\partial^2 u'}{\partial y^2}\right)$$

$$u'\frac{\partial v'}{\partial x'}+v'\frac{\partial v'}{\partial y'}=-\frac{\partial p'}{\partial y'}+\frac{1}{N_{RE}}\left(\frac{\partial^2 v'}{\partial x'^2}+\frac{\partial^2 v'}{\partial y'^2}\right)$$

Order of Magnitude Analysis

for δ, boundary layerthickness, $<< L$

$$u'=\frac{u}{U_\infty}0\rightarrow 1, v'=\frac{v}{U_\infty}0\rightarrow\delta$$

$$x'=\frac{x}{L}0\rightarrow 1, y'=\frac{y}{L}0\rightarrow\delta$$

Continuity

$$\frac{U_\infty}{L}\frac{\partial u'}{\partial x'}+\frac{U_\infty}{L}\frac{\partial v'}{\partial y'}=0$$

$1+\delta/\delta$

Momentum

$$u'\frac{\partial u'}{\partial x'}+v'\frac{\partial u'}{\partial y'}=-\frac{\partial p'}{\partial x'}+\frac{1}{N_{RE}}\left(\cancel{\frac{\partial^2 u'}{\partial x'^2}}+\frac{\partial^2 u'}{\partial y'^2}\right)$$

$1 \quad 1 \quad \delta \quad 1/\delta \qquad \delta^2 \quad (1+1/\delta^2)$ order of magnitude

$$u'\cancel{\frac{\partial v'}{\partial x'}}+v'\cancel{\frac{\partial v'}{\partial y'}}=-\frac{\partial p'}{\partial y'}+\frac{1}{N_{RE}}\left(\cancel{\frac{\partial^2 v'}{\partial x'^2}}+\cancel{\frac{\partial^2 v'}{\partial y'^2}}\right)$$

$1 \quad \delta \quad \delta \quad 1 \qquad \delta^2 \quad (\delta+1/\delta^2)$ order of magnitude

eliminating δ order terms and
switching back to dimensional varaiables.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\left(+\frac{\partial^2 u}{\partial y^2}\right)$$

at $y=0, u=0, v=0$ at $y=\delta, \quad u=U_\infty$

BLASIUS
PLAT PLATE SOLUTION
coordiate transformation
and numerical solutiuon,

$$\frac{u}{U_\infty}=f\left(y\sqrt{\frac{U_\infty\rho}{\mu x}}\right)$$

Table 10.1, Figure 10.2
curve fit,

$$\frac{u}{U_\infty}=2\left(\frac{y}{\delta}\right)-\left(\frac{y}{\delta}\right)^2$$

FLOW OVER BLUFF OBJECTS

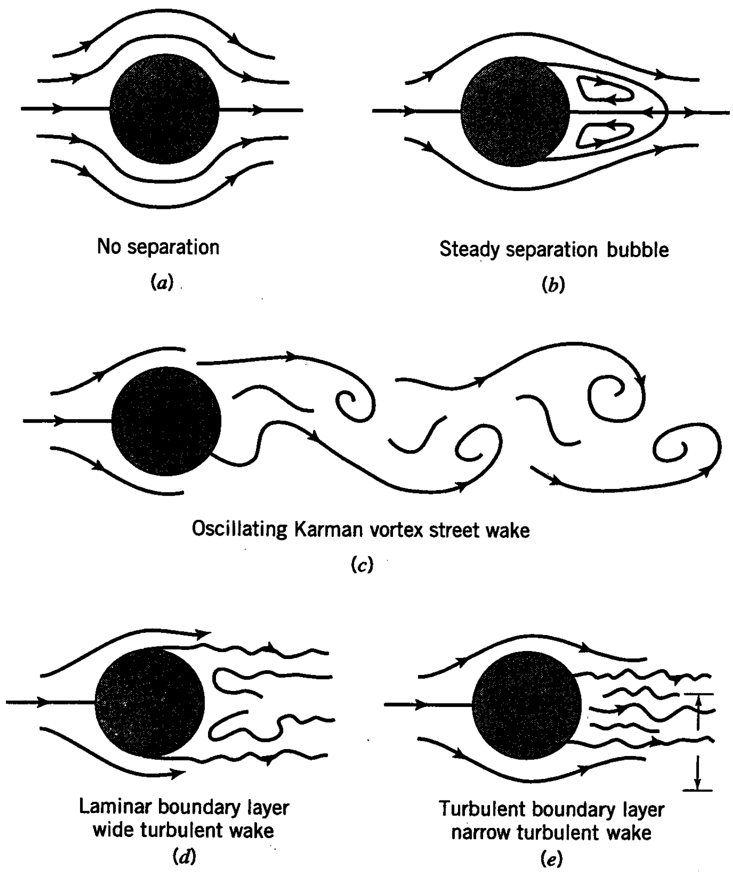


FIGURE 10.13 Flow patterns for flow over a cylinder. (*a*) Reynolds number = 0.2; (*b*) 12; (*c*) 120; (*d*) 30,000; and (*e*) 500,000. Patterns correspond to the points marked on Figure 10.12. From Munson, Young, & Okiishi, *Fundamentals of Fluid Mechanics,* John Wiley & Sons, 1998.

