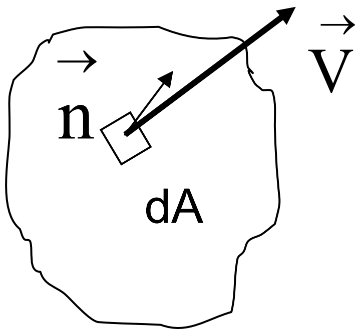
Control Volumes
Chapter 5.4, 5.1, 5.2, 3.4.5, 5.3, 3.6, 4.7 5.5

CONTINUITY EQUATION CONSERVATIVE INTEGRAL FORM

$\vec{V}$, velocity vector

$\vec{n}$, unit normal vector



control volume, 3D region in space
open thermodynamic system

$\oiint_A \rho\, \vec{n} \bullet \vec{V}\, dA$ net of mass entering and leaving the control volume. (Smist convention mass inflow is -)

$\frac{\partial}{\partial t} \oiiint_{vol} \rho \; d\,vol$ change in mass inside the control volume

Mass Balance

Net mass flow = Change in mass flow

$$\oiint_A \left(\rho\, \vec{n} \bullet \vec{V} \right) dA + \frac{\partial}{\partial t} \oiiint_{vol} \rho \; d\,vol = 0 \quad (5.3)$$ Continuity Equation in integral (conservative) form

Steady flow $\oiint_A \left(\rho\, \vec{n} \bullet \vec{V} \right) dA = 0$

Steady, 1D flow $w \int \rho\, V_x dy = 0$

Steady, 1D, constant density flow $\rho w \int V_x dy = 0$

$$\rho w V_{x2} - \rho w V_{x1} = 0$$

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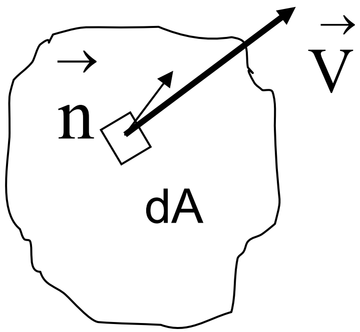

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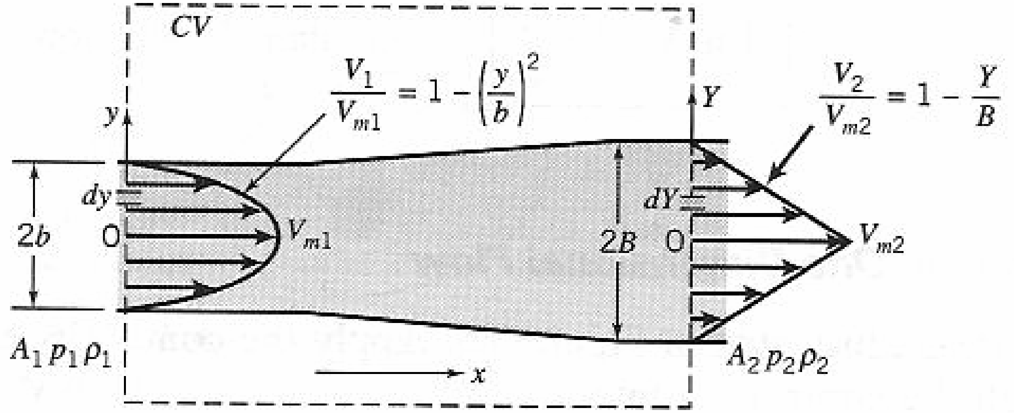


$$\oiint_A \left(\rho \vec{n} \bullet \vec{V} \right) dA + \frac{\partial}{\partial t} \oiiint_{vol} \rho \; d\,vol = 0 \quad (5.3)$$

steady

$$\oiint_A \left(\rho \vec{V} \right) dA = 0$$

steady, 1 D, variable density

$$\rho w \int V(x) dy = 0$$

$$2\rho_1 w \int_0^b V_1(x) dy + 2\rho_2 w \int_0^B V_2(x) dy = 0$$

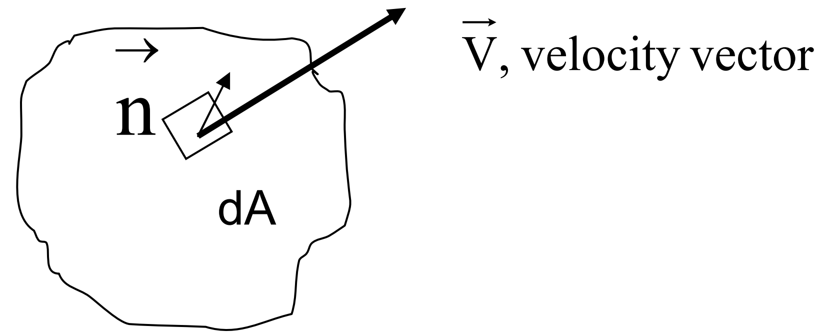
$$2\rho_1 w \int_0^b V_{m1} \left(1 + \left(\frac{y}{b} \right)^2 \right) dy + 2\rho_2 w \int_0^B Vm_2 \left(1 + \frac{Y}{B} \right) dy = 0$$

