POISEUILLE FLOW - flow between parallel infinite plates
1 Dimensional, steady, viscous, laminar, fully developed
Exact Solution Possible

\[ F_{\text{viscous}} + F_{\text{pressure}} = 0 \]

\[ F_{\text{viscous}} = \left( \tau_{\text{wall}} + \frac{\partial \tau_{\text{wall}}}{\partial y} \frac{dy}{2} \right) dx \, dz - \left( \tau_{\text{wall}} - \frac{\partial \tau_{\text{wall}}}{\partial y} \frac{dy}{2} \right) dx \, dz \]

\[ F_{\text{viscous}} = \frac{\partial \tau_{\text{wall}}}{\partial y} dx \, dy \, dz \]

since \( \tau_{xx} = \tau_{xzx} = 0 \)

\[ \frac{\partial \tau_{xy}}{\partial y} = \frac{d \tau_{xy}}{dy} \]

and since \( \tau_{xy} = \mu \left( \frac{du}{dy} \right) \)

\[ F_{\text{viscous}} = \frac{d}{dy} \left( \mu \left( \frac{du}{dy} \right) \right) dx \, dy \, dz \]

\[ F_{\text{viscous}} = \mu \frac{d^2u}{dy^2} dx \, dy \, dz \]
POISEUILLE FLOW
flow between parallel infinite plates
1-dimensional, steady, viscous, laminar, fully developed
POISEUILLE FLOW
1 Dimensional,
steady,
viscous,
laminar,
fully developed
Exact Solution Possible

\[ \rho \frac{DV}{Dt} = -\nabla p + \rho g + \mu \nabla^2 \vec{V} \]

\[ \frac{\partial p}{\partial x} = \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \]

\[ \frac{dp}{dx} = \frac{\mu}{\rho} \frac{d^2 u}{dy^2} \]

the momentum equations is satisfied only if,

\[ \frac{dp}{dx} = -K \quad \text{and} \quad \frac{\mu}{\rho} \frac{d^2 u}{dy^2} = -K \]

Boundary Conditions

\[ u = 0 \quad \text{at} \quad y = \pm \frac{D}{2} \]

\[ \frac{du}{dy} = 0 \quad \text{at} \quad y = 0 \]
VELOCITY DISTRIBUTION - parabolic

\[ \mu \frac{d^2 u}{dy^2} = -K \]

\[ \frac{d^2 u}{dy^2} = -\frac{K}{\mu} \]

\[ \frac{du}{dy} = -\frac{K}{\mu} y + C_1, \]

at \( t = 0, \frac{du}{dy} = 0 \)

\[ \frac{du}{dy} = -\frac{K}{\mu} 0 + C_1 = 0 \Rightarrow C_1 = 0 \]

\[ \frac{du}{dy} = -\frac{K}{\mu} y \]

\[ u = -\frac{K}{2\mu} y^2 + C_2 \]

\[ a t y = \pm \frac{D}{2}, u = 0 \]

\[ 0 = -\frac{K}{2\mu} \left( \frac{D}{2} \right)^2 + C_2 \Rightarrow C_2 = \frac{K}{2\mu} \left( \frac{D}{2} \right)^2 \]

\[ u = -\frac{K}{2\mu} y^2 + \frac{K}{2\mu} \left( \frac{D}{2} \right)^2 \]

\[ u = \frac{K}{2\mu} \left( \frac{D}{2} \right)^2 \left( 1 - \left( \frac{y}{D} \right)^2 \right) \]

\[ u = \frac{KD^2}{8\mu} \left( 1 - \left( \frac{y}{D} \right)^2 \right) \]

\[ u = \frac{KD^2}{8\mu} \left( 1 - \left( \frac{y}{D} \right)^2 \right) \]

\[ u = \frac{D^2}{8\mu} \left( -\frac{dp}{dx} \right) \left( 1 - \left( \frac{y}{D} \right)^2 \right) \]
MEAN FLOW VELOCITY, $V$

