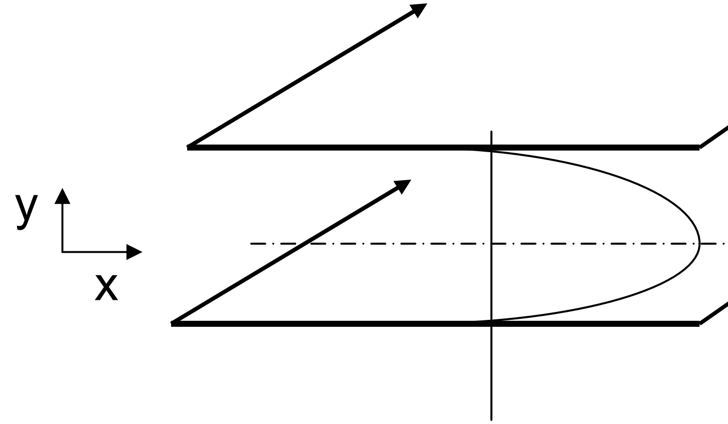
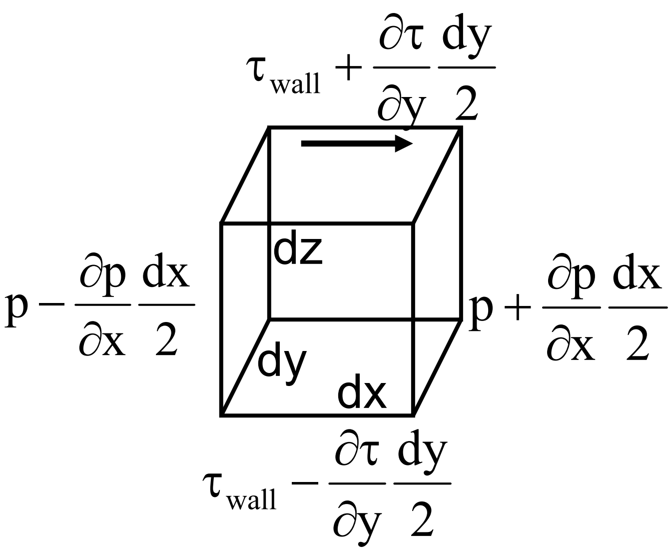
POISEUILLE FLOW -
flow between parallel infinite plates
1 Dimensional,
steady,
viscous,
laminar,
fully developed
Exact Solution Possible





$$F_{viscous} + F_{pressure} = 0$$

$$F_{viscous} = \left(\tau_{wall} + \frac{\partial \tau_{wall}}{\partial y}\frac{dy}{2}\right)dx\,dz - \left(\tau_{wall} - \frac{\partial \tau_{wall}}{\partial y}\frac{dy}{2}\right)dx\,dz$$

$$F_{viscous} = \frac{\partial \tau_{wall}}{\partial y}dx\,dy\,dz$$

since $\tau_{xx} = \tau_{xzx} = 0 \frac{\partial \tau_{xy}}{\partial y} = \frac{d\tau_{xy}}{dy}$

and since $\tau_{xy} = \mu\left(\frac{du}{dy}\right)$

$$F_{viscous} = \frac{d}{dy}\left(\mu\left(\frac{du}{dy}\right)\right)dx\,dy\,dz$$

$$F_{viscous} = \mu\frac{d^2u}{dy^2}dx\,dy\,dz$$

POISEUILLE FLOW

flow between parallel infinite plates
1dimensional,
steady,
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fully developed

$$F_{pressure} = \left(\left(p + \frac{\partial p}{\partial x}\frac{dx}{2}\right) - \left(p - \frac{\partial p}{\partial x}\frac{dx}{2}\right)\right) dy\, dz$$

$$F_{pressure} = -\frac{\partial p}{\partial x} dx\, dy\, dz$$

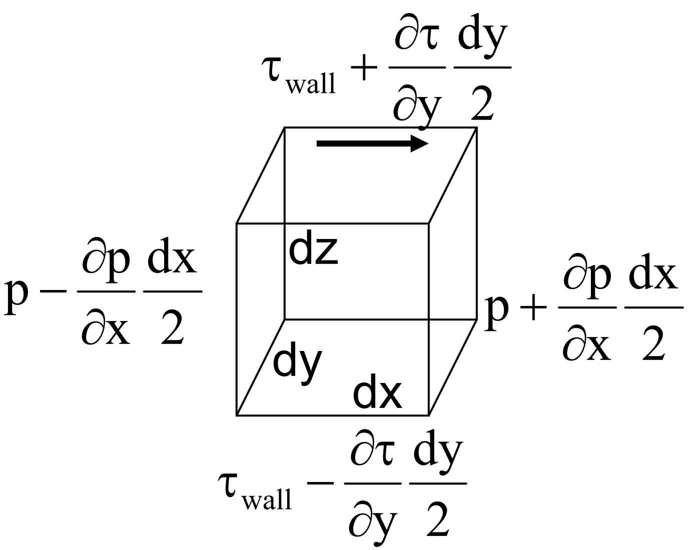
$$\sin ce \frac{\partial p}{\partial y} = \frac{\partial p}{\partial z} = 0 \frac{\partial p}{\partial y} = \frac{dp}{dy}$$

$$F_{pressure} = -\frac{dp}{dx} dx\, dy\, dz$$

$$F_{pressure} + F_{viscous} = 0$$

$$-\frac{dp}{dx} dxdydz + \mu \frac{d^2u}{dy^2} dx\, dy\, dz = 0$$

$$\frac{dp}{dx} = \mu \frac{d^2u}{dy^2}$$



POISEUILLE FLOW

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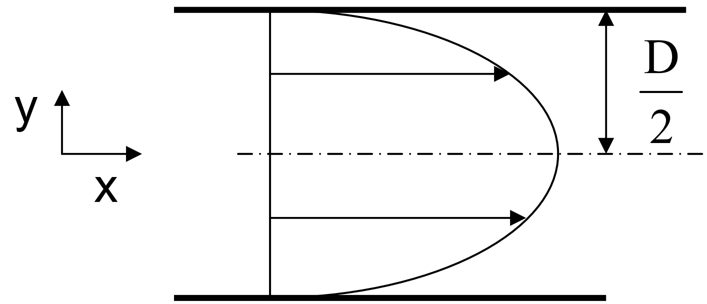
Exact Solution Possible



0, steady flow

$$\rho \frac{D\vec{V}}{Dt} = -\nabla p + \rho g + \mu \nabla^2 \vec{V}$$

0 0

$$\frac{\partial p}{dx} = \mu \left(\frac{\partial^2 u}{dx^2} + \frac{\partial^2 u}{dy^2} + \frac{\partial^2 u}{dz^2} \right)$$

$$\frac{dp}{dx} = \frac{\mu}{\rho} \frac{d^2 u}{dy^2}$$

the momentum equations is satisfied only if,

$$\frac{dp}{dx} = -K \quad \text{and} \quad \frac{\mu}{\rho} \frac{d^2 u}{dy^2} = -K$$