$$\rho_1 V_{m1} \left(y + \frac{y^3}{3b^2} \right)_0^b = \rho_2 Vm_2 \left(Y + \frac{Y^2}{2B} \right)_0^B$$

$$\rho_1 V_{m1} \frac{2b}{3} = \rho_2 Vm_2 \frac{B}{2}$$

$$\frac{V_{m2}}{V_{m1}} = \frac{4b}{3B} \frac{\rho_1}{\rho_2}$$

MONENTUM EQUATION - CONSERVATIVE INTEGRAL FORM



$$F = ma = \frac{d(mV)}{dt} = \text{change in momentum}$$

FORCES

Body Force $\iiint_{vol} \rho\, f\, d\,vol,$

where f is the body force constant

Pressure Force $\oiint_A p\, dA$

Viscous Force $\oiint_A \tau\, dA$

control volume
open thermodynamic system
region in space

Net of mometum in and outof CV $\oiint_A \left(\rho\, \vec{n} \bullet \vec{V} \right) \vec{V} dA$

Change of Momentum in CV $\iiint_{vol} \frac{\partial \left(\rho\, \vec{V} \right)}{\partial t} d\,vol$

FORCE BALANCE

Momentum Change + Body Force + Pressure Force + Viscous Force = 0

$$\oiint_A \left(\rho\, \vec{n} \bullet \vec{V} \right) V dA + \iiint_{vol} \frac{\partial \left(\rho\, \vec{V} \right)}{\partial t} d\,vol + \iiint_{vol} \rho\, f\, d\,vol + \oiint_A p\, dA + \oiint_A \tau\, dA = 0 \quad (5.16)$$

MONENTUM EQUATION - CONSERVATIVE INTEGRAL FORM

Unsteady, viscous, 3D, changing body force

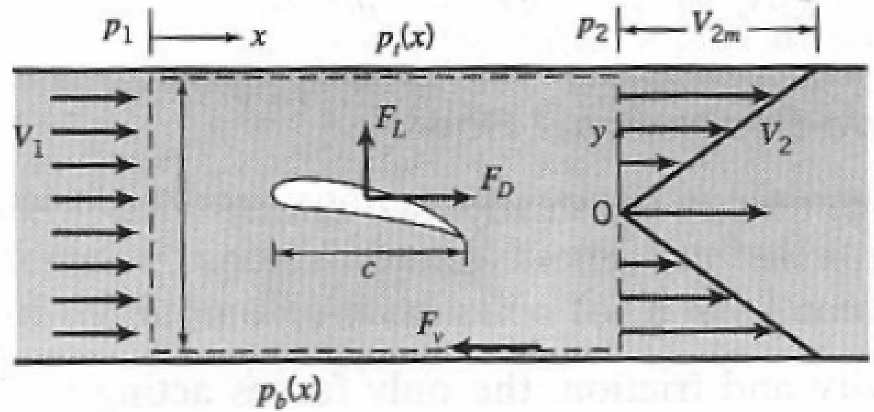
$$\oiint_A \left(\rho \vec{n} \bullet \vec{V} \right) V \, dA + \oiiint_{vol} \frac{\partial \left(\rho \vec{V} \right)}{\partial t} d\,vol + \oiiint_{vol} \rho f \, d\,vol + \oiint_A p \, dA + \oiint_A \tau \, dA = 0 \quad (5.16)$$

Steady, inviscid, no body force change, inviscid

$$\oiint_S \left(\rho \vec{n} \bullet \vec{V} \right) V dA + \oiint_A p \, dA = 0$$

Steady, inviscid, no body force change, inviscid, 1D (Q1D)

$$w \int (\rho V_x) V_x dy + w \int p dy = 0$$



steadyflow,1D, incompressible, inviscid

$$\sum F_x = 0$$

$$-F_D - \int p dA = \int (\rho V_x) V_x dA$$

$$-F_D + (p_1 - p_2)hw = -\rho V_{x1}^2 hw + 2w\int_0^{\frac{h}{2}} \rho V_{x2}^2 dy$$

$$F_D = (p_1 - p_2)hw + \rho V_{x1}^2 hw - \frac{1}{3}\rho V_{x2}^3 wh$$

$$\frac{F_D}{\rho V_{x1}^2 hw} = \frac{(p_1 - p_2)}{\frac{\rho V_{x1}^2}{2}} - \frac{2}{3}$$

Conventional Drag Coefficient

$$C_D = \frac{F_D}{\rho V_{x1}^2 A_{wing}} = \frac{F_D}{\rho V_{x1}^2 cw}$$

$$\sum F_y = 0, \text{ inviscid flow}$$

$$\text{Euler Normal Equation} \Rightarrow \frac{dp}{dy} = 0$$

$$-F_L - \int p dA = \int (\rho V_y) V_y dA$$

$$-F_L - \int p dA = 0$$

$$-F_L + w\int_1^2 p_{top} dx - w\int_1^2 p_{bottom} dx = 0$$

$$\frac{F_L}{\rho V_1^2 wh} = \frac{1}{\rho V_1^2 h}\left(\int_1^2 p_{top} dx - \int_1^2 p_{bottom} dx\right)$$

conventional lift coefficient

$$C_L = \frac{F_L}{\frac{1}{2}V_1^2 A_{wing}} = \frac{F_L}{\frac{1}{2}V_1^2 cw}$$

