Fluid Statics
Chapter 2.1-2.9
CONFINED FLUID (1.3.8 + 1.3.9)

A constant pressure from a force applied at a boundary is communicated throughout the fluid.

\[ \frac{F_2}{F_1} = \frac{\rho g A_2}{\rho g A_1} = \frac{A_2}{A_1} \]

HYDRAULIC JACK

NEGLIGENCE GRAVITY
A force of 445 N is exerted on lever $AB$. End $B$ is connected to a piston which fits into a cylinder having a diameter of 50 mm. What force $P$ must be exerted on the larger piston to prevent it from moving in its cylinder which has a 250 mm diameter?

$$A_1 = \pi r_1^2 = (\pi)(.025)^2 \quad A_2 = \pi r_2^2 = (\pi)(.125)^2$$

$$\frac{F_2}{F_1} = \frac{A_2}{A_1}$$

$$p_2 = [(2)(445)] \frac{\pi(.125)^2}{\pi(.025)^2} = 22,250 \text{ N} = 22.25 \text{ kN}$$
FLUID STATICS

Forces on submerged surfaces and on floating and submerged objects in a fluid at rest

Hydrostatic equation

\[ \Sigma F_y = b \Delta x \Delta y - \left( \rho + \frac{\partial \rho}{\partial y} \Delta y \right) \Delta x \Delta y - \gamma \Delta x \Delta y \Delta z = 0 \]

\[ \frac{\partial b}{\partial y} = -\gamma = -\rho g \]

\[ \Sigma F_x = 0 \Rightarrow \frac{\partial b}{\partial x} = 0 \quad \Sigma F_y = 0 \Rightarrow \frac{\partial b}{\partial y} = 0 \]

Pressure is constant on horizontal surface.

\[ \frac{\partial b}{\partial y} = -\gamma \] gives vertical variation
PRESSURE VARIATION FOR CONSTANT $\gamma$

$$\frac{dp}{dy} = -\gamma$$
$$p = -\gamma y + c$$
$$p_a = -\gamma h + c$$
$$p = p_a + \gamma(h-y)$$
$$p_0 = p_a + \gamma h$$
$$p = \text{ABSOLUTE} \quad p - p_a = \text{GAUGE} = p_g$$

ATMOSPHERIC PRESSURE VARIATION

FROM MEASUREMENTS $T = T_0 + m(y-y_0)$

$$\frac{dp}{dy} = -cg = -\frac{B}{RT} g$$
PERFECT GAS: $p = cRT$

$$\frac{dp}{p} = -\frac{g}{R} \frac{dy}{T_0 + m(y-y_0)}$$

$$\ln p \Bigg|_{p_0} = -\frac{g}{mR} \ln \left[ \frac{T_0 + m(y-y_0)}{T_0} \right]^{y-y_0}$$

$$\frac{p}{p_0} = \left[ \frac{T_0 + m(y-y_0)}{T_0} \right]^{-\frac{g}{mR}}$$
STANDARD ATMOSPHERE

AT SEA LEVEL

\[ T = 15 \, ^\circ C \]
\[ p = 101.325 \, kPa \]
\[ \rho = 1.2232 \, kg/m^3 \]

VARIATION WITH ELEVATION:
### TABLE C.5 Properties of the U.S. Standard Atmosphere (SI Units)*

<table>
<thead>
<tr>
<th>Altitude (m)</th>
<th>Temperature (°C)</th>
<th>Acceleration of gravity, $g$ (m/s²)</th>
<th>Pressure, $p$ [N/m²(abs)]</th>
<th>Density, $\rho$ (kg/m³)</th>
<th>Dynamic viscosity, $\mu$ (N·s/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1,000</td>
<td>21.50</td>
<td>9.810</td>
<td>1.139 E + 5</td>
<td>1.347 E + 0</td>
<td>1.821 E − 5</td>
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<tr>
<td>0</td>
<td>15.00</td>
<td>9.807</td>
<td>1.103 E + 5</td>
<td>1.225 E + 0</td>
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<tr>
<td>1,000</td>
<td>8.50</td>
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<td>2,000</td>
<td>2.00</td>
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<td>7.950 E + 4</td>
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<td>5.405 E + 4</td>
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<td>1.628 E − 5</td>
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<tr>
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<td>7.978 E + 1</td>
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</tr>
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<td>1.321 E − 5</td>
</tr>
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</table>