Boundary Conditions

$$u = 0 \text{ at } y = \pm \frac{D}{2}$$

$$\frac{du}{dy} = 0 \text{ at } y = 0$$

VELOCITY DISTRIBUTION - parabolic

$$\mu\frac{d^2u}{dy^2} = -K$$

$$\frac{d^2u}{dy^2} = \frac{-K}{\mu}$$

$$\frac{du}{dy} = \frac{-K}{\mu}y + C_1$$

$$at = 0, \frac{du}{dy} = 0$$

$$\frac{du}{dy} = \frac{-K}{\mu}0 + C_1 = 0 \Rightarrow C_1 = 0$$

$$\frac{du}{dy} = \frac{-K}{\mu}y$$

$$u = \frac{-K}{2\mu}y^2 + C_2$$

$$aty = \pm\frac{D}{2}, u = 0$$

$$0 = \frac{-K}{2\mu}\left(\frac{D}{2}\right)^2 + C_2 \Rightarrow C_2 = \frac{K}{2\mu}\left(\frac{D}{2}\right)^2$$

$$u = \frac{-K}{2\mu}y^2 + \frac{K}{2\mu}\left(\frac{D}{2}\right)^2$$

$$u = \frac{K}{2\mu}\left(\frac{D}{2}\right)^2\left(1 - \left(\frac{y}{D}\right)^2\right)$$

$$u = \frac{KD^2}{8\mu}\left(1 - \left(\frac{y}{D}\right)^2\right)$$

$$u = \frac{K}{2\mu}\left(\frac{D}{2}\right)^2\left(1 - \left(\frac{y}{D}\right)^2\right)$$

$$u = \frac{KD^2}{8\mu}\left(1 - \left(\frac{y}{D}\right)^2\right)$$

$$u = \frac{D^2}{8\mu}\left(-\frac{dp}{dx}\right)\left(1 - \left(\frac{y}{D}\right)^2\right)$$

MEAN FLOW VELOCITY, V

$$V = \frac{1}{w\,D}\int_{-D/2}^{D/2} w\,u\,dy = \frac{1}{D}\int_{-D/2}^{D/2}\frac{D^2}{8\mu}\left(-\frac{dp}{dx}\right)\left(1-\left(\frac{y}{D}\right)^2\right)dy$$

$$V = \frac{1}{D}\frac{D^2}{8\mu}\left(-\frac{dp}{dx}\right)\int_{-D/2}^{D/2}\left(1-\left(\frac{y}{D}\right)^2\right)dy$$

$$V = \frac{1}{D}\frac{D^2}{8\mu}\left(-\frac{dp}{dx}\right)\left[y-\left(\frac{2}{D}\right)^2\frac{y^3}{3}\right]_{-D/2}^{D/2}$$

$$V = \frac{1}{D}\frac{D^2}{8\mu}\left(-\frac{dp}{dx}\right)\left[\frac{D}{2}-\frac{4}{D^2}\frac{D^3}{3\times 8}\right]-\left[-\frac{D}{2}+\frac{4}{D^2}\frac{D^3}{3\times 8}\right]$$

$$V = \frac{D^2}{12\mu}\left(-\frac{dp}{dx}\right)$$

VELOCITY DISTRIBUTION

$$\frac{u}{V} = \frac{\frac{D^2}{8\mu}\left(-\frac{dp}{dx}\right)\left(1-\left(\frac{y}{D}\right)^2\right)}{\frac{D^2}{12\mu}\left(-\frac{dp}{dx}\right)} = \frac{3}{2}\left(1-\left(\frac{y}{D}\right)^2\right)$$

FRICTION FACTOR

$$h_1 = f\,\frac{L}{D}\frac{V^2}{\rho 2}$$

$$f = -\frac{dp}{dx}\frac{2D}{\rho V^2}$$

$$\text{since}\ V = \frac{D^2}{12\mu}\left(-\frac{dp}{dx}\right)$$

$$-\frac{dp}{dx} = \frac{12V\mu}{D^2}$$

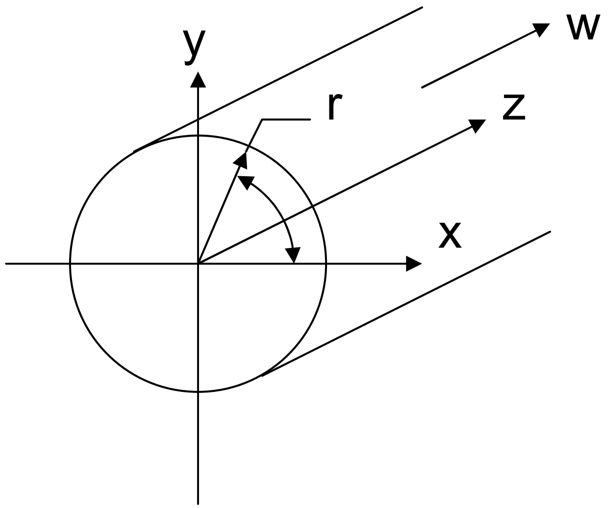
$$f = \frac{12V\mu}{D^2}\frac{2D}{\rho V^2}$$

$$f = \frac{24\mu}{\rho V D}$$

$$f = \frac{24}{N_{RE}}$$

LAMINAR FLOW

LAMINAR PIPE FLOW
steady, 1D, viscous, constant density
Exact Solution Possible



$$\rho \frac{D\vec{V}}{Dt} = -\nabla p + \rho g + \mu \nabla^2 \vec{V}$$

z direction momentun equation

$$\frac{\partial p}{dz} = \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

$\nabla^2 \vec{V}$ in cylindrical coordinates p 478

$$\frac{\partial p}{dz} = \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

$$\frac{dp}{dz} = \frac{1}{r}\frac{d}{dr}\left(r\mu\frac{dw}{dr}\right)$$

$$\frac{dp}{dz} = \frac{1}{r}\frac{d(r\tau)}{dr}$$

VELOCITY DISTRIBUTION - parabolic

$$\frac{1}{r}\frac{d(r\tau)}{dr} = -K$$

$$d(r\tau) = -Krdr$$

$$r\tau = -K\frac{r^2}{2}$$

$$\tau =$$

$$\frac{d^2u}{dy^2} = \frac{-K}{\mu}$$

$$\frac{du}{dy} = \frac{-K}{\mu}y + C_1$$

$$\text{at} = 0, \frac{du}{dy} = 0$$

$$\frac{du}{dy} = \frac{-K}{\mu}0 + C_1 = 0 \Rightarrow C_1 = 0$$

$$\frac{du}{dy} = \frac{-K}{\mu}y$$

$$u = \frac{-K}{2\mu}y^2 + C_2$$

$$\text{at} \quad y = \pm\frac{D}{2}, u = 0$$

$$0 = \frac{-K}{2\mu}\left(\frac{D}{2}\right)^2 + C_2 \Rightarrow C_2 = \frac{K}{2\mu}\left(\frac{D}{2}\right)^2$$

$$u = \frac{-K}{2\mu}y^2 + \frac{K}{2\mu}\left(\frac{D}{2}\right)^2$$

$$u = \frac{K}{2\mu}\left(\frac{D}{2}\right)^2\left(1 - \left(\frac{2y}{D}\right)^2\right)$$

$$u = \frac{KD^2}{8\mu}\left(1 - \left(\frac{2y}{D}\right)^2\right)$$

$$u = \frac{D^2}{8\mu}\left(-\frac{dp}{dx}\right)\left(1 - \left(\frac{2y}{D}\right)^2\right)$$