$$V = \frac{1}{w} \int_{-D/2}^{D/2} w \, u \, dy = \frac{1}{D} \int_{-D/2}^{D/2} \frac{D^2}{8 \mu} \left( - \frac{dp}{dx} \right) \left( 1 - \left( \frac{y}{D} \right)^2 \right) \, dy$$

$$V = \frac{1}{D} \frac{D^2}{8 \mu} \left( - \frac{dp}{dx} \right) \int_{-D/2}^{D/2} \left( 1 - \left( \frac{y}{D} \right)^2 \right) \, dy$$

$$V = \frac{1}{D} \frac{D^2}{8 \mu} \left( - \frac{dp}{dx} \right) \left[ y - \left( \frac{2}{D} \right) y^3 \right]_{-D/2}^{D/2}$$

$$V = \frac{1}{D} \frac{D^2}{8 \mu} \left( - \frac{dp}{dx} \right) \left[ \frac{D}{2} - \frac{4 \, D^3}{3 \times 8} \right] - \left[ -\frac{D}{2} + \frac{4 \, D^3}{3 \times 8} \right]$$

$$V = \frac{D^2}{12 \mu} \left( - \frac{dp}{dx} \right)$$

VELOCITY DISTRIBUTION

$$u \, \frac{V}{V} = \frac{D^2}{8 \mu} \left( - \frac{dp}{dx} \right) \left( 1 - \left( \frac{y}{D} \right)^2 \right) = \frac{3}{2} \left( 1 - \left( \frac{y}{D} \right)^2 \right)$$

FRICTION FACTOR

$$h_1 = f \frac{L \, V^2}{D \, \rho \, 2}$$

$$f = - \frac{dp}{dx} \frac{2D}{\rho V^2}$$

since $V = \frac{D^2}{12\mu} \left( - \frac{dp}{dx} \right)$

$$- \frac{dp}{dx} = \frac{12V \mu}{D^2}$$

$$f = \frac{12V \mu}{D^2} \frac{2D}{\rho V^2}$$

$$f = \frac{24 \mu}{\rho V D}$$

$$f = \frac{24}{N_{RE}}$$

LAMINAR FLOW
LAMINAR PIPE FLOW
steady, 1D, viscous, constant density
Exact Solution Possible

\[ \rho \frac{D \vec{V}}{Dt} = -\nabla p + \rho g + \mu \nabla^2 \vec{V} \]

z direction momentum equation

\[ \frac{\partial p}{\partial z} = \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \]

\( \nabla^2 \vec{V} \) in cylindrical coordinates p 478

\[ \frac{\partial p}{\partial z} = \mu \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial w}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} + \frac{\partial^2 w}{\partial z^2} \right) \]

\[ \frac{dp}{dz} = \frac{1}{r} \frac{d}{dr} \left( r \mu \frac{dw}{dr} \right) \]

\[ \frac{dp}{dz} = \frac{1}{r} \frac{d(r\tau)}{dr} \]
VELOCITY DISTRIBUTION - parabolic

\[ \frac{1}{r} \frac{d}{dr} (r \tau) = -K \]
\[ d(r \tau) = -Krdr \]
\[ r \tau = -K \frac{r^2}{2} \]
\[ \tau = \frac{d^2 u}{dy^2} = -\frac{K}{\mu} \]
\[ \frac{du}{dy} = -\frac{K}{\mu} y + C_1, \]
\[ \text{at } y = 0, \frac{du}{dy} = 0 \]
\[ \frac{du}{dy} = -\frac{K}{\mu} 0 + C_1 = 0 \Rightarrow C_1 = 0 \]
\[ \frac{du}{dy} = -\frac{K}{\mu} y \]
\[ u = -\frac{K}{2\mu} y^2 + C_2 \]
\[ \text{at } y = \pm \frac{D}{2}, u = 0 \]
\[ 0 = -\frac{K}{2\mu} \left( \frac{D}{2} \right)^2 + C_2 \Rightarrow C_2 = \frac{K}{2\mu} \left( \frac{D}{2} \right)^2 \]
\[ u = -\frac{K}{2\mu} y^2 + \frac{K}{2\mu} \left( \frac{D}{2} \right)^2 \]
\[ u = \frac{K}{2\mu} \left( \frac{D}{2} \right)^2 \left[ 1 - \left( \frac{2y}{D} \right)^2 \right] \]
\[ u = \frac{KD^2}{8\mu} \left[ 1 - \left( \frac{2y}{D} \right)^2 \right] \]
\[ u = \frac{D^2}{8\mu} \left( -\frac{dp}{dx} \right) \left[ 1 - \left( \frac{2y}{D} \right)^2 \right] \]
Balance Unit Exponents
for M, \( 1 = a + b \)
for L, \(-1 = -3a - b + c + d + e + f \)
for T, \(-2 = -b - c \)