MATH REVIEW

Substantial or Total Deriviative (Appendix A – 9 page 481)

Eulerian prespective

$$\frac{D(\,)}{Dt} = \left(\frac{d(\,)_x}{dt} + u\frac{d(\,)_x}{dx} + v\frac{d(\,)_x}{dy} + w\frac{d(\,)_x}{dz} \right)\vec{i}$$

$$+ \left(\frac{d(\,)_y}{dt} + u\frac{d(\,)_y}{dx} + v\frac{d(\,)_y}{dy} + w\frac{d(\,)_y}{dz} \right)\vec{j}$$

$$+ \left(\frac{d(\,)_z}{dt} + u\frac{d(\,)_z}{dx} + v\frac{d(\,)_z}{dy} + w\frac{d(\,)_z}{dz} \right)\vec{k}$$

$$\frac{D}{Dt} = \frac{\partial}{dt} + \vec{V} \bullet \nabla$$

$$\vec{V} \bullet \nabla = u\frac{\partial}{dx} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

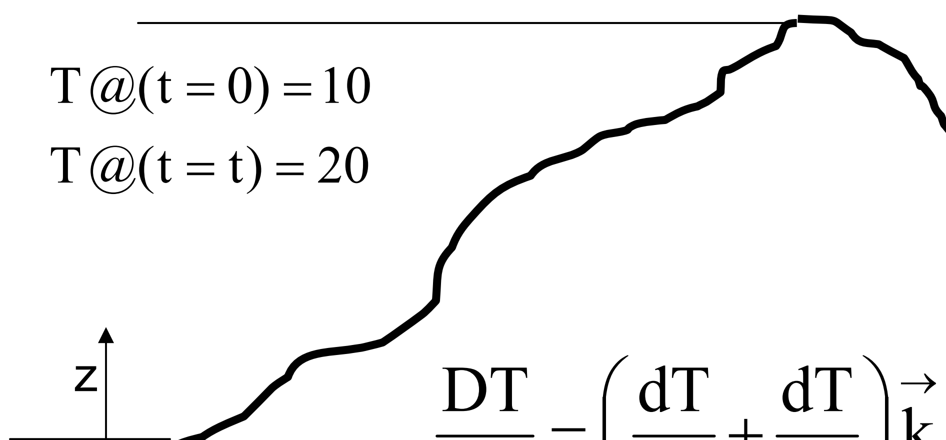
Use:

relate Eulerian to Lagrangian

particle path to control voume

$T@(t=0)=10$

$T@(t=t)=20$



$T@(t=0)=50$

$T@(t=t)=60$

$$\frac{DT}{dt} = \left(\frac{dT}{dt} + \frac{dT}{dz} \right)\vec{k}$$

REYNOLDS TRANSPORT THEORM

Continuity $$\frac{\partial}{\partial t}\oiiint_{vol} \rho \; d\,vol + \oiint_{A} \left(\rho\, \vec{n} \bullet \vec{V}\right) dA = 0$$

rate of change of mass in CV — net flow of mass in CV

Momentum $$\frac{\partial}{\partial t}\oiiint_{vol} \left(\rho\, \vec{V}\right) d\,vol + \oiint_{A} \left(\rho\, \vec{n} \bullet \vec{V}\right) \vec{V}\, dA = -\oiiint_{vol} \rho\, f \; d\,vol - \oiint_{A} p\, dA - \oiint_{A} \tau\, dA$$

rate of change of momentum in CV — net flow of momentum in CV

$$B = \text{property valune}, \quad b = \frac{B}{m}\ \text{property value per mass}$$

$$\frac{D(B)}{Dt} = \frac{\partial}{\partial t}\oiiint_{vol} \rho\,(b)\, d\,vol + \oiint_{A} \left(\rho\, \vec{V}\right)(b)\; dA$$

rate of change of total B following a particle	rate of change of b in CV	net flow of b thru CV

Eulerian	Lagarngian
follow particle	control volume

$$\frac{D(B)}{Dt} = \frac{\partial}{\partial t}\oiiint_{vol} \rho\,(b)\,d\,vol + \oiint_{A} \left(\rho\,\vec{V}\right)(b)\,dA$$