VORTEX SHEDDING

$$\text{FREQUENCY} = f(V, D, \rho, \mu)$$

$$\frac{fD}{V} = F\left(\frac{\rho V D}{\mu}\right)$$

$$\frac{fD}{V} = \text{STROUHAL NUMBER}$$

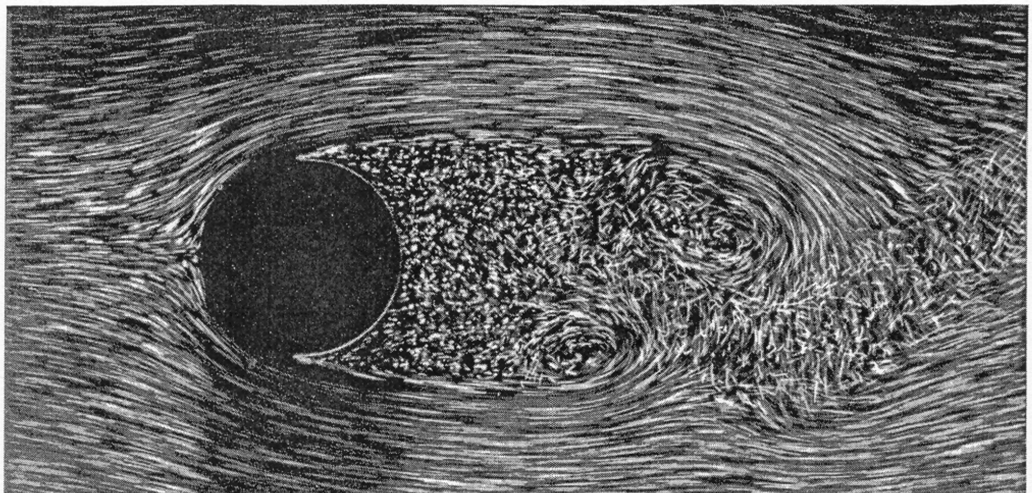


FIGURE 10.8 Flow over a single cylinder at a Reynolds number of 2000, visualized using small bubbles in water. ONERA photograph, Werlé & Gallon, 1972 *Aéronaut. Astronaut.*, **34,** 31–33.

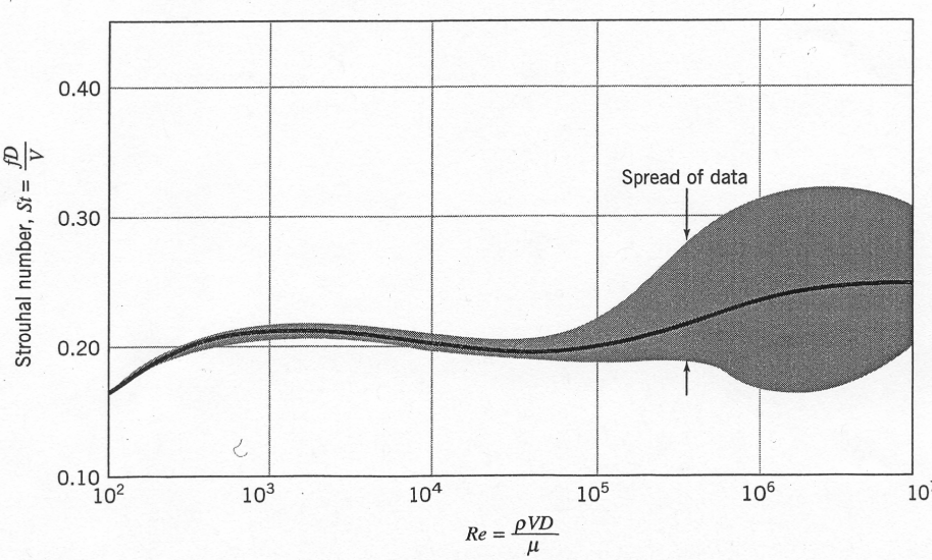


FIGURE 10.9 Dimensionless shedding frequency from a circular cylinder (Strouhal number) as a function of Reynolds number. Adapted from A. Roshko, *Turbulent Wakes from Vortex Streets,* NACA Rept. 1191, 1954.

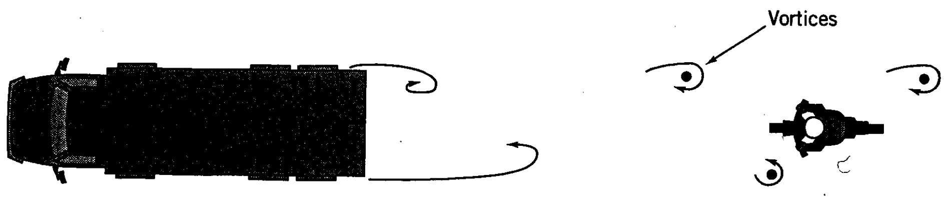


FIGURE 10.11 Periodic vortex formation in the wake of a large truck. Adapted from *Race Car Aerodynamics,* J. Katz, Robert Bentley Publishers, 1995, with permission.

EXAMPLE 10.2 *Vortex Shedding*

In an exposed location, telephone wires will "sing" when the wind blows across them. Find the frequency of the note when the wind velocity is 30 *mph,* and the wire diameter is 0.25 *in.* For air, we assume that $\nu = 15 \times 10^{-6} m^2/s$.

Solution: First we need to know the Reynolds number, *Re,* where

$$Re = \frac{VD}{\nu}$$

$$= \frac{\left(30\frac{mi}{hr} \times 5280\frac{ft}{mi} \times 12\frac{in.}{ft} \times 0.0254\frac{m}{in.} \times \frac{1}{3600}\frac{hr}{s}\right)\left(0.25\,in. \times 0.0254\frac{m}{in.}\right)}{15 \times 10^{-6}\frac{m^2}{s}}$$

$$= 2774$$

From Figure 10.9, we see that the Strouhal number is approximately equal to 0.21. That is,

$$St = \frac{fD}{V} = 0.21$$

so that

$$f = \frac{0.21V}{D} Hz$$

$$= \frac{\left(0.21 \times 30\frac{mi}{hr} \times 5280\frac{ft}{mi} \times 12\frac{in.}{ft} \times 0.0254\frac{m}{in.} \times \frac{1}{3600}\frac{hr}{s}\right)}{0.25\,in. \times 0.0254\frac{m}{in.}} Hz$$