BAROMETER DEVICE FOR MEASURING ATMOSPHERIC PRESSURE

\[ p_a = p_v + \gamma h \]
\[ h = \frac{p_a - p_v}{\gamma} = \frac{(14.7 - 0.2)144}{13.6(62.4)} = 2.46' = 29.5'' \]
\[ p_a = 29.5 \text{ in. Hg} \]

FOR WATER \[ h = \frac{(14.7 - 0.4)144}{62.4} = 33' \]
MANOMETER: A FLUID-STATIC DEVICE FOR MEASURING PRESSURE DIFFERENCES

\[
\frac{dp}{dz} = -\gamma \\
p_2 - p_B = -\gamma (z_2 - z_1) \\
p_B = p_A = p_i \\
p_i - p_2 = \gamma (z_2 - z_1) = \gamma \delta h
\]

DETERMINE \( p_A \)

\( A \)

Pressure tank

\( p_2 - p_i = -\gamma (z_2 - z_1) \)
\( p_A - p_M = -\gamma d_1 \)
\( p_a - p_N = -\gamma H_d d_2 \)
\( p_M = p_N \)
\( p_A + \gamma d_1 = p_a + \gamma H_d d_2 \)
\( \therefore p_A - p_a = p_{Aq} = \gamma H_d d_2 - \gamma d_1 \)
\[ p_1 + \rho_1 g(z_1 - z_2) + \rho_2 g(z_2) = p_4 + \rho_3 g(z_4 - z_3) + \rho_2 g(z_3) \]

\[ p_1 + \rho_1 g(z_1 - z_2) = p_4 + \rho_3 g(z_4 - z_3) + \rho_2 g(z_3 - z_2) \]
TANK OPEN TO THE ATMOSPHERE

\[ p = p_a + \rho g (h - z) \]
CONSTANT PRESSURE

\( p_{\text{atm}} = \text{constant} \)

\[
F = p_{\text{atm}} \int dA = p_{\text{atm}} w \int_0^h dz = p_{\text{atm}} w h
\]

\( M_O = F \times \text{moment arm} \)

\[
M_O = \int p z \, dz = p_{\text{atm}} w \int_0^h z \, dz = p_{\text{atm}} w \frac{h^2}{2}
\]

\( M_O = p_{\text{atm}} w h \times \frac{h}{2} = F \times \text{distance to center of area} \)

LINEAR PRESSURE VARIATION

\[ p = \rho g (h - z) \]

\[
F = \int p \, dA = \int_0^h \rho g (h - z) \times w \, dz
\]

\[ F = \rho g \frac{h^2}{2} \times w \]

\[ F = p_{\text{ave}} \times \text{area} \]

\[ M_O = F \times \text{moment arm} \]

\[
M_O = \int p \times z \, dz
\]

\[
M_O = \int_0^h \rho g (h - z) \times z \times w \, dz
\]

\[
M_O = w \rho g \int_0^h (h - z) \, z \, dz = w \rho g \left[ h z - \frac{z^2}{2} \right]_0^h
\]

\[
M_O = w \rho g \frac{h^3}{6} = w \rho g \frac{h^2}{2} \times \frac{h}{3}
\]

\[ M_O = F \times \text{distance to centroid} \]
CLOSED PRESSURIZED TANK

\[ p = p_{\text{tank}} + \rho g (h - z) \]
**INCLINED SURFACE**

**FORCE**
\[ dA = w \, ds \]
\[ ds = \frac{dz}{\sin \theta}, \quad s = \frac{h}{\sin \theta} \]
\[ dA = \frac{w \, dz}{\sin \theta} \]
\[ p = \rho \, g \, (h - z) \]
\[ F = \int p \, dA = \int \rho \, g \, (h - z) \times w \, ds \]
\[ F = \int \rho \, g \, (h - z) \times w \, \frac{dz}{\sin \theta} \]
\[ F = \frac{w \, \rho \, g \, h}{\sin \theta} \left[ h \, z - \frac{z^2}{2} \right]_0^h \]
\[ F = \frac{w \, \rho \, g \, h}{\sin \theta} \, \frac{2}{2} \left[ \frac{w \, h}{\sin \theta} \right] \times \left[ \frac{\rho \, g \, h}{2} \right] \]
\[ F = A \times P_{ave} \]