For $B = m, \quad b = \frac{B}{m} = \frac{m}{m} = 1$

left side of the continuity equation (5.3) results

If $B = \text{momentum}, m\vec{V}, \; b = \frac{B}{m} = \frac{m\vec{V}}{m} = \vec{V}$

left side of the the monentum equation (5.16) results

B can be any flow property transported by the fluid,

mass - momentum - enthalpy

kinetic energy - internal energy

ENERGY EQUATION FROM THE TRANSPORT THEORM

1st Law of Thermodynamics

$$Q + W = \Delta E$$

$$Q + W = \frac{DE}{DT}$$

$$\text{for } E = m\left(u + pv + \frac{V^2}{2} + gz\right), \quad e = \frac{E}{m}$$

$$\frac{D(B)}{Dt} = \frac{\partial}{\partial t}\oiiint_{vol} \rho\,(b)\,d\,vol + \oiint_{A} \left(\rho\,\vec{V}\right)(b)\,dA$$

$$Q + W = \frac{\partial}{\partial t}\oiiint_{vol} \rho\left(u + pv + \frac{V^2}{2} + gz\right) d\,vol + \oiint_{A}\left(\rho\,\vec{V}\right)\left(u + pv + \frac{V^2}{2} + gz\right) dA$$

rate change of energy in CV — net flow of energy in CV

W includes $W_{\text{pressure and}}$, W_{viscous}

ENERGY EQUATION CONSERVATIVE INTEGRAL FORM

$$\text{First Law}\quad Q = \Delta E + W = \Delta E + W_{shaft} + W_{viscous} + W_{pressure} + W_{body}$$

$$\text{Work} = \text{Force} \times \text{Velocity}$$

$$W_{shaft}$$

$$\text{Work}_{pressure} \quad \oiint_A \left(\frac{p}{\rho}\, dA \right) \vec{V}$$

$$\text{Work}_{body} \quad \oiiint_{Vol} (\rho\, f\, d\, vol) \vec{V}$$

$$\text{Work}_{viscous} \quad \oiint_A (\tau\, dA) \vec{V}$$

$$\text{Net Energy into control volume} \quad \oiint_A (\rho\, V\, dA) \left(u + \frac{V^2}{2} \right) d\, vol$$

$$\text{Change in energy inside the control volume} \quad \frac{\partial}{\partial t} \oiiint_{Vol} \rho \left(u + \frac{V^2}{2} \right) d\, vol$$

$$\text{Heat addition} \quad \oiint_A \vec{q}\, dA$$

$$\text{Internal energy,} \quad u = c_v T$$

ENERGY EQUATION - CONSERVATIVE INTEGRAL FORM

First Law of Thermodynamics

$$Q = \Delta E - W$$

Q and W transfered to control volume = + by Smits convention

$$Q = \Delta E_{\substack{\text{net energy in}\\ \text{control}\\ \text{volume}}} + \Delta E_{\substack{\text{change in}\\ \text{energy in}\\ \text{control volume}}} - W_{shaft} - W_{viscous} + W_{pressure} + W_{body}$$

$$Q = \oiint_A \left(\rho\,\vec{V}\,dA\right)\left(u + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)dA + \frac{\partial}{\partial t}\iiint_{vol} \rho\left(u + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)d\,vol$$

$$- W_{shaft} - \oiint_A \left(\tau\,dA\right)\vec{V}$$

3D, inviscid, no change in body force, $W_{shaft} = 0$, variable density

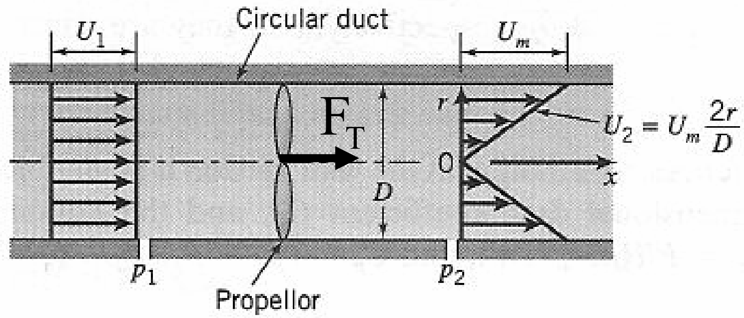
$$Q + W = \oiint_A \left(\rho\,\vec{V}\right)\left(u + \frac{p}{\rho} + \frac{V^2}{2}\right)dA + \frac{\partial}{\partial t}\iiint_{vol} \rho\left(u + \frac{p}{\rho} + \frac{V^2}{2}\right)d\,vol \qquad (5.22)$$

$Q + W$ = net energy across CV boundary + change in energy in CV

Comparing to the Reynolds Transport Theorm,

$$B = \left(u + \frac{p}{\rho} + \frac{V^2}{2}\right) \text{ the transported fluid property}$$

5.17 A propeller is placed in a constant area circular duct of diameter D, as shown in Figure P5.17. The flow is steady and the fluid has a constant density ρ. The pressure p_1 and p_2 are uniform across the entry and exit areas, and the velocity profiles are as shown. Find the thrust T produced by the propeller on the fluid in terms of U_1, U_m, ρ and D, and p_1 and p_2.