Balance Unit Exponents

for M, $1 = a + b$

for L, $-1 = -3a - b + c + d + e + f$

for T, $-2 = -b - c$

from M $a = 1 - b$

fromT $c = 2 - b$

substituting into the L equation,

$-1 = -3 + 3b - b + 2 - b + d + e + f$

$b = -d - e - f$

$c = 2 - b = 2 + d + e + f$

$$\Delta p = f\left(\rho^a \times \mu^b \times V^c \times L^d \times D^e \times e^f\right)$$

$$\Delta p = f(\rho^{(1+d+e+f)} \times \mu^{(-d-e-f)} \times V^{(2+d+e+f)} \times L^d \times D^e \times e^f)$$

$$\Delta p = \rho V^2 \left(\frac{\rho VL}{\mu}\right)^d \left(\frac{\rho VD}{\mu}\right)^e \left(\frac{\rho Ve}{\mu}\right)^f$$

PIPE FLOW

hypothesis: $\Delta P = f(\rho, \mu, V, L, D, e)$

	Δp	ρ	μ	V	L	D	e
M	1	1	1	0	0	0	0
L	-1	-3	-1	1	1	1	1
T	-2	0	-1	-1	0	0	0

Multiplication or division of one dimensionless number by another result in a dimensionless number

$$\frac{\Delta p}{\rho V^2} = f\left(\frac{\left(\frac{\rho VL}{\mu}\right)}{\left(\frac{\rho VD}{\mu}\right)} \left(\frac{\rho VD}{\mu}\right) \frac{\left(\frac{\rho Ve}{\mu}\right)}{\left(\frac{\rho VD}{\mu}\right)}\right)$$

$$\frac{\Delta p}{\rho V^2} = f\left(\frac{\rho VD}{\mu}, \frac{L}{D}, \frac{e}{D}\right)$$

MEAN FLOW VELOCITY, V

$$\bar{w} = \frac{1}{A}\int w\, dA = \frac{1}{A}\int w\, 2\pi r\, dr$$

$$\bar{w} = \frac{1}{\frac{\pi D^2}{4}} \int_0^{D/2} \frac{D^2\left(-\frac{dp}{dx}\right)}{16\mu}\left(1-\left(\frac{2r}{D}\right)^2\right) 2\pi r\, dr$$

$$\bar{w} = \frac{4}{\pi D^2}\frac{D^2\left(-\frac{dp}{dx}\right)}{16\mu} 2\pi \int_0^{D/2}\left(1-\left(\frac{2r}{D}\right)^2\right) r\, dr$$

$$\bar{w} = \frac{2\times 4}{D^2}\frac{D^2\left(-\frac{dp}{dx}\right)}{16\mu}\int_0^{D/2}\left(r-\frac{4r^3}{D^2}\right)dr$$

$$\bar{w} = \frac{1}{2\mu}\left(-\frac{dp}{dx}\right)\left(\frac{r^2}{2}-\frac{4r^4}{4D^2}\right)_0^{D/2}$$

$$\bar{w} = \frac{1}{2\mu}\left(-\frac{dp}{dx}\right)\left(\frac{D^2}{2\times 4}-\frac{D^4}{16D^2}\right)$$

$$\bar{w} = \frac{1}{2\mu}\left(-\frac{dp}{dx}\right)\frac{D^2}{16}$$

$$\bar{w} = \frac{1}{32}\frac{D^2}{\mu}\left(-\frac{dp}{dx}\right)$$

FRICTION FACTOR

from dimensional analysis

$$\frac{\Delta p}{\rho V^2} = f\left(\frac{\rho V D}{\mu}, \frac{L}{D}, \frac{e}{D}\right)$$

$$\frac{p_1 - p_2}{\rho g} = \frac{\Delta p}{\rho g} = h_l \qquad \text{ft or meters of flowing fluid}$$

$$\Delta p = \left(-\frac{dp}{dx}\right)L$$

$$\left(-\frac{dp}{dx}\right) = \frac{32\,\mu\, V}{D^2}, \qquad \text{letting } \bar{V} = V$$

$$h_l = \left(-\frac{dp}{dx}\right)\frac{L}{\rho g} = \frac{32\,\mu\, V}{D^2}\frac{L}{\rho g}$$

multiply by 2V/2V

$$h_l = \frac{2\times 32\,\mu}{D\, V}\frac{L}{\rho g}\frac{\bar{V}^2}{D\, 2}$$

$$h_l = \frac{64}{\frac{\rho V D}{\mu}}\frac{L\,\bar{V}^2}{D\, 2\, g} = \left(\frac{64}{N_{RE}}\right)\frac{L}{D}\frac{V^2}{2\, g}$$

$$h_l = f\frac{L}{D}\frac{V^2}{2g}, \qquad f = \frac{64}{N_{RE}} \text{ LAMINAR FLOW}$$

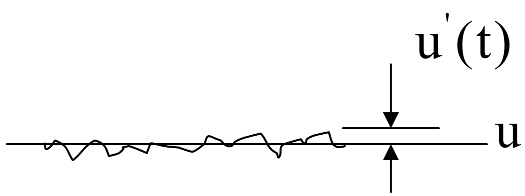
TURBULENT PIPE FLOW

$$\frac{dp}{dz} = \frac{1}{r}\frac{d}{dr}\left(r\mu\frac{du}{dr}\right)$$

$$u = \bar{u} + u'(t)$$

$$p = \bar{p} + p'(t)$$

additional equations required



No Exact Solution

by Dimensional Analysis

$$l_f = \text{function}\left(\frac{\rho VD}{\mu}, \frac{L}{D}, \frac{e}{D}\right)$$

$$\frac{\rho VD}{\mu} = N_{RE}$$

$$l_f = f\frac{L}{D}\frac{V^2}{2g}$$

$$f = \text{function}\left(\frac{\rho VD}{\mu}, \frac{L}{D}, \frac{e}{D}\right)$$