$$= 444\,Hz$$

which is very close to the note middle C (= 440 *Hz*). ■

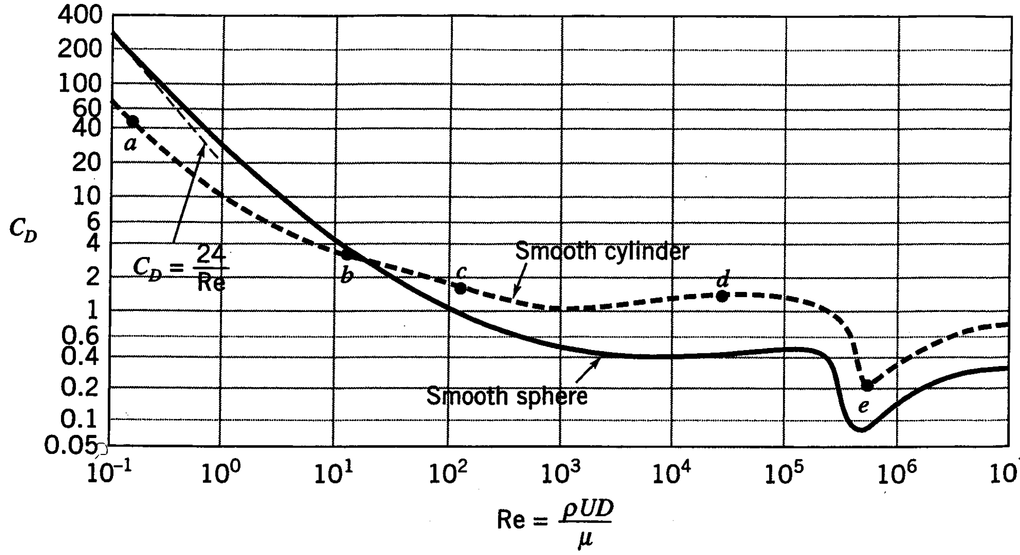


FIGURE 10.12 Drag coefficient as a function of Reynolds number for smooth circular cylinders and smooth spheres. From Munson, Young, & Okiishi, *Fundamentals of Fluid Mechanics,* John Wiley & Sons, 1998.

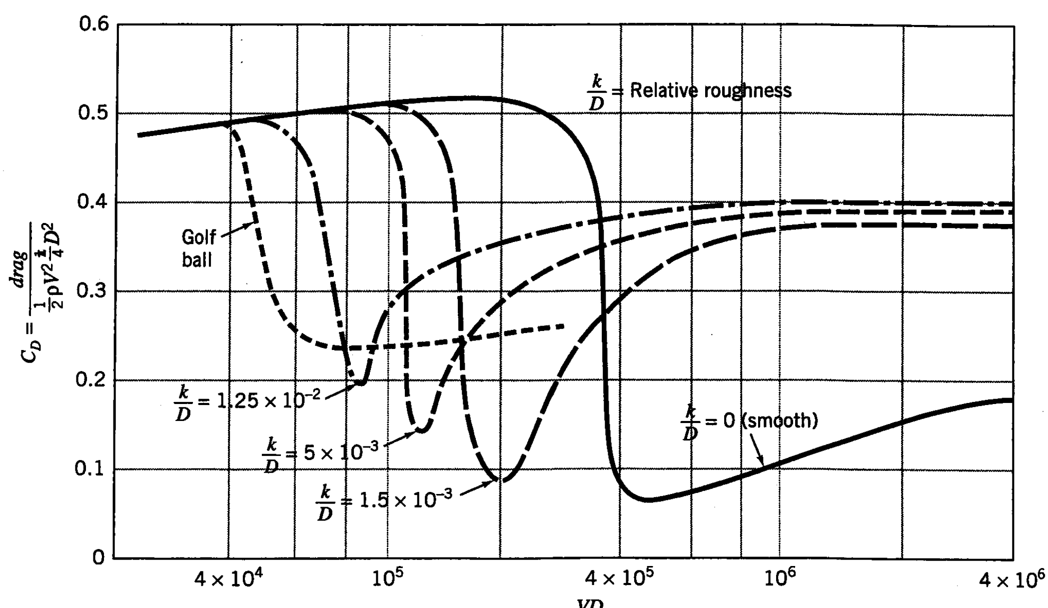


FIGURE 10.16 Drag coefficient as a function of Reynolds number for spheres with different degrees of roughness. *k* is the equivalent roughness height, and *D* is the sphere diameter. From Munson, Young, & Okiishi, *Fundamentals of Fluid Mechanics,* John Wiley & Sons, 1998.