$$A_1 = A_2 = \int dA = 2\int_0^R \pi r dr = \pi R^2 = \pi \frac{D^2}{4}$$

$$dA = 2\pi r dr$$

Continuity Equation

steady flow, constant density

$$\oiint_A \left(\rho \vec{V}\right) dA = 0$$

$$\int_0^R \rho\, U_1 dA = \int_0^R \rho_2 U_2 dA$$

$$\rho\, U_1 \pi \frac{D^2}{4} = \int_0^R \rho \left(\frac{U_m 2r}{D}\right) 2\pi r dr$$

$$\rho\, U_1 \pi \frac{D^2}{4} = \rho\, \frac{U_m 4\pi}{D} \int_0^{\frac{D}{2}} r^2 dr$$

$$\rho\, U_1 \pi \frac{D^2}{4} = \rho\, \frac{U_m 4\pi}{D} \frac{D^3}{3\times 8}$$

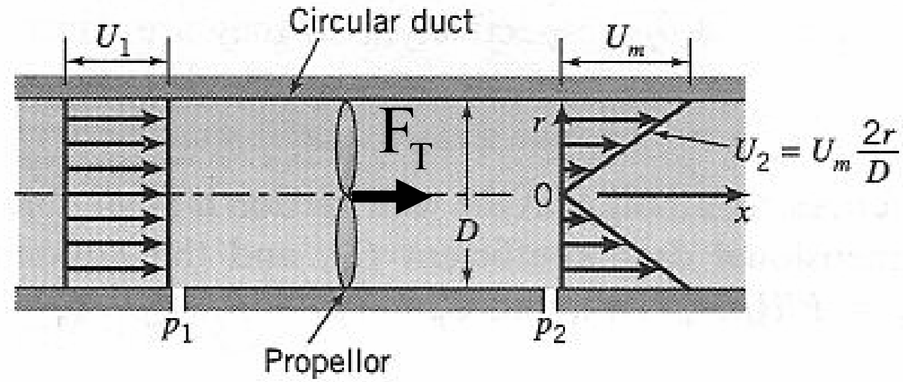
$$U_m = \frac{3}{2} U_1$$

5.17

Momentum Equation

$$\oint\!\!\oint_S \left(\rho\, \vec{n} \bullet \vec{V}\right) V dA + \oint\!\!\oint_A p\, dA + F_T = 0$$

1 D inviscid, no body force change, inviscid



$$\int p_2 dA - \int p_1 dA + \int_0^{\frac{D}{2}} (\rho U_2) V_2 dA - \int_0^{\frac{D}{2}} (\rho U_1) U_1 dA - F_T = 0$$

F_T - force on fluid

$$\frac{\pi D^2}{4}(p_2 - p_1) + \int_0^{\frac{D}{2}} \rho \left(\frac{U_m 2r}{D}\right)^2 2\pi r dr - \rho \frac{\pi D^2}{4} U_1^2 - F_T = 0$$

$$A_1 = A_2 = \int dA = 2\int_0^R \pi r dr$$

$$= \pi R^2 = \pi \frac{D^2}{4}$$

$$\frac{\pi D^2}{4}(p_2 - p_1) + \rho 2\pi \left(\frac{U_m 2}{D}\right)^2 \int_0^{\frac{D}{2}} r^3 dr - \rho \frac{\pi D^2}{4} U_1^2 \text{ - } F_T = 0$$

$$\frac{\pi D^2}{4}(p_2 - p_1) + \rho 2\pi \left(\frac{U_m 2}{D}\right)^2 \frac{D^4}{4 \times 8} - \rho \frac{\pi D^2}{4} U_1^2 + F_T - F_T = 0$$

$$\frac{\pi D^2}{4}\left((p_2 - p_1) + \frac{\rho U_m^2}{4} - \rho U_1^2\right) - F_T = 0$$

$$F_T = \frac{\pi D^2}{4}\left((p_2 - p_1) + \frac{\rho U_m^2}{4} - \rho U_1^2\right)$$

5.22 The entrance region of a parallel, rectangular duct flow is shown in Figure P5.22. The duct has a width W and a height D. The fluid density is constant, and the flow is steady. The velocity variation in the boundary layer of thickness δ at station 2 is assumed to be linear, and the pressure at any cross-section is uniform. Ignore the flow over the side walls of the duct (that is, $W \gg D$).

(a) Using the continuity equation, show that $U_1/U_2 = 1 - \delta/D$.

(b) Find the pressure coefficient $C_p = (p_1 - p_2)/(\frac{1}{2}\rho U_1^2)$.

(c) Show that

$$\frac{-F_v}{\frac{1}{2}\rho U_1^2 WD} = \frac{U_2^2}{U_1^2}\left(1 - \frac{8}{3}\frac{\delta}{D}\right) - 1$$

where F_v is the total viscous force acting on the walls of the duct.