Laminar Flow – analytical solution

$$f = \frac{64}{N_{RE}}$$

Turbulent Flow – semi – empirical

Prandtl for smooth pipe

$$\frac{1}{\sqrt{f}} = 2.0\log\left(N_{RE}\sqrt{f}\right) - .08$$

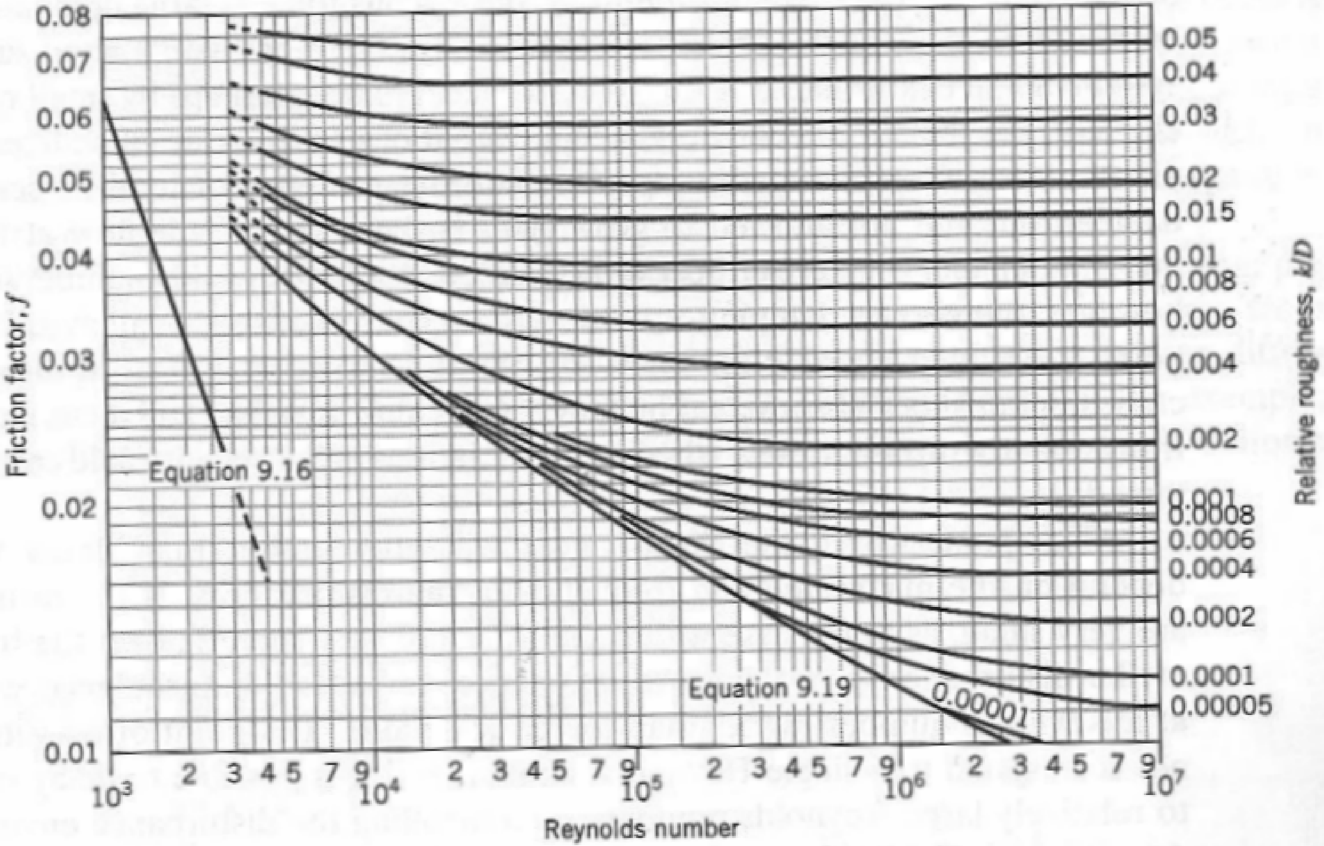


FIGURE 9.7 Moody diagram for fully developed flow in circular pipes. Laminar flow is described by equation 9.16. Prandtl's universal law of friction for turbulent smooth pipes is given by equation 9.19. Adapted from Moody, L. F. "Friction Factors for Pipe Flow," *Trans. of the ASME,* **66,** 671–684, 1944, with permission.

PIPE PRESSURE DROP (english)

250gpm (.947 m^3/min) of water at 60 F (15.739 C) is flowing through a 4 in pipe (4.026 in ID .10226 m) with roughness e/D = .0004, 200ft (60.976 m)long. What is the head loss in ft and meters of the flowing fluid and in psi and kPa?

$$@\ 60\ \text{F}\ \rho = 1.938\frac{\text{slugs}}{\text{ft}^3}, \mu = 1.21\frac{\text{ft}^2}{\text{sec}}, \upsilon = 1.210\times10^{-5}\ \frac{\text{ft}^2}{\text{sec}}$$

$$A = \pi\frac{D^2}{4} = \frac{3.1416}{4}\left(\frac{4.026}{12}\right)^2 = .0884\ \text{ft}^2$$

$$V = \frac{\text{GPM}\times.1337}{60\times A} = \frac{250\ \text{gpm}\times.1337\text{ft}^3/\text{gpm}}{60\ \text{min/hr}\times\text{ft}^2\times.0884\ \text{ft}^2} = 6.30\ \text{ft/sec}$$

$$N_{RE} = \frac{VD}{\upsilon} = \frac{6.30\ \text{ft/sec}}{\upsilon = 1.210\times10^{-5}\ \frac{\text{ft}^2}{\text{sec}}}\left(\frac{4.026}{12}\right) = 174{,}682$$

$$\text{Figure 9.7 @ at } N_{RE} \text{ and } \frac{e}{D} = .0004, \quad f = .019$$

$$h_l = f\frac{L}{D}\frac{V^2}{2g} = .019\frac{200\ \text{ft}}{\left(\frac{4.026}{12}\right)\text{ft}}\frac{6.30\ \text{ft/sec}}{2\times32.2\ \text{ft/sec}^2} = 6.89\ \text{ft water}$$

$$\Delta p = h_l\times\frac{\rho g}{144} = 6.89\times1.938\times32.2/144 = 2.99\ \text{psi}$$