**MOMENT**
\[ \text{Moment} = F \times \text{moment arm} = F \times s \]
\[ M = \frac{w}{\sin \theta} \int \rho \, g \, (h - z) \frac{z}{\sin \theta} \, dz \]
\[ M = \frac{w \, \rho \, g \, (h - z) \, z}{\sin \theta^2} \, dz \]
\[ M = \frac{w \, \rho \, g}{\sin \theta^2} \left[ \frac{h \, z^2}{2} - \frac{z^3}{3} \right]_0^h \]
\[ M = \frac{w \, \rho \, g \, h^3}{\sin \theta^2} \, \frac{2}{6} \]
\[ M = \left[ \frac{w \, \rho \, g \, h^2}{\sin \theta \, 2} \right] \times \left[ \frac{h}{3 \sin \theta} \right] \]
\[ M = F \times \frac{s}{3} = F \times \text{centroid of pressure} \]
2.48 A certain volume of water is contained in the square vessel shown in Figure P2.48. Where the sealing edges of the inclined plate come into contact with the vessel walls, the reactive force normal and parallel to the wall is zero.

(a) Find the magnitude and direction of the single force $F$ required to hold the plate in position. The weight of the plate may be neglected.

(b) Where does this force $F$ act?

(c) If the inclined plate is replaced by a horizontal one, find the relative position of the new plate if the force used to maintain its position has the same magnitude as before. The volume of fluid remains the same as before.

FIGURE P2.48
2.48

a) \( dA = w \, ds \)
\[
ds = \frac{dz}{\cos \theta}
\]
\[
dA = w \, \frac{dz}{\cos \theta}
\]
\[
p = \rho \, g \, (h + z)
\]
\[
F = \int p \, dA = \int [\rho \, g \, (h + z)] \left[ w \, \frac{dz}{\cos \theta} \right]
\]
\[
F = \frac{w \, \rho \, g}{\cos \theta} \int_0^z (h + z) \, dz
\]
\[
F = \frac{w \, \rho \, g}{\cos \theta} \left[ h \, z + \frac{z^2}{2} \right]_0^5
\]
\[
F = \frac{1.95 \times 32.2 \times 5}{0.707} \left[ 25 + 5^2 \right]
\]
\[
F = 16,652 \, \text{lb}_f
\]

b) \( F = p \times A = \rho \, g \, (h + z) \times 5^2 = 16,652.2 \)
\[
1.95 \times 32.2 \times (5+z) = 16,652.2
\]
\[
(5+z) = 10.608 \, \text{ft}
\]
10.608 ft below the surface

\[
p_{ave} = p @ 7.5 \, \text{feet} = 1.95 \times 32.2 \times 7.5
\]
\[
p_{ave} = 470.93 \, \frac{\text{lb}_f}{\text{ft}^2}
\]
\[
F = p_{ave} \times A = 470.93 \times 5 \times \frac{5}{\cos 45}
\]
\[
F = 16652.2 \, \text{lb}_f
\]
2.47 If the weightless gate shown in Figure P2.47 is just on the point of opening, find an expression for $B$ in terms of $h$. 

**FIGURE P2.47**
Vertical Force
\[ p = \rho g (h - z) \]
\[ p_{\text{net}} = \rho g \left[ \left( h - \frac{3h}{4} \right) - \frac{h}{2} \right] = -\rho g \frac{h}{4} \]
\[ F_{\text{vert}} = p \times A = -\rho g \frac{h}{4} \times w \times B \]
\[ \text{momentarm} = B/2 \]
\[ \text{Moment}_{\text{vert}} = -w \rho g h \frac{B}{8} \]

Horizontal Force
\[ p_{\text{ave}} = \rho g \frac{h}{2} \]
\[ F_{\text{horiz}} = p \times A = \rho g \frac{h}{2} \times w \times h \]
\[ \text{momentarm} = \frac{2h}{3} - \frac{h}{2} = h \]
\[ \text{Moment}_{\text{horiz}} = \rho g \frac{h}{2} \times w \times h \times h \]
\[ \text{Moment}_{\text{horiz}} - \text{Moment}_{\text{vert}} = 0 \]
\[ w \rho \frac{h^3}{2} - w \rho g h \frac{B^2}{8} = 0 \]
\[ B^2 = \frac{8h^3}{2h} \]
\[ B = 2h \]
2.45 A rectangular gate of width $w$ and height $h$ is placed in the vertical side wall of a tank containing water. The top of the gate is located at the surface of the water, and a rectangular container of width $w$ and breadth $b$ is attached to the gate, as shown in Figure P2.45. Find $d$, the depth of the water required to be put into the container so that the gate is just about to open, in terms of $h$ and $b$. The top and sides of the tank and container are open to the atmosphere. Neglect the weight of the container.
INSIDE