from M \( a = 1 - b \)
from T \( c = 2 - b \)

substituting into the L equation,
\(-1 = -3 + 3b - b + 2 - b + d + e + f \)
b = \(-d - e - f \)
c = \(2 - b = 2 + d + e + f \)

\[ \Delta p = f(\rho^a \times \mu^b \times V^c \times L^d \times D^e \times e^f) \]
\[ \Delta p = f(\rho^{(1+d+e+f)} \times \mu^{(-d-e-f)} \times V^{(2+d+e+f)}) \times L^d \times D^e \times e^f \]
\[ \Delta p = \rho V^2 \left( \frac{\rho VL}{\mu} \right)^d \left( \frac{\rho VD}{\mu} \right)^e \left( \frac{\rho Ve}{\mu} \right)^f \]

**PIPE FLOW**

**hypothesis:** \( \Delta P = f(\rho, \mu, V, L, D, e) \)

<table>
<thead>
<tr>
<th></th>
<th>( \Delta p )</th>
<th>( \rho )</th>
<th>( \mu )</th>
<th>( V )</th>
<th>( L )</th>
<th>( D )</th>
<th>( e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>L</td>
<td>-1</td>
<td>-3</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>T</td>
<td>-2</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Multiplication or division of one dimensionless number by another result in a dimensionless number

\[ \frac{\Delta p}{\rho V^2} = f \left( \frac{\rho VL}{\mu}, \frac{\rho VD}{\mu}, \frac{\rho Ve}{\mu} \right) \]

\[ \frac{\Delta p}{\rho V^2} = f \left( \frac{\rho VD}{\mu}, L, D, e \right) \]
MEAN FLOW VELOCITY, V

\[
\bar{w} = \frac{1}{A} \int w \, dA = \frac{1}{A} \int w \, 2\pi r \, dr
\]

\[
\bar{w} = \frac{1}{\pi D^2} \int_{0}^{D/2} D^2 \left( -\frac{dp}{dx} \right) \left( 1 - \left( \frac{2r}{D} \right)^2 \right) 2\pi r \, dr
\]

\[
\bar{w} = \frac{4}{\pi D^2} \left( -\frac{dp}{dx} \right) \frac{D^2}{16\mu} \int_{0}^{D/2} \left( 4r^3 - 4r \right) dr
\]

\[
\bar{w} = \frac{2 \times 4}{D^2} \left( -\frac{dp}{dx} \right) \frac{D^2}{16\mu} \int_{0}^{D/2} \left( r - 4r^3 \right) dr
\]

\[
\bar{w} = \frac{1}{2\mu} \left( -\frac{dp}{dx} \right) \left( \frac{r^2}{2} - \frac{4r^4}{4D^2} \right)_{0}^{D/2}
\]

\[
\bar{w} = \frac{1}{2\mu} \left( -\frac{dp}{dx} \right) \left( \frac{D^2}{2 \times 4} - \frac{D^4}{16D^2} \right)
\]

\[
\bar{w} = \frac{1}{2\mu} \left( -\frac{dp}{dx} \right) \frac{D^2}{16}
\]

\[
\bar{w} = \frac{1}{32\mu} \left( -\frac{dp}{dx} \right)
\]