PIPE PRESSURE DROP (metric)

$$@\,15.739\,\mathrm{C} \quad \rho = 998.8\,\frac{\mathrm{kg}}{\mathrm{m}^3}, \quad \mu = 1.287\times10^{-3}\,\frac{\mathrm{N\,sec}}{\mathrm{m}^2}, \quad \upsilon = 1.1331\times10^{-6}\,\frac{\mathrm{m}^2}{\mathrm{sec}}$$

$$A = \pi\frac{D^2}{4} = \frac{3.1416}{4}\times.10266^2 = .00821\,\mathrm{m}^2$$

$$V = \frac{Q}{60\times A} = \frac{.947\,\frac{\mathrm{m}^3}{\mathrm{sec}}}{60\times.00821\,\mathrm{m}^2} = 1.921\,\frac{\mathrm{m}}{\mathrm{sec}}$$

$$N_{RE} = \frac{VD}{\upsilon} = \frac{1.921\times.10226\,\mathrm{m}}{1.1331\times10^{-6}\,\frac{\mathrm{m}^2}{\mathrm{sec}}} = 173{,}367$$

$$\text{Figure 9.7 @ } N_{RE} \text{ and } \frac{e}{d} = .0004,\ f = .019$$

$$h_1 = f\,\frac{L}{D}\,\frac{V^2}{2g} = .019\,\frac{60.976}{.10226}\,\frac{1.921}{2\times9.81} = 2.13\,\mathrm{m}$$

$$\Delta p = h_1\times\rho\,g = 2.13\,\mathrm{m}\times998.8\,\frac{\mathrm{kg}}{\mathrm{m}^3}\times9.81 = 20{,}870\ \mathrm{Pa} = 20.870\ \mathrm{kPa}$$

FITTING LOSSES

LOSS COEFFICIENT

$$l_f = f_t \frac{L}{D}\left(\frac{V^2}{2g}\right) = \frac{ft}{ft} \times \frac{\frac{ft^2}{sec^2}}{\frac{ft}{sec^2}} = ft$$

$$l_{eq} = K\left(\frac{V^2}{2g}\right)$$

$$K = f_t \times \left(\frac{L}{D}\right) = \text{resistance coefficient}$$

loss is K "velocity heads"

Loss coefficients are reported in catalogues by fitting manufacturers from test data.

EQUIVALENT LENGHT

Equivalent length is reported in catalogues by fitting manufacturers from test data.

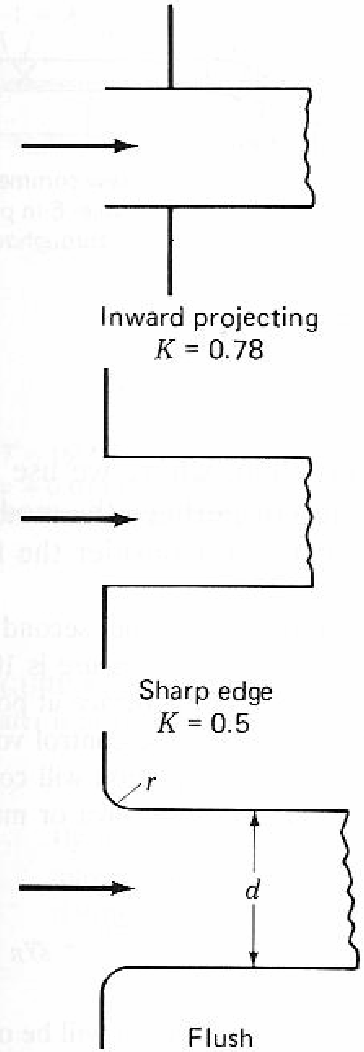
Loss in a length of pipe,
of the same diameter as the fitting,
equivalnet to the loss in the fitting.

LOSS COEFFICIENTS

TABLE 9.2
K factors for fittings*

	Nominal diameter, in											
	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{2}$	2	3	4	5	6	8–10	12–16	18–24
Gate valve (open)	0.22	0.20	0.18	0.16	0.15	0.14	0.14	0.13	0.12	0.11	0.10	0.096
Globe valve (open)	9.2	8.5	7.8	7.1	6.5	6.1	5.8	5.4	5.1	4.8	4.4	4.1
Standard elbow (screwed) 90°	0.80	0.75	0.69	0.63	0.57	0.54	0.51	0.48	0.45	0.42	0.39	0.36
Standard elbow (screwed) 45°	0.43	0.40	0.37	0.34	0.30	0.29	0.27	0.26	0.24	0.22	0.21	0.19
Standard tee (flow through)	0.54	0.50	0.46	0.42	0.38	0.36	0.34	0.32	0.30	0.28	0.26	0.24
Standard tee (flow branched)	1.62	1.50	1.38	1.26	1.14	1.08	1.02	0.96	0.90	0.84	0.78	0.72

*Data from Crane C., "Flow of Fluids," Tech. Paper 410, 1979.



r/d	0.02	0.04	0.06	0.10	0.15 and up
K	0.28	0.24	0.15	0.09	0.04

PIPING SYSTEM CHARACTERISTIC

$$h_l = f\frac{L}{D}\frac{V^2}{2g}$$

$$Q = V \times A$$

$$V = \frac{Q}{A}$$

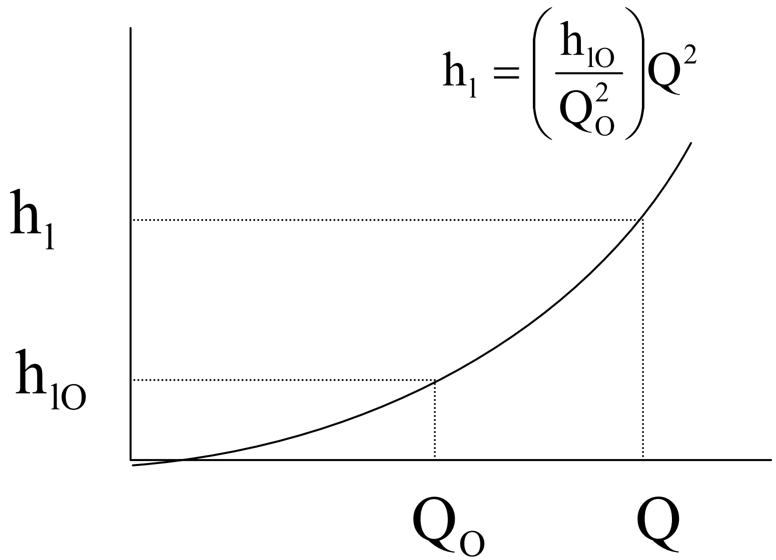
$$h_l = f\frac{L}{D}\frac{\left(\frac{Q}{A}\right)^2}{2g}$$