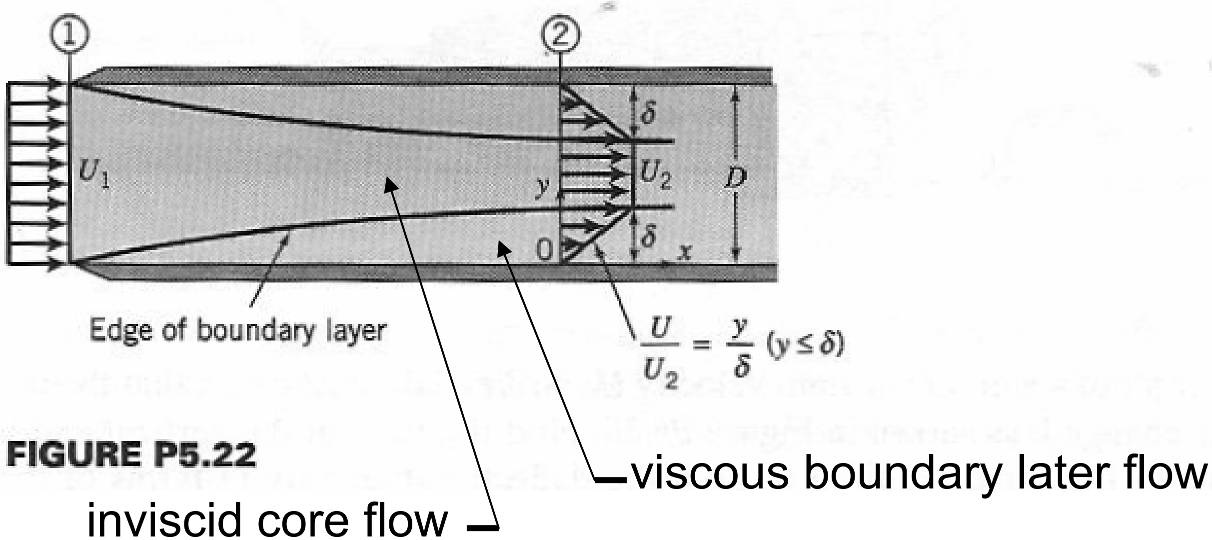


FIGURE P5.22

Contunity Equation

$$\oiint_A \left(\rho \vec{V}\right) dA = 0$$

$$- UDw + 2\left(U_2 \frac{\delta}{2}\right)w + U_2\left(D - 2\delta\right)w = 0$$

$$U_1 = \frac{\delta}{D} U_2 + U_2\left(1 - \frac{2\delta}{D}\right)$$

$$\frac{U_1}{U_2} = 1 - \frac{\delta}{D}$$

In the inviscid flow
outside the boundary layer
no viscosity $\rightarrow$ Bernoulli Equation

$$p_1 + \frac{\rho U_1^2}{2} = p_2 + \frac{\rho U_2^2}{2}$$

$$p_1 - p_2 = \frac{U_2^2}{U_1^2} - 1$$

Momentum Equation

$$\oiint_A \left(\rho \vec{n} \bullet \vec{V}\right) V dA + \iiint_{vol} \frac{\partial\left(\rho \vec{V}\right)}{\partial t} d\,vol + \iiint_{vol} \rho\, f\, d\,vol + \oiint_A p\, dA + \oiint_A \tau\, dA \quad (5.16)$$

1D, steady, no body force change, inviscid

$$\oiint_A p\, dA + \oiint_A \left(\rho \vec{n} \bullet \vec{V}\right) V dA = 0$$

$$-F_V + \left(P_1 - P_2\right) = -\rho U_1^2 Dw + \rho U_2^2 (D - 2\delta) + 2\rho w \int_0^\delta U_2^2 dy$$

$$F_V = \frac{1}{2} \rho U_1^2 Dw \left(1 - \frac{U_2^2}{U_1^2}\left(1 - \frac{8\delta}{3D}\right)\right)$$

FLUX – transport

across CV boundaries

fluid properties

by flow

per unit time

volume, mass, momentum, kinetic energy, internal energy

$$\text{total volume flux} = \int \vec{n} \bullet \vec{V}\, dA, \qquad \text{x direction flux} = V \times A$$

$$\text{total momentum flux} = \int \left(\rho\, \vec{n} \bullet \vec{V} \right) \vec{V}\, dA$$

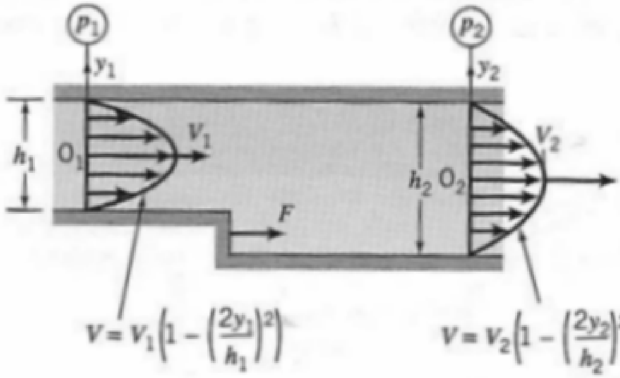
$$\text{x direction momentum flux} = w \int \left(\rho V_x \right) V_x\, dy$$

sometimes it is conceptually or computationally simpler to consider fluxes rather the mass, energy or momentum equations as a whole.