FRICITION FACTOR

from dimensional analysis

\[
\frac{\Delta p}{\rho V^2} = f \left( \frac{\rho V D}{\mu}, \frac{L}{D}, \frac{e}{D} \right)
\]

\[
\frac{p_1 - p_2}{\rho g} = \frac{\Delta p}{\rho g} = h_1 \text{ ft or meters of flowing fluid}
\]

\[
\Delta p = \left( -\frac{dp}{dx} \right) L
\]

\[
\left( -\frac{dp}{dx} \right) = \frac{32\mu V}{D^2}, \text{ letting } \bar{V} = V
\]

\[
h_1 = \left( -\frac{dp}{dx} \right) \frac{L}{\rho g} = \frac{32\mu V}{D^2} \frac{L}{\rho g}
\]

Multiply by \(2V/2V\)

\[
h_1 = \frac{2 \times 32\mu}{DV} \frac{L}{\rho g} \frac{\bar{V}^2}{D^2}
\]

\[
h_1 = \frac{64}{\rho V D} \frac{L V^2}{2g} = \left( \frac{64}{N_{RE}} \right) \frac{L V^2}{D 2g}
\]

\[
h_1 = f \frac{L V^2}{D 2g}, \quad f = \frac{64}{N_{RE}} \text{ LAMINAR FLOW}
\]
TURBULENT PIPE FLOW

\[ \frac{dp}{dz} = \frac{1}{r} \frac{d}{dr} \left( \frac{r \mu}{r} \frac{du}{dr} \right) \]

\[ u = u + u'(t) \]

\[ p = p + p'(t) \]

additional equations required

by Dimensional Analysis

\[ l_f = \text{function} \left( \frac{\rho VD}{\mu}, \frac{L}{D}, \frac{e}{D} \right) \]

\[ \frac{\rho VD}{\mu} = N_{RE} \]

\[ l_f = f \frac{L V^2}{D 2g} \]

\[ f = \text{function} \left( \frac{\rho VD}{\mu}, \frac{L}{D}, \frac{e}{D} \right) \]

Laminar Flow – analytical solution

\[ f = \frac{64}{N_{RE}} \]

Turbulent Flow – semi – empirical

Prandtl for smooth pipe

\[ \frac{1}{\sqrt{f}} = 2.0 \log \left( N_{RE} \sqrt{f} \right) - 0.08 \]
PIPE PRESSURE DROP (english)

250 gpm (0.947 m³/min) of water at 60 F (15.739 C) is flowing through a 4 inch pipe (4.026 in ID .10226 m) with roughness e/D = .0004, 200 ft (60.976 m) long. What is the head loss in ft and meters of the flowing fluid and in psi and kPa?

@ 60 F ρ = 1.938 slugs/ft³, μ = 1.21 ft²/sec, v = 1.210 × 10⁻⁵ ft²/sec

\[
A = \pi \frac{D^2}{4} = 3.1416 \left(\frac{4.026}{12}\right)^2 = .0884 \text{ ft}^2
\]

\[
V = \frac{\text{GPM} \times .1337}{60 \times A} = \frac{250 \text{ gpm} \times .1337 \text{ ft}^3/\text{gpm}}{60 \text{ min/hr} \times \text{ ft}^2 \times .0884 \text{ ft}^2} = 6.30 \text{ ft/sec}
\]

\[
N_{RE} = \frac{V D}{\nu} = \frac{6.30 \text{ ft/sec}}{1.210 \times 10^{-5} \frac{\text{ft}^2}{\text{sec}}} \left(\frac{4.026}{12}\right) = 174,682
\]

Figure 9.7 @ at N_{RE} and \(\frac{e}{D} = .0004\), f = .019

\[
h_1 = f \frac{L V^2}{D 2g} = .019 \frac{200 \text{ ft}}{4.026 \frac{\text{ft}}{12}} \left(\frac{6.30 \text{ ft/sec}}{2 \times 32.2 \frac{\text{ft/sec}^2}}\right) = 6.89 \text{ ft water}
\]

\[
\Delta p = h_1 \times \frac{\rho g}{144} = 6.89 \times 1.938 \times 32.2/144 = 2.99 \text{ psi}
\]
PIPE PRESSURE DROP (metric)