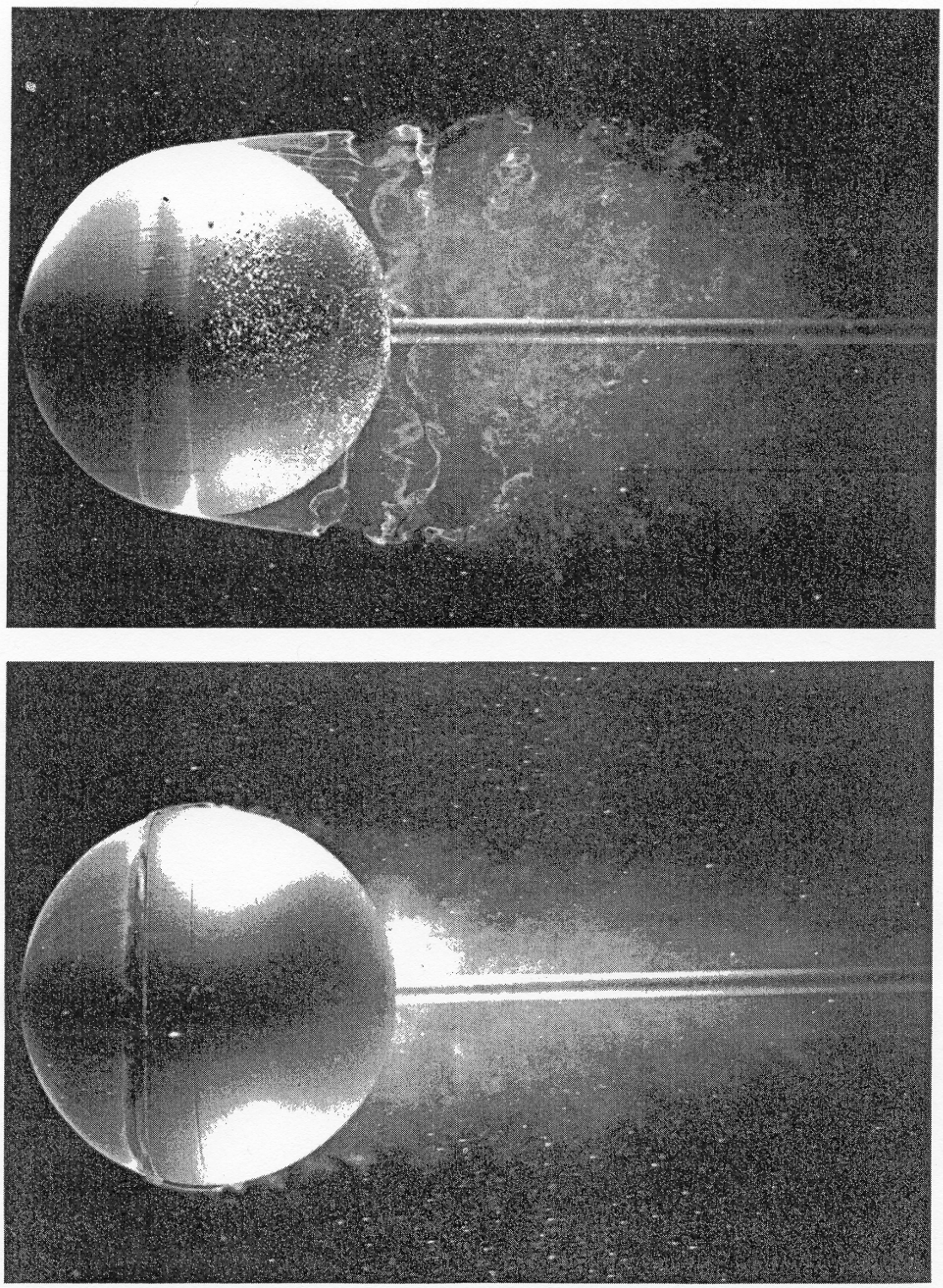


FIGURE 10.15 Flow over a sphere. (*a*) Reynolds number = 15,000 (laminar separation). (*b*) Reynolds number = 30,000, with trip wire (turbulent separation). From Van Dyke, *Album of Fluid Motion,* Parabolic Press, 1982. Original photographs by Werlé, ONERA, 1980.

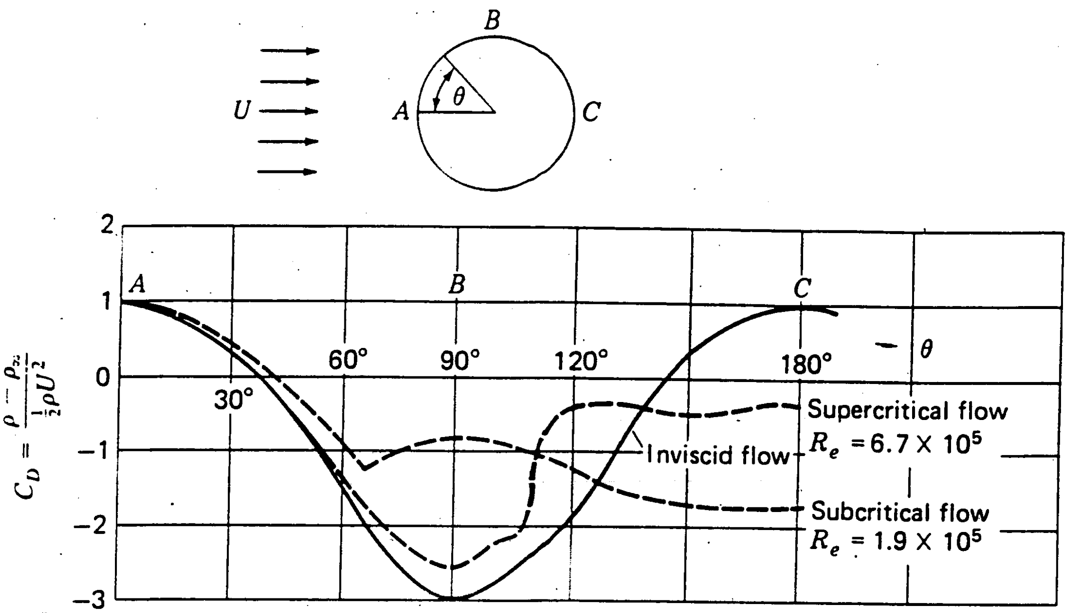
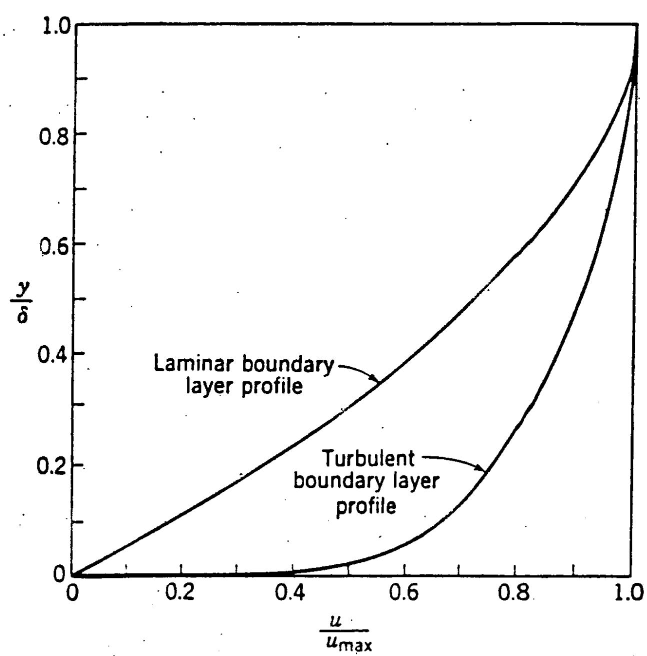