\[ p_{\text{ave}} = \rho g (h - z) = \rho g \frac{h}{2} \]

\[ F = p \times A = \rho g \frac{h}{2} \times w h \]

\[ M_o = F \times \text{moment arm} \]

\[ \text{moment arm} = \frac{2h}{3} \]

\[ M_o = \rho g w \frac{h^2}{2} \times \frac{2h}{3} \]

\[ M_o = \rho g w \frac{h^3}{3} \]

OUTSIDE

\[ p = \rho g d \]

\[ F = p \times A = \rho g d \times b w \]

\[ \text{moment arm} = \frac{b}{2} \]

\[ M_o = \rho g d b w \times \frac{b}{2} \]

\[ M_{o\text{inside}} = M_{o\text{outside}} \]

\[ \rho g w \frac{h^3}{3} = \rho g d b w \times \frac{b}{2} \]

\[ d = \frac{2h^3}{3b^2} \]
MOMENT OF INERTIA

\[ I_{yy} = \int x^2 \, d \]
\[ I_{xx} = \int y^2 \, d \]
\[ I_{xx} = \iint y^2 \, dx \, dy \]
\[ I_{x'x'} = I_{xx} + Ad^2 \]

\[ I_{xx} = \frac{b \left( \frac{h}{2} \right)^3}{2} - \frac{b \left( -\frac{h}{2} \right)^3}{2} \]
\[ I_{xx} = \frac{bh^3}{12} \]
\[ I_{x'x'} = \frac{bh^3}{12} + bh \frac{h^2}{4} = \frac{bh^3}{3} \]
### Areas and Moments of Inertia of Areas Around Centroidal Axes

#### Rectangle
- Area: \( A = bh \)
- Centroid: \( \frac{b}{2}, \frac{h}{2} \)
- Moment of Inertia:
  - About the x-axis: \( I_x = \frac{bh^3}{12} \)
  - About the y-axis: \( I_y = \frac{b^3h}{12} \)

#### Circle
- Area: \( A = \pi R^2 \)
- Centroid: \( O \)
- Moment of Inertia:
  - About the x-axis: \( I_x = \frac{\pi R^4}{4} \)
  - About the y-axis: \( I_y = \frac{\pi R^4}{4} \)

#### Triangle
- Area: \( A = \frac{bh}{2} \)
- Centroid: \( \frac{b}{3}, \frac{h}{3} \)
- Moment of Inertia:
  - About the x-axis: \( I_x = \frac{bh^3}{36} \)

#### Semicircle
- Area: \( A = \frac{\pi R^2}{2} \)
- Centroid: \( O \)
- Moment of Inertia:
  - About the x-axis: \( I_x = \frac{4R^4}{3\pi} \)
  - About the y-axis: \( I_y = 0.110R^4 \)

#### Thin Tube
- Area: \( A = 2\pi R_{avg} t \)
- Centroid: \( \frac{d_{avg}}{2}, R_{avg} \)
- Moment of Inertia:
  - About the x-axis: \( I_x = \frac{2R_{avg}^4 t}{3\pi} \)

#### Half of Thin Tube
- Area: \( A = \pi R_{avg} t \)
- Centroid: \( \frac{d_{avg}}{2}, \frac{R_{avg}}{2} \)
- Moment of Inertia:
  - About the x-axis: \( I_x = \frac{2R_{avg}^4 t}{\pi} \)
  - About the y-axis: \( I_y = 0.065\pi R_{avg}^4 t \)

### Areas and Centroids of Areas

#### Triangle
- Area: \( A = \frac{bh}{2} \)
- Centroid: \( \frac{b}{2}, \frac{h}{2} \)

#### Parabola
- Area: \( A = \frac{2ah}{3} \)
- Centroid: \( \frac{b}{2}, \frac{h}{3} \)
- Moment of Inertia:
  - About the x-axis: \( I_x = \frac{2}{3} bh \)