@ 15.739°C  \( \rho = 998.8 \frac{\text{kg}}{\text{m}^3} \),  \( \mu = 1.287 \times 10^{-3} \frac{\text{N sec}}{\text{m}^2} \),  \( \nu = 1.1331 \times 10^{-6} \frac{\text{m}^2}{\text{sec}} \)

\[ A = \pi \frac{D^2}{4} = \frac{3.1416}{4} \times .10266^2 = .00821 \text{m}^2 \]

\[ V = \frac{Q}{60 \times A} = \frac{.947 \text{ m}^3}{60 \times .00821 \text{ m}^2} = 1.921 \frac{\text{m}}{\text{sec}} \]

\[ N_{RE} = \frac{VD}{\nu} = \frac{1.921 \times .10226 \text{ m}}{1.1331 \times 10^{-6} \frac{\text{m}^2}{\text{sec}}} = 173,367 \]

Figure 9.7 @ \( N_{RE} \) and \( \frac{e}{d} = .0004, f = .019 \)

\[ h_1 = f \frac{L \cdot V^2}{D \cdot 2g} = .019 \frac{60.976}{.10226} \frac{1.921}{2 \times 9.81} = 2.13 \text{ m} \]

\[ \Delta p = h_1 \times \rho \cdot g = 2.13 \text{ m} \times 998.8 \frac{\text{kg}}{\text{m}^3} \times 9.81 = 20,870 \text{ Pa} = 20.870 \text{ kPa} \]
FITTING LOSSES

LOSS COEFFICIENT

\[ l_r = f_t \frac{L}{D} \left( \frac{V^2}{2g} \right) = \frac{ft}{ft} \times \frac{ft^2}{\sec^2} = ft \]

\[ l_{eq} = K \left( \frac{V^2}{2g} \right) \]

\[ K = f_t \times \left( \frac{L}{D} \right) = \text{resistance coefficient} \]

loss is K "velocity heads"

EQUIVALENT LENGTH

Equivalent length is reported in catalogues by fitting manufacturers from test data.

Loss in a length of pipe, of the same diameter as the fitting, equivalent to the loss in the fitting.

Loss coefficients are reported in catalogues by fitting manufacturers from test data.
# LOSS COEFFICIENTS

## Table 9.2

**K factors for fittings**

<table>
<thead>
<tr>
<th>Nominal diameter, in</th>
<th>$\frac{1}{2}$</th>
<th>$\frac{3}{4}$</th>
<th>1</th>
<th>1½</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>8–10</th>
<th>12–16</th>
<th>18–24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gate valve (open)</td>
<td>0.22</td>
<td>0.20</td>
<td>0.18</td>
<td>0.16</td>
<td>0.15</td>
<td>0.14</td>
<td>0.14</td>
<td>0.13</td>
<td>0.12</td>
<td>0.11</td>
<td>0.10</td>
<td>0.096</td>
</tr>
<tr>
<td>Globe valve (open)</td>
<td>9.2</td>
<td>8.5</td>
<td>7.8</td>
<td>7.1</td>
<td>6.5</td>
<td>6.1</td>
<td>5.8</td>
<td>5.1</td>
<td>4.8</td>
<td>4.4</td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td>Standard elbow (screwed) 90°</td>
<td>0.80</td>
<td>0.75</td>
<td>0.69</td>
<td>0.63</td>
<td>0.57</td>
<td>0.54</td>
<td>0.51</td>
<td>0.48</td>
<td>0.45</td>
<td>0.42</td>
<td>0.39</td>
<td>0.36</td>
</tr>
<tr>
<td>Standard elbow (screwed) 45°</td>
<td>0.43</td>
<td>0.40</td>
<td>0.37</td>
<td>0.34</td>
<td>0.30</td>
<td>0.29</td>
<td>0.27</td>
<td>0.26</td>
<td>0.24</td>
<td>0.22</td>
<td>0.21</td>
<td>0.19</td>
</tr>
<tr>
<td>Standard tee (flow through)</td>
<td>0.54</td>
<td>0.50</td>
<td>0.46</td>
<td>0.42</td>
<td>0.38</td>
<td>0.36</td>
<td>0.34</td>
<td>0.32</td>
<td>0.30</td>
<td>0.28</td>
<td>0.26</td>
<td>0.24</td>
</tr>
<tr>
<td>Standard tee (flow branched)</td>
<td>1.62</td>
<td>1.50</td>
<td>1.38</td>
<td>1.26</td>
<td>1.14</td>
<td>1.02</td>
<td>0.96</td>
<td>0.90</td>
<td>0.84</td>
<td>0.78</td>
<td>0.72</td>
<td></td>
</tr>
</tbody>
</table>