FIGURE 13.26
Pressure distributions around a cylinder for subcritical, supercritical, and completely inviscid flows.

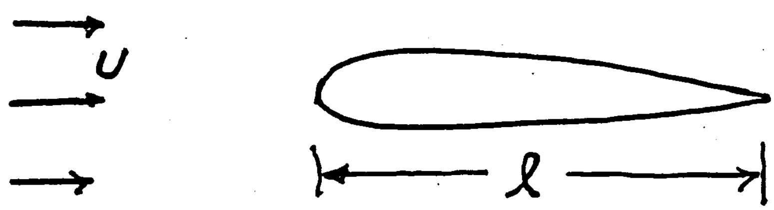


TURBULENT BOUNDARY LAYER:

HIGHER SHEAR STRESS – BUT SEPARATES LATER.

DRAG

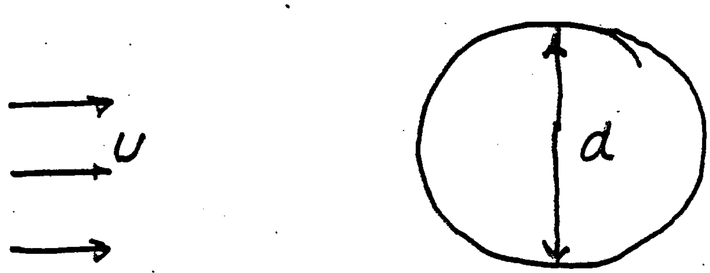
STREAMLINED OBJECTS



$$C_D = \frac{D}{\underbrace{\tfrac{1}{2}\rho U^2 b\ell}}$$

PLAN AREA (bℓ)

BLUNT OBJECTS



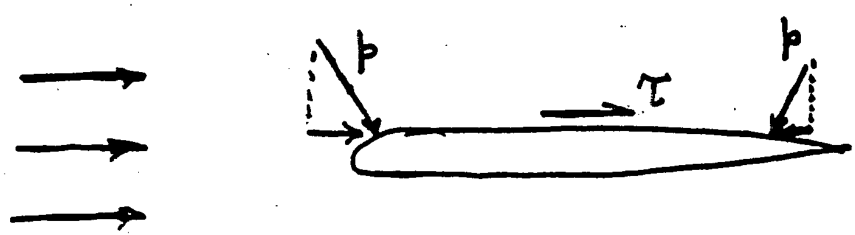
$$C_D = \frac{D}{\underbrace{\tfrac{1}{2}\rho U^2 bd}}$$

CROSS-SECTIONAL AREA ⊥ TO FLOW (bd)

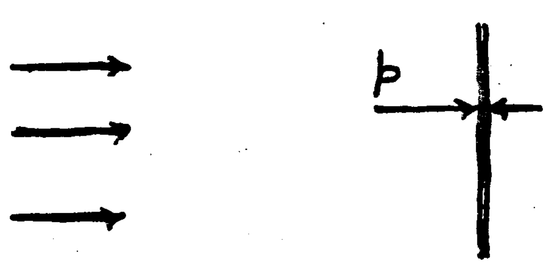
DRAG

ALL FRICTION DRAG

FRICTION + PRESSURE DRAG



ALL PRESSURE DRAG



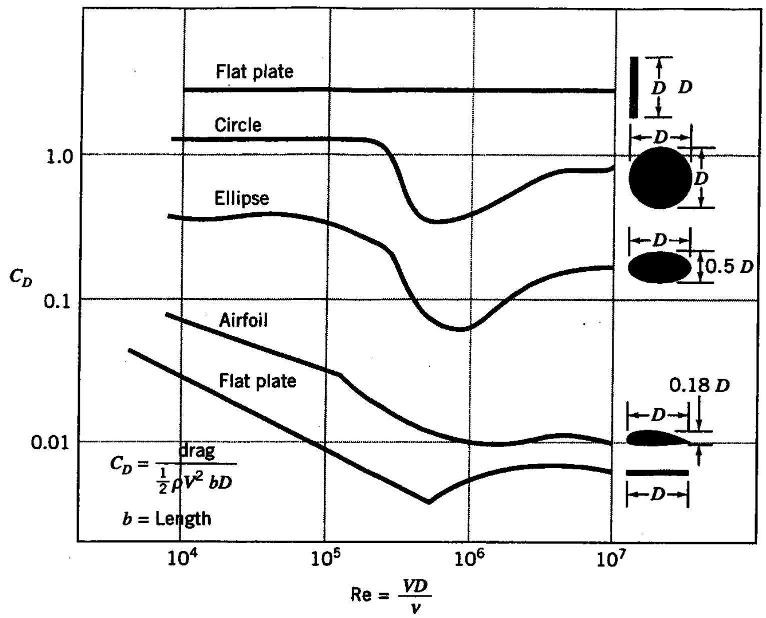
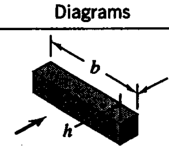
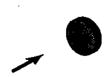
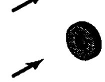
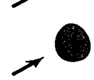
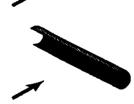
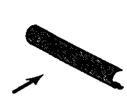


FIGURE 10.14 Drag coefficients of bluff and streamlined bodies. From Munson, Young, & Okiishi, *Fundamentals of Fluid Mechanics,* John Wiley & Sons, 1998.