5.19 A fluid of constant density ρ enters a duct of width W and height h_1, with a parabolic velocity profile with a maximum value of V_1, as shown in Figure P5.19. At the exit plane, the duct has height h_2 and the flow has a parabolic velocity profile with a maximum value of V_2. The pressures at the entry and exit stations are p_1 and p_2, respectively, and they are uniform across the duct.

(a) Find V_2 in terms of V_1, h_1, and h_2.

(b) Find the magnitude and direction of the horizontal force F exerted by the fluid on the step in terms of ρ, V_1, W, p_1 and p_2, h_1 and h_2. Ignore friction. Note that at the point where the flow separates off the step, the flow streamlines can be assumed to be parallel: this observation provides information about the pressure on the vertical face of the step.



$$V_1\left[y_1 - \frac{4y_1^3}{3h_1^2}\right]_0^{\frac{h_1}{2}} = V_2\left[y_2 - \frac{4y_2^3}{3h_2^2}\right]_0^{\frac{h_2}{2}}$$

$$V_1\left[h_1 - \frac{4h_1^3}{8\times 3h_1^2}\right] = V_2\left[h_2 - \frac{4h_2^3}{8\times 3h_2^2}\right]V_1$$

$$V_2 = V_1\frac{h_1}{h_2}$$

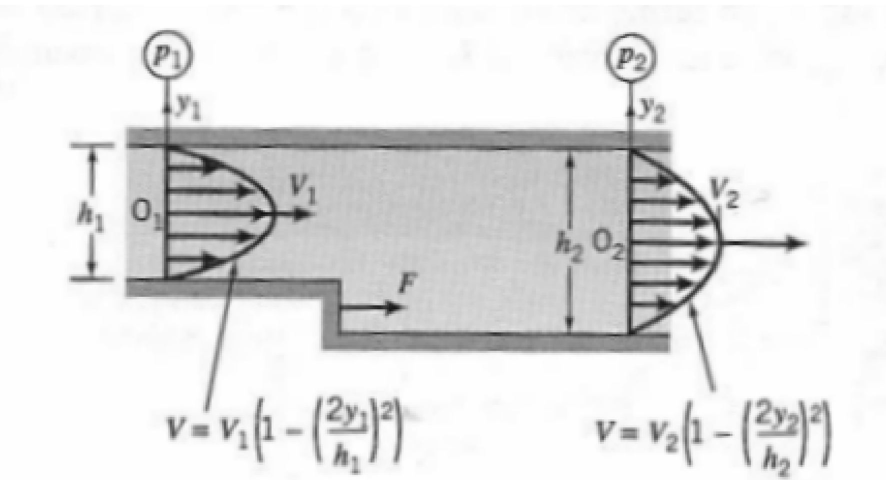
CONTINUITY EQUATION

$$\int \rho \vec{n}\,\vec{V}\,dA = 0$$

$$\rho w\int V_x\,dA = 0, \qquad dA = w\,dy$$

$$\rho w\int V_1\left[1-\left(\frac{2y_1}{h_1}\right)^2\right]dy = \rho w\int V_2\left[1-\left(\frac{2y_2}{h_2}\right)^2\right]dy$$

5.19



$$\text{Momentum Flux}_1 = 2\int_0^{\frac{h_1}{2}} (\rho V_1) V_1 dA = 2\rho\rho \int_0^{\frac{h_1}{2}} V_1^2 dy$$

$$= 2\rho\rho w_1^{\ 2} \int_0^{\frac{h_1}{2}} \left[1-\left(\frac{2y_1}{h_1}\right)^2\right]^2 dy = 2\rho\rho \int_0^{\frac{h_1}{2}} \left[1-\frac{8y_1^2}{h_1^2}+\frac{16y_1^4}{h_1^4}\right]$$

$$= 2\rho\rho w_1^{\ 2}\left[y_1-\frac{8y_1^3}{3h_1^2}+\frac{16y_1^5}{5h^4}\right]_0^{\frac{h_1}{2}}$$

$$= 2\rho\rho w_1^{\ 2}\left[\frac{h_1}{2}{}_1-\frac{h_1^3}{3h_1^2}+\frac{16h_1^5}{5\times 32h_1^4}\right]$$

$$\text{Momentum Flux}_1 = \rho w V_1^2\left[\frac{8h_1}{15}\right]$$

$$\text{Momentum Flux}_2 = \rho w V_2^2\left[\frac{8h_2}{15}\right]$$

$$F = w\left(p_1h_1 - p_2h\ \right) + \rho w V_1^2\left[\frac{8h_1}{15}\right] - \rho w V_2^2\left[\frac{8h_2}{15}\right]$$

$$F = w\left(p_1h_1 - p_2h\ \right) + \frac{8}{15}\rho w V_1^2 h_1\left[1-\frac{V_2^2}{V_x^2}\frac{h_2}{h_1}\right]$$