$$h_l = f\left(\frac{L}{D}\frac{}{2gA^2}\right)Q^2$$

$$h_l = \text{cons}\tan\text{t} \times Q^2$$

$$\text{cons}\tan\text{t} = \frac{h_{lO}}{Q_O^2} = \frac{h_l}{Q^2}$$

$$h_l = \left(\frac{h_{lO}}{Q_O^2}\right)Q^2$$



$$h_l = \left(\frac{h_{lO}}{Q_O^2}\right) \times Q^2$$

$$A: \quad h_l = \left(\frac{20}{50^2}\right) Q^2 = .008Q^2$$

$$Q_A = \left(\frac{h_{1A}}{.008}\right)^5$$

$$B: \quad h_l = \left(\frac{25}{30^2}\right) Q^2 = .0278Q^2$$

$$Q_B = \left(\frac{h_{lB}}{.0278}\right)^5$$

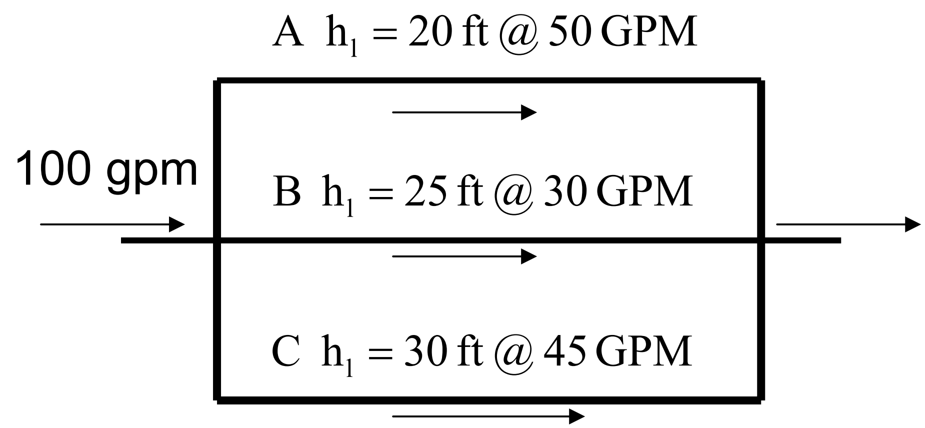
$$C: \quad h_l = \left(\frac{00}{45^2}\right) Q^2 = .01481Q^2$$

$$Q_C = \left(\frac{h_{lC}}{.01481}\right)^5$$

$$Q_{total} = Q_A + Q_B + Q_C, h_{1A} = h_{lB} = h_{lC} = h_l$$

$$Q_{total} = \left(\left(\frac{h_{lA}}{.008}\right)^{.5} + \left(\frac{h_{lB}}{.0278}\right)^{.5} + \left(\frac{h_C}{.01481}\right)^{.5}\right) = 25.397h_l^{.5}$$

$$h_l = \left(\frac{Q_{total}^2}{25.397}\right)$$



$$Q_A?,\ Q_B?,\ Q_C?$$

at 100 gpm, $\quad h_l = \left(\frac{100^2}{25.397}\right) = 15.5$ ft

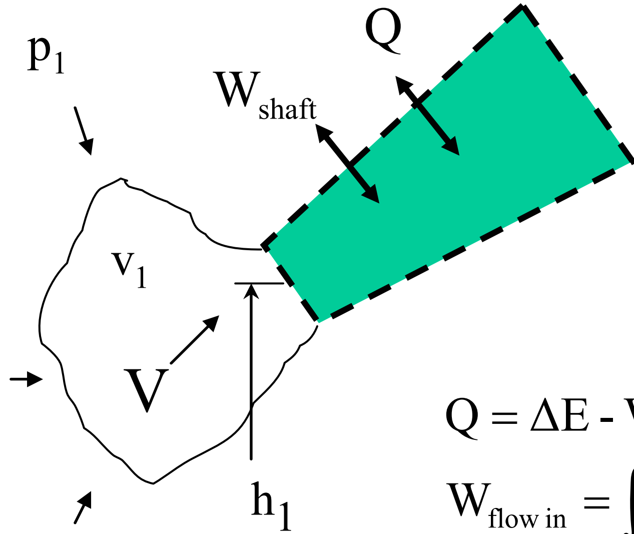
$$Q_A = Q_A = \left(\frac{h_{1A}}{.008}\right)^5 = \left(\frac{15.5}{.008}\right)^{.5} = 44.\text{ gpm}$$

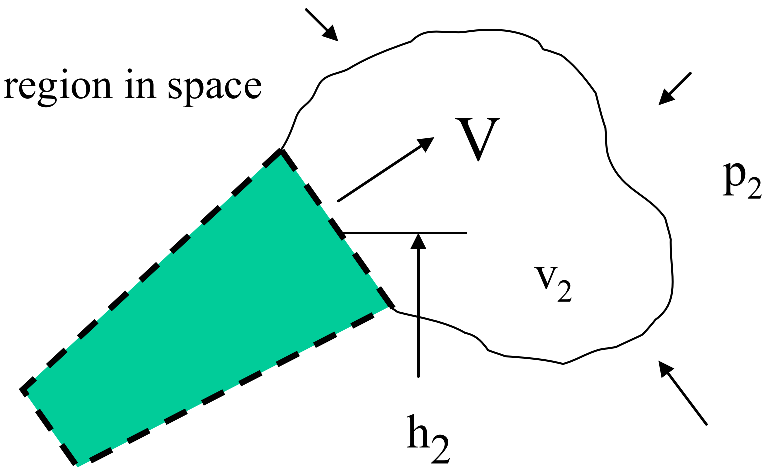
$$Q_B = Q_B = \left(\frac{h_{1B}}{.0278}\right)^5 = \left(\frac{15.5}{.0278}\right)^{.5} = 23.6.\text{ gpm}$$

$$Q_c = Q_c = \left(\frac{h_{1c}}{.01481}\right)^5 = \left(\frac{15.5}{.01481}\right)^{.5} = 32.4\text{ gpm}$$

Steady Flow Energy Equation

Open, steady flow thermodynamic system - a region in space





$Q = \Delta E - W$ First Law q and in is + by Smits convention

$$W_{\text{flow in}} = \int p dV = p_1\left(V_{1\ \text{initial}} - V_{1\ \text{final}}\right) = p_1 V_1 = m\, p_1 v_1$$

$$W_{\text{flow out}} = m\, p_2 v_2$$

$$W = W_{\text{shaft}} + W_{\text{flow in}} + W_{\text{flow out}} = W_{\text{shaft}} + m\, p_1 v_1 - m\, p_2 v_2$$

$$E = U(T) + KE + PE = U(T) + \frac{V^2}{2g} + h$$

$$Q = m(u_1 + p_1 v_1 + \frac{V^2}{2} + h_1) - m(u_2 + p_2 v_2 + \frac{V^2}{2g} + h_2) - W_{\text{shaft}}$$

$$Q = m\Delta(u + pv + \frac{V^2}{2} + h) - W_{\text{shaft}}$$

FIRST LAW

$$Q = m \times \Delta(u + pv + \frac{V^2}{2g} + z) - W_{shaft}$$

$$\mu = 0 \Rightarrow \Delta T = 0 \Rightarrow c_v \Delta T = u = 0$$

$$Q = m \times \Delta(\frac{p}{\rho} + \frac{V^2}{2g} + z) - W_{shaft}$$

frictional head losses end up as heat

$$l_f = \Delta\left(\frac{p}{\rho} + \frac{V^2}{2g} + z\right) - \frac{W_{shaft}}{m} \qquad (10-1a)$$

work done on the system is negative

BERNOULLI'S EQUATION

$$l_f = \Delta\left(\frac{p}{\rho} + \frac{V^2}{2g} + z\right) - \frac{W_{shaft}}{m}$$