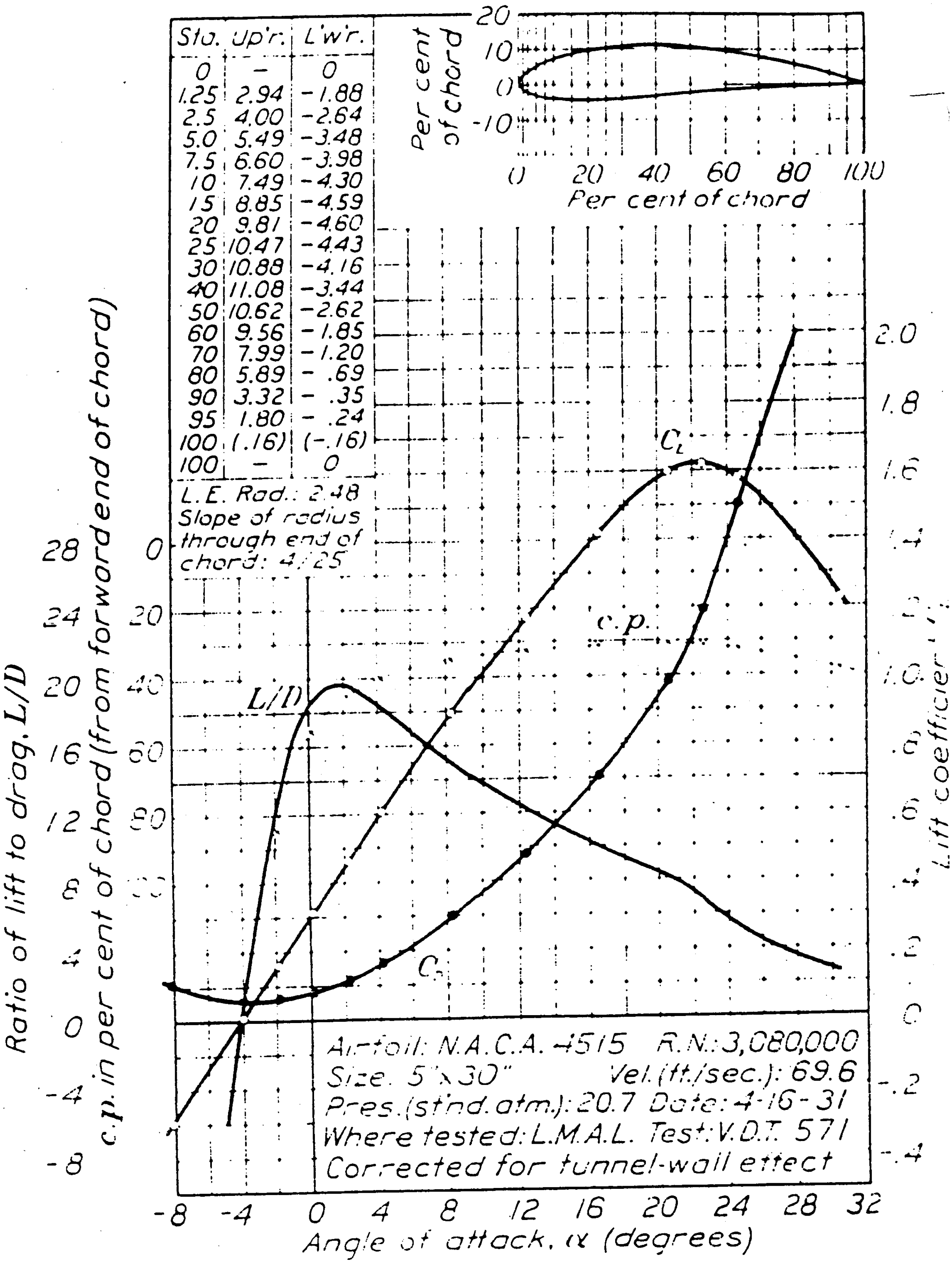
TABLE 10.2 Drag Coefficient Data for Sharp-Edged Bodies[a]

Object	Diagrams		C_D ($Re \gtrsim 10^{-1}$)
Square cylinder		$b/h = \infty$ $b/h = 1$	2.05 1.05
Disk			1.17
Ring			1.20[b]
Hemisphere (open end facing flow)			1.42
Hemisphere (open end facing downstream)			0.38
C-section (open side facing flow)			2.30
C-section (open side facing downstream)			1.20

From Fox & McDonald, *Introduction to Fluid Mechanics,* 4th edition, John Wiley & Sons, 1992.

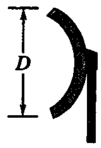
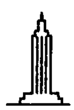
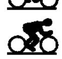
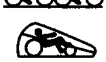
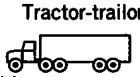
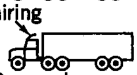
[a] Original data from Hoerner, Fluid-Dynamic Drag, 2nd edition, Midland Park, NJ, published by the author.

[b] Based on area of ring.