since, $V_2^2 = V_1^2\,\frac{h_1^2}{h_2^2}$,

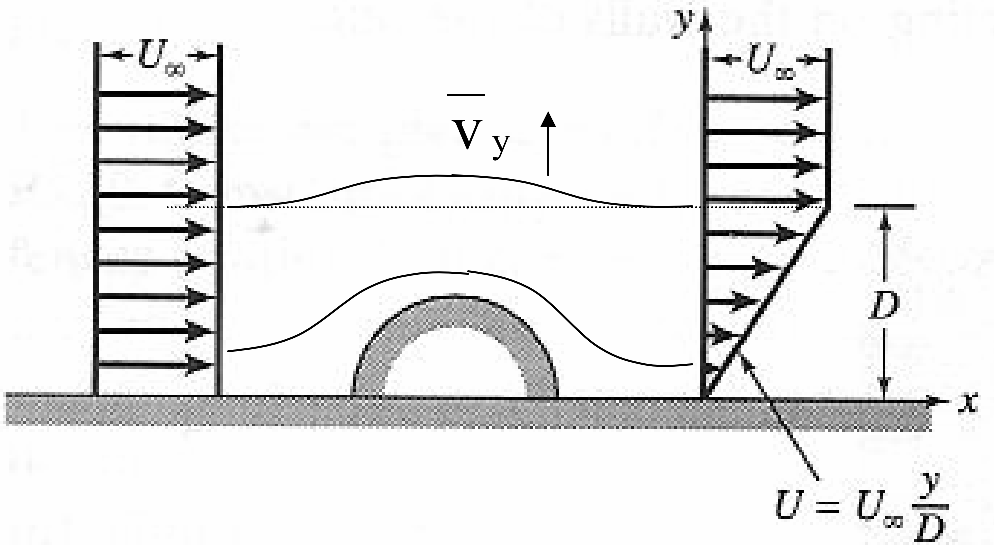
$$F = w\left(p_1h_1 - p_2h\ \right) + \frac{8}{15}\rho w V_1^2 h_1\left[1-\frac{h_1}{h_2}\right]$$

5.25 A model of a two-dimensional semicircular hut was put in a wind tunnel, and the downstream velocity profile was found to be as shown in Figure P5.25. Here, U_∞ is the freestream velocity, ρ is the air density, and D is the hut diameter. Assume that viscous effects and pressure variations can be neglected.

(a) Draw the flow pattern over the hut (remember that continuity must be satisfied).

(b) Find the average velocity in the y-direction over the horizontal plane located at $y = D$.

(c) Calculate the nondimensional force coefficient C_D, where $C_D = F/(\frac{1}{2}\rho U_\infty^2 D)$, and F is the force acting on the hut per unit span.



CONTINUITY EQUATION

$$\int \rho \vec{n} \bullet \vec{V} dA = 0$$

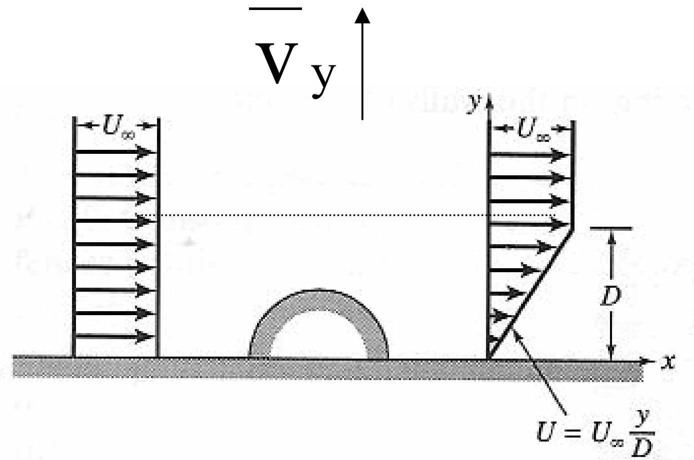
(mass in is – by convention)

$$-wDU_\infty + wL\bar{v}_y + w\int_0^D \left(U_\infty \frac{y}{D}\right) dy = 0$$

$$-DU_\infty + L\bar{v}_y - U_\infty D = 0$$

$$\bar{v}_y = \frac{U_\infty D}{2L}$$

5.25



MOMENTUM EQUATION

$$-F_x = \int (\rho U_\infty V_x) V_x dA$$

$$-F_x = -\rho U_\infty^2 Dw + w\int_0^L \left(\rho\, \bar{v}_y\right) U_\infty dx + w\int_0^D \left(U_\infty \frac{y^2}{D^2}\right) dy$$

$$F_x = \rho U_\infty^2 Dw\left[1 - \frac{1}{U_\infty^2 D}\,\bar{v}_y\, L - \frac{1}{D}\left(\frac{y^3}{3D^2}\right)_0^D\right]$$

$$F_x = \rho U_\infty^2 Dw\left[1 - \frac{1}{U_\infty^2 D}\left(\frac{U_\infty D}{2L}\right) - \frac{1}{D}\left(\frac{D^3}{3D^2}\right)\right]$$

$$F_x = \rho U_\infty^2 Dw\left[1 - \frac{1}{2} - \frac{1}{3}\right] = \frac{1}{6}\rho U_\infty^2 Dw$$

$$\frac{F}{\rho U_\infty^2 Dw} = \frac{1}{6}$$