The area for any segment of a parabola is \( A = \frac{bh}{3} \).
\[ F_R \]

\[ h \]

\[ \theta \]

\[ x' \]

\[ y' \]

\[ x' \]

\[ y' \]

\[ y'_{c} \]

\[ y'_{R} \]

\[ dA = dx'\,dy' \]

\[ p = \rho g(h - z) \]

\[ (h - z) = y'\sin \theta \]
\[(h - z) = y' \sin \theta \]
\[p = \rho \ g \ (h - z) = \rho \ g \ y' \sin \theta \]
\[F_{\text{inside}} = \int p \, dA\]
\[F_{\text{inside}} = \rho \ g \int \int (h - z) \, dx' \, dy'\]
\[F_{\text{inside}} = \rho \ g \ \int \int y' \, dx' \, dy'\]
\[\int \int y' \, dx' \, dy' = \text{centroid} \times A = y'_{\text{centroid}} \ A\]
\[F_{\text{inside}} = \rho \ g \ \sin \theta \ y'_{\text{centroid}} \ A = p_{\text{centroid}} \times A\]

\[M_O = \int \text{moment arm} \times F\]
\[M_O = \int \int y' p \, dx' \, dy' = \int \int y' \rho \ g \ y' \sin \theta \, dy' \, dx'\]
\[M_O = \rho \ g \ \sin \theta \ \int \int y'^2 \, dy' \, dx'\]
\[\int \int y'^2 \, dy' \, dx' \text{ is the second moment of area around the x' axis}\]
\[M_O = \rho \ g \ \sin \theta \ I_{x'x'}\]

**GENERAL PLANE SURFACE**

\[M_{OR} = M_{O_{\text{inside}}}\]
\[p_c \ y' A = \rho \ g \ I_{x'x'} \ \sin \theta \]
\[I_{x'x'} = I_{xx} + A \ y_c^2\]
\[p_c y' A = \rho \ g \ \sin \theta \ \left(I_{xx} + A y_c^2\right)\]
\[p_c y' A = \rho \ g \ \sin \theta \ I_{xx} + \rho \ g \ \sin \theta \ A y_c^2\]

Since \[p_c = \rho \ g \ y_c \sin \theta \ i \ \rho \ g \ \sin \theta = \frac{p_c}{y_c}\]

\[p_c y' A = \rho \ g \ \sin \theta \ I_{xx} + \frac{p_c}{y_c} A \ y_c^2\]
\[p_c \ y' A - p_c \ y_c A = \rho \ g \ \sin \theta \ I_{xx}\]
\[y' - y_c = \frac{\rho \ g \ \sin \theta \ I_{xx}}{p_c \ A}\]
Vertical Surface Analysis

\[ p = \rho g z \]
\[ F = p_{\text{centroid}} A \]
\[ F = \rho g \frac{h}{2} \text{wh} = w \rho g \frac{h^2}{2} \]
\[ M_o = F \times y_{\text{centroid}} = w \rho g \frac{h^2}{2} \frac{2h}{3} \]
\[ M_o = w \rho g \frac{h^3}{3} \]
\[ z_R - z_c = \frac{2h}{3} - \frac{h}{2} = \frac{h}{6} \]

General Surface Analysis

\[ \sin \theta = 1, \]
\[ I_{xx} = \frac{w h^3}{12} \]
\[ A = w h \]

\[ p = \rho g z, \quad p_c = \frac{\rho g h}{2} \]
\[ F = p_{\text{centroid}} \times A = \frac{\rho g h}{2} \text{wh} = w \rho g \frac{h^2}{2} \]
\[ M_o = \rho g \sin \theta I_{x'x'} = \rho g (I_{xx} + Ad^2) \]
\[ M_o = \rho g \frac{wh^3}{12} + wh \frac{h^2}{4} = w \rho g \frac{h^3}{3} \]
\[ y' - y_c = \frac{\rho g \sin \theta I_{xx}}{p_c A} \]
\[ y' - y_c = \rho g \frac{wh^3}{12} \frac{1}{\rho g h w h} \frac{1}{6} = \frac{h}{6} \]
**FORCES ON A CURVED SURFACE**

**HORIZONTAL FORCE**

\[
F_x = F \sin \theta = w \rho g \int (h - z) \sin \theta \, ds
\]

\[
F_x = w \rho g \int (h - z) \, dz
\]

\[
F_x = \text{force on a vertical surface}
\]