$$\text{for} \quad l_f = 0, \qquad W = 0$$

$$0 = \Delta\left(\frac{p}{\rho} + \frac{V^2}{2g} + z\right)$$

weight flow = 10 lb_m/sec

$$\frac{p_1}{\rho}+\frac{V_1^2}{2}+z_1=\frac{p_2}{\rho}+\frac{V_2^2}{2}+z_2-\frac{W}{m}$$

$$\frac{(17+14.7)\times 144}{62.4}+\frac{V_1^2}{2\times 32.2}+0$$

$$=\frac{(8+14.7)\times 144}{62.4}+\frac{V_2^2}{2\times 32.2}+12\times 25-\frac{W}{m}$$

$$73.15=52.38+300-\frac{W}{m}$$

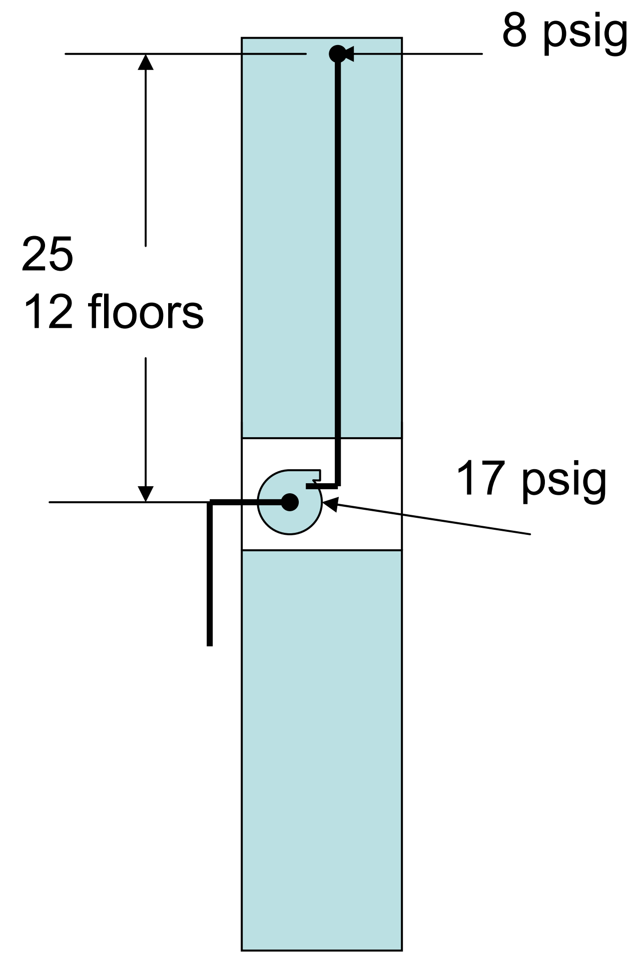
$$\frac{W}{m}=-279.23\,\frac{\text{ft lb}_f}{\text{lb}_m}$$

$$W=-279.23\times 10=-2792.3\,\frac{\text{ft lb}}{\text{sec}}$$

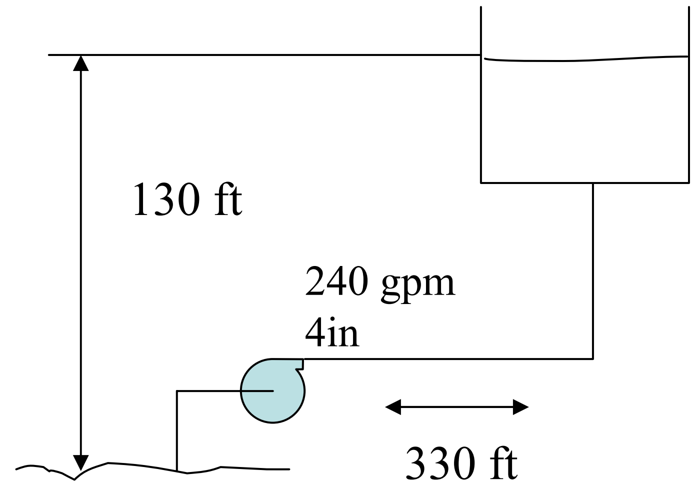
$$W=-\frac{2792.3\,\frac{\text{ft lb}}{\text{sec}}}{550\,\frac{\text{ft lb}}{\text{sec HP}}}=\text{-5.08 HP}$$

work added to system

$$W=-3.8\text{ KW}$$



10-1. The system shown in Fig. 10-50 transfers water to the tank at a rate of 240 gpm (0.015 m^3/s) through standard commercial steel 4 in. pipe. The total equivalent length of the pipe is 330 ft (100 m). The increase in elevation is 130 ft (40 m). Compute (a) the work done on the water and (b) the power delivered to the water, and (c) sketch the system characteristic.



10-1

assume 60° F water

$$\mu = 2.713\,\frac{lb_m}{ft\,hr},\ \rho = 62.37\,\frac{lb}{ft^3}\quad \text{page 586 Table A}-1a$$

$$\nu = \frac{\mu}{\rho} = \frac{lb_m}{ft\,hr}\times\frac{1}{3600\,sec/hr}\times\frac{ft^3}{lb_m} = \frac{ft^2}{sec}$$

$$m = \frac{240\,gpm\times.1337 ft^3/gpm\times 62.37\,lb/ft^3}{60\,sec/hr} = 33.36\,lb/sec$$

$$A = \frac{\pi D}{4} = \frac{3.1416}{4}\times\left(\frac{4.026}{12}\right)^2 = .0884\,ft^2$$

$$V = \frac{Q}{A} = \frac{240\,gpm\times.1337\,ft^3/gpm}{60\,sec/hr\times.0884\,ft^2} = 6.05\,ft/sec$$

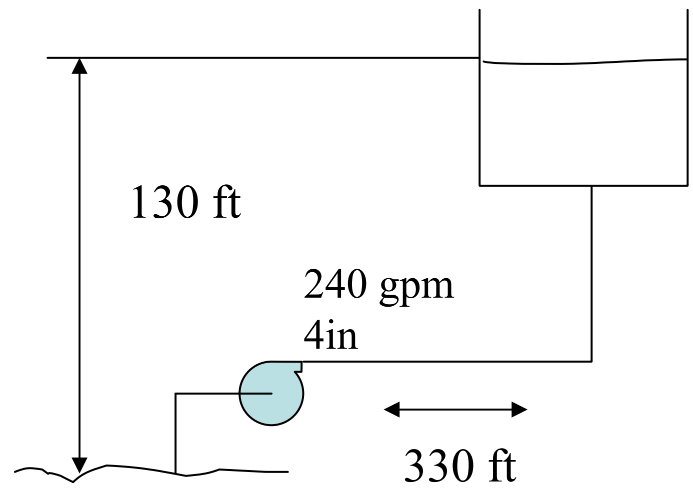
$$R_e = \frac{\rho VD}{\mu} = \frac{62.37\,lb/ft^3\times 6.05\,ft/sec\times\left(\frac{4.026}{12}\right)}{2.713\,lb_m/ft\,hr\times 3600\,sec/hr}$$

$$R_e = 167{,}750$$

$$\frac{\varepsilon}{D} = .0004\quad \text{Table 10-1 Commercial Steel}$$

$$@\ R_e \text{ and } \frac{\varepsilon}{D},\quad f = .019\quad \text{Figure 10}-1$$

$$l_f = f\,\frac{L}{D}\left(\frac{V^2}{2g}\right) = .19\times\left(\frac{330ft}{4.026/12}\right)\times\frac{6.05^2}{2\times 32.2} = 10.62\ \ ft$$