Sta.	Up'r.	L'w'r.
0	–	0
1.25	2.94	-1.88
2.5	4.00	-2.64
5.0	5.49	-3.48
7.5	6.60	-3.98
10	7.49	-4.30
15	8.85	-4.59
20	9.81	-4.60
25	10.47	-4.43
30	10.88	-4.16
40	11.08	-3.44
50	10.62	-2.62
60	9.56	-1.85
70	7.99	-1.20
80	5.89	-.69
90	3.32	-.35
95	1.80	-.24
100	(.16)	(-.16)
100	–	0

TABLE 10.3 Drag Coefficient Data for Selected Objects

Shape	Reference area	Drag coefficient C_D
 Parachute	Frontal area $A = \frac{\pi}{4}D^2$	1.4
 Porous parabolic dish	Frontal area $A = \frac{\pi}{4}D^2$	Porosity: 0 / 0.2 / 0.5 → 1.42 / 1.20 / 0.82 ← 0.95 / 0.90 / 0.80 Porosity = open area/total area
 Average person	Standing Sitting Crouching	$C_DA = 9\ ft^2$ $C_DA = 6\ ft^2$ $C_DA = 2.5\ ft^2$
 Fluttering flag	$A = lD$	l/D : C_D 1 : 0.07 2 : 0.12 3 : 0.15
 Empire State Building	Frontal area	1.4
 Six-car passenger train	Frontal area	1.8
Bikes		
 Upright commuter	$A = 5.5\ ft^2$	1.1
 Racing	$A = 3.9\ ft^2$	0.88
 Drafting	$A = 3.9\ ft^2$	0.50
 Streamlined	$A = 5.0\ ft^2$	0.12
Tractor-trailor trucks		
 Standard	Frontal area	0.96
Fairing  With fairing	Frontal area	0.76
Gap seal  With fairing and gap seal	Frontal area	0.70
 Tree $U = 10$ m/s $U = 20$ m/s $U = 30$ m/s	Frontal area	0.43 0.26 0.20
 Dolphin	Wetted area	0.0036 at Re = 6×10^6 (flat plate has $C_{Df} = 0.0031$)
 Large birds	Frontal area	0.40

SOURCE: From Munson, Young & Okiishi, *Fundamentals of Fluid Mechanics*, John Wiley & Sons, 1998.

AUTOMOBILE AERODYNAMICS

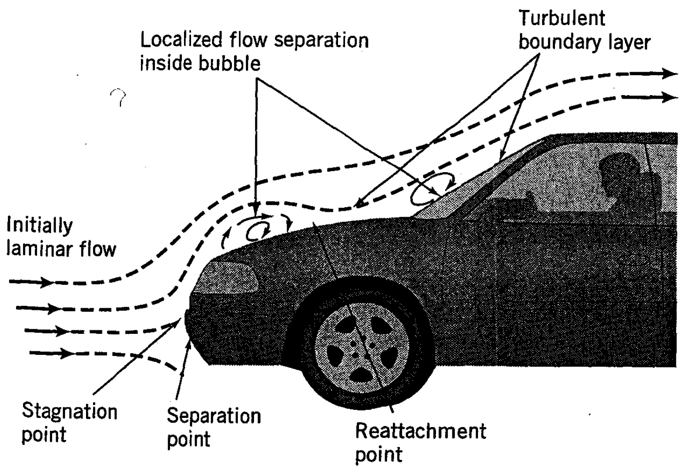


FIGURE 10.7 Sketch of the flow over the front of a car, showing points of separation and reattachment. From *Race Car Aerodynamics,* J. Katz, Robert Bentley Publishers, 1995, with permission.

TABLE 10.4 Typical Lift and Drag Coefficients

			C_L	C_D
1	Circular plate		0	1.17
2	Circular cylinder $L/D < 1$		0	1.15
3	Circular cylinder $L/D > 2$		0	0.82
4	Low drag body of revolution		0	0.04
5	Low drag vehicle near the ground		0.18	0.15
6	Generic automobile		0.32	0.43
7	Prototype race car		–3.00	0.75

SOURCE: From *Race Car Aerodynamics,* J. Katz, Robert Bentley Publishers, 1995, with permission.

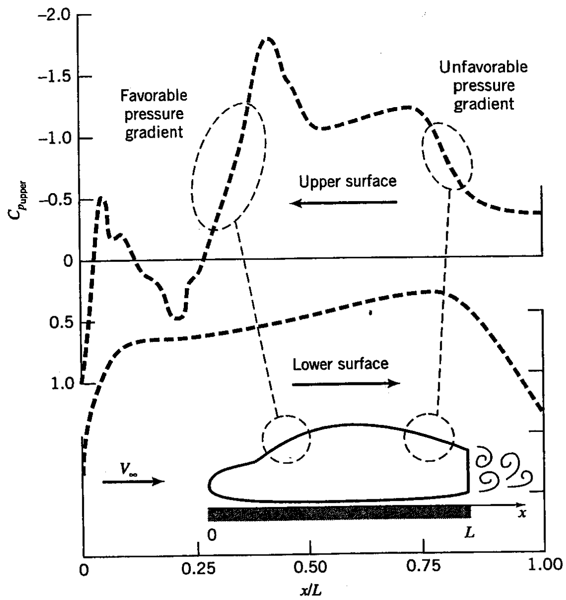


FIGURE 10.17 Distribution of measured pressure coefficients over a two-dimensional automobile shape. From *Race Car Aerodynamics,* J. Katz, Robert Bentley Publishers, 1995, with permission.

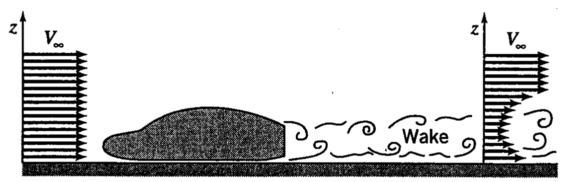


FIGURE 10.10 Wake flow behind a road vehicle (with flow separation and vortex shedding in the base area). Adapted from *Race Car Aerodynamics,* J. Katz, Robert Bentley Publishers, 1995, with permission.