PIPING SYSTEM CHARACTERISTIC

\[ h_1 = f \left( \frac{L}{D} \frac{V^2}{2g} \right) \]

\[ Q = V \times A \]

\[ V = \frac{Q}{A} \]

\[ h_1 = f \left( \frac{L}{D} \frac{\left( \frac{Q}{A} \right)^2}{2g} \right) \]

\[ h_1 = f \left( \frac{L}{D} \frac{A^2}{2g} \right) Q^2 \]

\[ h_1 = \text{const} \tan t \times Q^2 \]

\[ \text{const} \tan t = \frac{h_{10}}{Q_o^2} = \frac{h_1}{Q^2} \]

\[ h_1 = \left( \frac{h_{10}}{Q_o^2} \right) Q^2 \]
\[ h_1 = \left( \frac{h_{1O}}{Q_{O}^2} \right) \times Q^2 \]

A: \[ h_1 = \left( \frac{20}{50^2} \right) Q^2 = .008Q^2 \]

\[ Q_A = \left( \frac{h_{1A}}{.008} \right)^5 \]

B: \[ h_1 = \left( \frac{25}{30^2} \right) Q^2 = .0278Q^2 \]

\[ Q_B = \left( \frac{h_{1B}}{.0278} \right)^5 \]

C: \[ h_1 = \left( \frac{00}{45^2} \right) Q^2 = .01481Q^2 \]

\[ Q_C = \left( \frac{h_{1C}}{.01481} \right)^5 \]

\[ Q_{\text{total}} = Q_A + Q_B + Q_C, h_{1A} = h_{1B} = h_{1C} = h_1 \]

\[ Q_{\text{total}} = \left( \frac{h_{1A}}{.008} \right)^5 + \left( \frac{h_{1B}}{.0278} \right)^5 + \left( \frac{h_{1C}}{.01481} \right)^5 = 25.397h_1^5 \]

\[ h_1 = \left( \frac{Q_{\text{total}}}{25.397} \right) \]

A: \[ h_1 = 20 \text{ ft} @ 50 \text{ GPM} \]

B: \[ h_1 = 25 \text{ ft} @ 30 \text{ GPM} \]

C: \[ h_1 = 30 \text{ ft} @ 45 \text{ GPM} \]

\[ Q_A?, \ Q_B?, \ Q_C? \]

at 100 gpm, \[ h_1 = \left( \frac{100^2}{25.397} \right) = 15.5 \text{ ft} \]

\[ Q_A = Q_A = \left( \frac{h_{1A}}{.008} \right)^5 = \left( \frac{15.5}{.008} \right)^5 = 44. \text{ gpm} \]

\[ Q_B = Q_B = \left( \frac{h_{1B}}{.0278} \right)^5 = \left( \frac{15.5}{.0278} \right)^5 = 23.6. \text{ gpm} \]

\[ Q_C = Q_C = \left( \frac{h_{1C}}{.01481} \right)^5 = \left( \frac{15.5}{.01481} \right)^5 = 32.4 \text{ gpm} \]
Steady Flow Energy Equation

Open, steady flow thermodynamic system - a region in space

\[ Q = \Delta E - W \quad \text{First Law} \quad q \text{ and in is } + \text{ by Smits convention} \]

\[ W_{\text{flow in}} = \int p \, dV = p_1 (V_{1 \text{ initial}} - V_{1 \text{ final}}) = p_1 V_1 = m \, p_1 \, v_1 \]

\[ W_{\text{flow out}} = m \, p_2 \, v_2 \]

\[ W = W_{\text{shaft}} + W_{\text{flow in}} + W_{\text{flow out}} = W_{\text{shaft}} + m \, p_1 \, v_1 - m \, p_2 \, v_2 \]