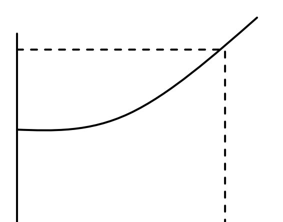
$$\frac{p_1}{\rho_1}+\frac{V_1^2}{2g_c}+z_1\frac{g}{g_c} = \frac{p_2}{\rho_2}+\frac{V_2^2}{2g_c}+z_2\frac{g}{g_c}+W+\frac{g}{g_c}l_f$$

$$W = m\times h = m\times(z_2 - z_1 + l_f)$$

$$W = 33.36\,lb(130\,ft + 10.62\,ft) = 4692\,ftlb/sec$$

$$W = \frac{4692\,ftlb/sec}{550\,ft\,lb/\,sec/\,HP} = 8.529\,HP$$

$$W = 8.529\,HP\times.7457\,KW/HP = 6.36\,KW$$



$$h = 130 + \left(\frac{h_o}{Q_o^2}\right)Q^2$$

$$h = 130 + .0001844\,Q^2$$

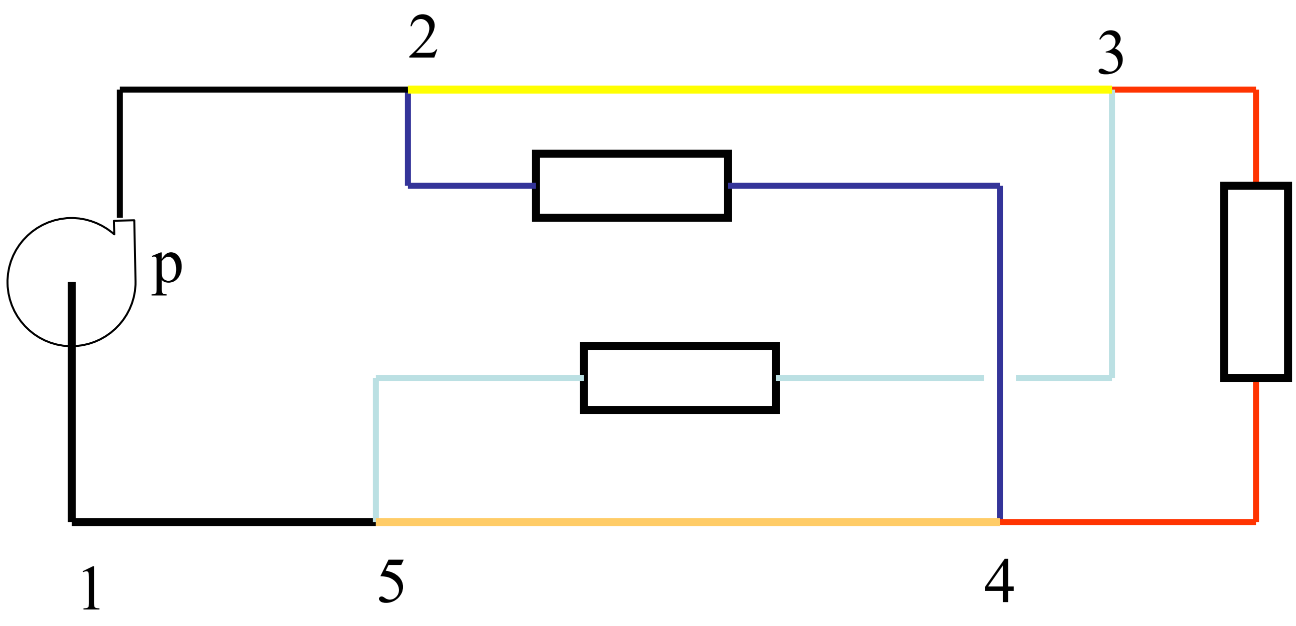
A 3-zone heating system uses hot water passing through the piping network shown. The heater increases water temperature 20 F. All pipes are copper type L.

a) What is the total head added by the pump?

b) Assuming a pump efficiency of 45%, what size electric motor should be used?

C) What is the heat flow rate into the water?

Circuits	l eq, ft	D,in	Q, gpm
5-1-p-2	40	2.5	60
2-4	70	1.5	20
2-3	55	2.0	40
3-4	65	1.5	20
3-5	60	1.5	20
4-5	50	2.0	40



heating $\Rightarrow T = 140^{\circ}F$, $\mu = 1.129$, $\rho = 61.38 lb/ft^3$

Section	L	D	Q	A	V	N_{re}	f	h_f	$Q \times H$
				$\frac{3.14 D^2}{4 \times 144}$	$\frac{Q \times .1337}{A}$	$\frac{VD}{\nu}$			
5−1−P−2	40.	2.495	60	.03395	3.93	1.58×10^5	.022	.761	45.66
2−4	70.	1.527	20	.01272	3.50	8.6×10^5	.0185	1.936	38.72
2−3	55.	2.009	40	.02382	3.74	1.2×10^5	.0186	1.249	49.96
3−4	65.	1.527	20					1.798	35.96
3−5	60.	1.527	20					1.659	33.18
4−5	50.	2.009	40					1.135	45.40
									248.88

a) Head/section

$P-2-4-1 = .761 + 1.936 + 1.135 = 3.071 \text{ ft}$

$P-3-4-5-1 = .761 + 1.249 + 1.659 = 3.669 \text{ ft}$

$P-2-3-5 = .761 + 1.249 + 1.798 + 1.135 = 4.943 \text{ ft maximum head}$

b)

$$\text{Power} = \sum m \times H = \sum \frac{Q \times .1337 \times \rho}{60 \text{ sec/min}} \times H = \frac{.1337 \times 61.4}{60} \sum QH$$

$$\text{Ideal Power} = \frac{.1337 \times 61.4}{60} \times \frac{248.88 \times .7457 \text{ KW/HP}}{550 \text{ ftlb/HP}} = .0462 \text{ KW}$$

$$\text{ActualPower} = \frac{\text{Ideal Power}}{\eta} = \frac{.0462 \text{HP}}{.45} = .103 \text{ KW}$$

c) $$Q = m(h_2 - h_1) = \frac{60 \text{gpm} \times .1337 \text{ft}^3/\text{gal} \times 61.4(117.89 - 97.9)}{60 \text{sec/min}} = 590{,}751 \text{BTU/hr}$$