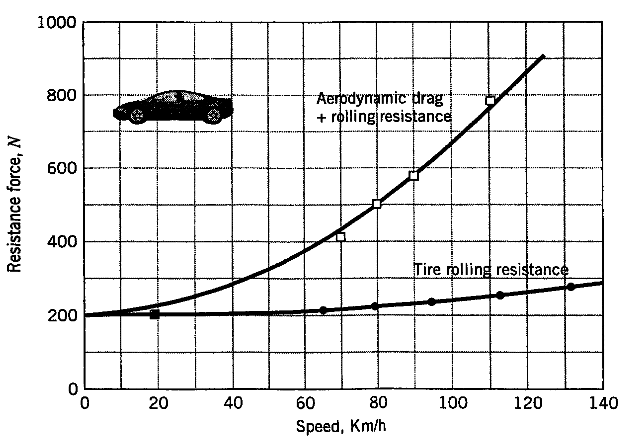


FIGURE 10.18 Aerodynamic and rolling resistances for an average sedan car. From *Race Car Aerodynamics,* J. Katz, Robert Bentley Publishers, 1995, with permission.

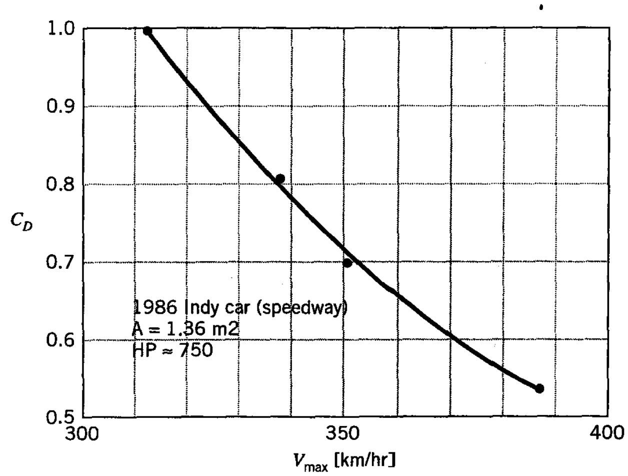


FIGURE 10.19 Effect of the drag coefficient on the maximum speed of an Indy-class speedway car. From *Race Car Aerodynamics,* J. Katz, Robert Bentley Publishers, 1995, with permission.

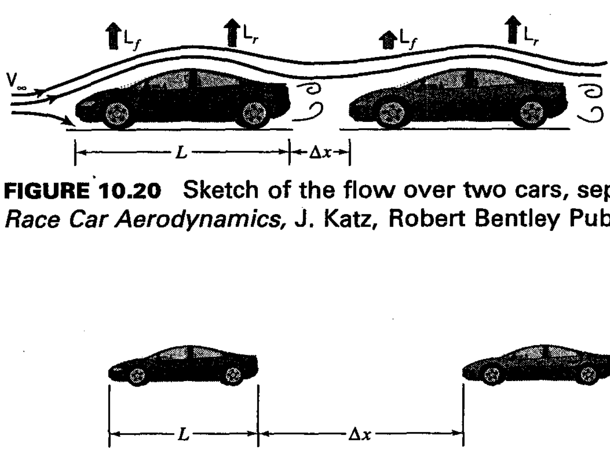


FIGURE 10.20 Sketch of the flow over two cars, separated by a distance Δx. From *Race Car Aerodynamics,* J. Katz, Robert Bentley Publishers, 1995, with permission.

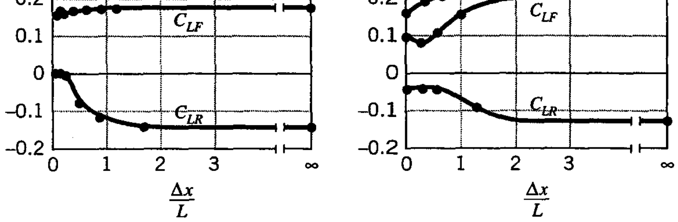


FIGURE 10.21 Lift and drag coefficients for two cars separated by a distance Δx (for notation, See Figure 10.20). From *Race Car Aerodynamics,* J. Katz, Robert Bentley Publishers, 1995, with permission.

CAR $U = 60$ mph $C_D = 0.5$

$$D = \frac{1}{2} \rho U^2 A \; C_D$$

$$\frac{1}{2}\rho U^2 = \frac{1}{2}(.00238)(88)^2 = 9.22 \text{ lb/ft}^2$$

$$A = 7 \times 4.5 = 31.5 \text{ ft}^2$$

$$D = 9.22(31.5)(.5) = 145 \text{ lbs}$$

$$P = D \cdot U = 145(88) = 12760 \frac{\text{ft-lb}}{\text{sec}}$$
$$= 23.2 \text{ HP}$$

$$P = 12760 \frac{\text{ft-lb}}{\text{sec}} \frac{\text{Btu}}{778 \text{ ft lb}} = 16.4 \frac{\text{Btu}}{\text{sec}}$$

Heat of Combustion ~ 18000 Btu / lbm

$$16.4 \frac{\text{Btu}}{\text{sec}} \frac{\text{lbm}}{18000 \text{ Btu}} \frac{\text{ft}^3}{50 \text{ lb}} \frac{7.5 \text{ gal}}{\text{ft}^3} \frac{3600 \text{ sec}}{\text{hr}}$$

$= 0.49 \frac{\text{gal}}{\text{hr}}$ = FUEL CONSUMPTION FOR DRAG

TOTAL FUEL CONSUMPTION ≅ 30 Mi/GAL

$$\frac{\text{GAL}}{30 \text{ mi}} \frac{60 \text{ mi}}{\text{hr}} = 2.0 \text{ Gal/hr}$$

SOURCES OF DRAG

FRICTION

PRESSURE

WAVE DRAG

- LIQUID SURFACE
- COMPRESSIBLE

DRAG DUE TO LIFT

(THREE DIMENSIONAL
FLOW LOSSES ON WINGS)