\[ E = U(T) + KE + PE = U(T) + \frac{V^2}{2g} + h \]

\[ Q = m(u_1 + p_1 \, v_1 + \frac{V^2}{2} + h_1) - m(u_2 + p_2 \, v_2 + \frac{V^2}{2g} + h_2) - W_{\text{shaft}} \]

\[ Q = m \Delta (u + pv + \frac{V^2}{2} + h) - W_{\text{shaft}} \]
FIRST LAW

\[ Q = m \times \Delta(u + pv + \frac{V^2}{2g} + z) - W_{\text{shaft}} \]

\[ \mu = 0 \Rightarrow \Delta T = 0 \Rightarrow c_v \Delta T = u = 0 \]

\[ Q = m \times \Delta\left(\frac{p}{\rho} + \frac{V^2}{2g} + z\right) - W_{\text{shaft}} \]

frictional head losses end up as heat

\[ 1_f = \Delta\left(\frac{p}{\rho} + \frac{V^2}{2g} + z\right) - \frac{W_{\text{shaft}}}{m} \quad (10 - 1a) \]

work done on the system is negative

BERNOULLI’S EQUATION

\[ 1_f = \Delta\left(\frac{p}{\rho} + \frac{V^2}{2g} + z\right) - \frac{W_{\text{shaft}}}{m} \]

for \( 1_f = 0, \quad W = 0 \)

\[ 0 = \Delta\left(\frac{p}{\rho} + \frac{V^2}{2g} + z\right) \]
weight flow = 10 \, \text{lb}_m/\text{sec}

\frac{p_1}{\rho} + \frac{V_1^2}{2} + z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + z_2 - \frac{W}{m}

\frac{(17 + 14.7) \times 144}{62.4} + \frac{V_1^2}{2 \times 32.2} + 0

= \frac{(8 + 14.7) \times 144}{62.4} + \frac{V_2^2}{2 \times 32.2} + 12 \times 25 - \frac{W}{m}

73.15 = 52.38 + 300 - \frac{W}{m}

\frac{W}{m} = -279.23 \, \text{ft lb}_f

\frac{\text{lb}_m}{\text{lb}_m}

W = -279.23 \times 10 = -2792.3 \, \text{ft lb/} \text{sec}

\frac{2792.3 \, \text{ft lb}}{\text{sec}} = -5.08 \, \text{HP}

\frac{550 \, \text{ft lb}}{\text{sec HP}}

work added to system

W = -3.8 \, \text{KW}
10-1. The system shown in Fig. 10-50 transfers water to the tank at a rate of 240 gpm (0.015 m³/s) through standard commercial steel 4 in. pipe. The total equivalent length of the pipe is 330 ft (100 m). The increase in elevation is 130 ft (40 m). Compute (a) the work done on the water and (b) the power delivered to the water, and (c) sketch the system characteristic.
assume 60°F water

\[
\mu = 2.713 \frac{\text{lb}_m}{\text{ft hr}}, \quad \rho = 62.37 \frac{\text{lb}}{\text{ft}^3}
\]

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\[
v = \frac{\mu}{\rho} = \frac{\frac{\text{lb}_m}{\text{ft hr}}}{3600 \text{sec/hr}} \times \frac{1}{\frac{\text{ft}^3}{\text{lb}_m}} = \frac{\text{ft}^2}{\text{sec}}
\]

\[
m = \frac{240 \text{ gpm} \times .1337 \text{ ft}^3/\text{gpm} \times 62.37 \text{ lb/ft}^3}{60 \text{ sec/hr}} = 33.36 \text{ lb/sec}
\]

\[
A = \frac{\pi D}{4} = \frac{3.1416}{4} \times \left(\frac{4.026}{12}\right)^2 = .0884 \text{ ft}^2
\]

\[
V = \frac{Q}{A} = \frac{240 \text{ gpm} \times .1337 \text{ ft}^3/\text{gpm}}{60 \text{ sec/hr} \times .0884 \text{ ft}^2} = 6.05 \text{ ft/sec}
\]

\[
R_e = \frac{\rho V D}{\mu} = \frac{62.37 \text{ lb/ft}^3 \times 6.05 \text{ ft/sec} \times \left(\frac{4.026}{12}\right)}{2.713 \text{ lb}_m/\text{ft hr} \times 3600 \text{ sec/hr}} = 167,750
\]

\[
\frac{\varepsilon}{D} = .0004 \quad \text{Table 10-1 Commercial Steel}
\]

at \( R_e \) and \( \frac{\varepsilon}{D}, \quad f = .019 \quad \text{Figure 10-1}

\[
l_f = f \frac{L}{D} \left(\frac{V^2}{2g}\right) = .19 \times \left(\frac{330 \text{ft}}{4.026/12}\right) \times \frac{6.05^2}{2 \times 32.2} = 10.62 \text{ ft}
\]

\[
\frac{p_1}{\rho_1} + \frac{V_1^2}{2g_c} + z_1 \frac{g_c}{g} = \frac{p_2}{\rho_2} + \frac{V_2^2}{2g_c} + z_2 \frac{g_c}{g} + W + \frac{g}{g_c} l_f
\]

\[
W = m \times h = m \times (z_2 - z_1) + l_f
\]

\[
W = 33.36 \text{ lb}(130 \text{ ft} + 10.62 \text{ ft}) = 4692 \text{ ft lb/sec}
\]

\[
W = \frac{4692 \text{ ft lb/sec}}{550 \text{ ft lb/sec}/\text{HP}} = 8.529 \text{ HP}
\]

\[
W = 8.529 \text{ HP} \times .7457 \text{ KW/HP} = 6.36 \text{ KW}
\]

\[
h = 130 + \left(\frac{h_o}{Q_o^2}\right) Q^2
\]

\[
h = 130 + .0001844 Q^2
\]
A 3-zone heating system uses hot water passing through the piping network shown. The heater increases water temperature 20 F. All pipes are copper type L.

a) What is the total head added by the pump?
b) Assuming a pump efficiency of 45%, what size electric motor should be used?
C) What is the heat flow rate into the water?

<table>
<thead>
<tr>
<th>Circuits</th>
<th>l eq, ft</th>
<th>D,in</th>
<th>Q, gpm</th>
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<tr>
<td>5-1-p-2</td>
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<tr>
<td>4-5</td>
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<td>2.0</td>
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</tbody>
</table>
heating \( \Rightarrow T = 140^\circ F, \ \mu = 1.129, \ \rho = 61.38 \text{lb/ft}^3 \)

<table>
<thead>
<tr>
<th>Section</th>
<th>L</th>
<th>D</th>
<th>Q</th>
<th>A</th>
<th>V</th>
<th>( N_{re} )</th>
<th>f</th>
<th>( h_f )</th>
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<td>1.135</td>
<td>45.40</td>
</tr>
</tbody>
</table>

\( \sum Q \times H = 248.88 \)

a) Head/section

\[ P = 2 - 4 - 1 = .761 + 1.936 + 1.135 = 3.071 \text{ ft} \]

\[ P = 3 - 4 - 5 - 1 = .761 + 1.249 + 1.659 = 3.669 \text{ ft} \]

\[ P = 2 - 3 - 5 = .761 + 1.249 + 1.798 + 1.135 = 4.943 \text{ ft maximum head} \]

b) Power

\[ \text{Power} = \sum m \times H = \sum \frac{Q \times 1.337 \times \rho}{60 \text{ sec/min}} \times H = \frac{1.337 \times 61.4}{60} \sum QH \]

Ideal Power

\[ \text{Ideal Power} = \frac{1.337 \times 61.4 \times 248.88 \times 0.7457}{550 \text{ ftlb/HP}} = .0462 \text{ KW} \]

Actual Power

\[ \text{Actual Power} = \frac{\text{Ideal Power}}{\eta} = \frac{.0462 \text{HP}}{.45} = .103 \text{ KW} \]

c)\[ Q = m(h_2 - h_1) = \frac{60 \text{gpm} \times 1.337 \text{ft}^3/\text{gal} \times 61.4(117.89 - 97.9)}{60 \text{sec/min}} = 590,751 \text{BTU/hr} \]