**VERTICAL FORCE**

\[
dF = p \omega ds = w \rho g (h - z) ds
\]

\[
dx = ds \cos \theta
\]

\[
dz = ds \sin \theta
\]

\[
dF = \rho \omega ds = w \rho g (h - z) ds
\]

\[
F_y = F \cos \theta = w \rho g \int (h - z) \cos \theta \, ds
\]

\[
F_y = w \rho g \int (h - z) \, dx
\]

\[
F_y = (p g) \times (w (h - z) dx)
\]

\[
F_y = \rho g \times \text{volume above surface}
\]
VERTICAL FORCE LOCATION

\[ M_o = -\int x \rho w \, dx = -\int x \rho g (h - z) w \, dx \]

\[ M_o = -\rho g \int_0^L x \times (h - z) w \, dx = \rho g \int_0^L x \, d\text{Volume} \]

\[ M_o = -\rho g \text{Volume} x_c \]

\[ M_o = -F_z x_c \]

The vertical force component acts along the x coordinate direction of the centroid of the volume of fluid above the surface.

RESULTANT

The horizontal force acts through the centroid of the pressure – depth diagram.

\[ F_x = \rho g \frac{h}{2} \text{wh} \]

\[ M_o = -\frac{h}{3} F_x - x_c |F_z| \]

\[ |F_z| = \rho g \text{Volume} \]
Example: Find the force to hold the quarter-circular wall

Width = 3 m

\( \gamma_c = 0.424 \text{ kPa} \)

\( \gamma_{H_2O} = 9800 \text{ N/m}^2 \)
From the Hydrostatic Equation a pressure difference exists between the top and bottom of a submerged body

\[
\begin{align*}
\text{d}F_z &= (p_1 - p_2)\text{d}A_z = (p_a + \rho g(h - z_1)) - (p_a + \rho g(h - z_2)) \\
\text{d}F_z &= \rho g(z_2 - z_1)\text{d}A_z = \rho g \text{Volume}_{\text{object}} \\
F_{\text{bouyancy}} &= \text{weight of fluid displaced}
\end{align*}
\]

Moment about y axis
\[
\text{d}M = x\text{d}F = x \times \rho g \text{d}V
\]
\[
M = \rho g \int x\text{d}V = \rho g x_c V = x_{\text{centroid}} F_y
\]

The bouyant force acts through the centroid of the body volume
A float and lever system is used to open a drain valve, as shown in Figure P2.42. The float has a volume \( V \) and a density \( \rho_f \). The density of the water is \( \rho_w \). Find the maximum force available to open the drain valve, given that the hinge is frictionless. Hint: first consider the forces acting on the float, then consider the free-body diagram of the gate.

**FIGURE P2.42**

max force with ball submerged

\[
F_{\text{float}} = V \rho_w g - V \rho_{\text{float}} g
\]

\[
M_{\text{Ofloat}} = F_{\text{float}} \times L = L \ V \ g \left( \rho_w - \rho_{\text{float}} \right)
\]

\[
M_{\text{valve}} = F_{\text{float}} \times \frac{L}{3}
\]

\[
M_{\text{Ofloat}} = M_{\text{valve}}
\]

\[
F_{\text{float}} \times \frac{L}{3} = L \ V g \left( \rho_w - \rho_{\text{float}} \right)
\]

\[
F_{\text{float}} = 3 \ V \ g \left( \rho_w - \rho_{\text{float}} \right)
\]
A rectangular barge of length $L$ floats in water (density $\rho_w$) and when it is empty, it is immersed to a depth $D$, as shown in Figure P2.35. Oil of density $\rho_o$ is slowly poured into the barge until it is about to sink. Find a relationship for the depth of oil at this point in terms of $H$, $D$, $\rho_w$, and $\rho_o$.

![Diagram of barge with water and oil](image)

**FIGURE P2.35**

- **Barge weight** = $L W D \rho_w g$
- "sinking" barge weight = $L W H \rho_w g$
- Oil weight = $L W d_{oil} \rho_{oil} g$

"sinking" weight = barge weight + oil weight

$L W H \rho_w g = L W D \rho_w g + L W d_{oil} \rho_{oil} g$

$H \rho_w = D \rho_w + d_{oil} \rho_{oil}$

$d_{oil} = \frac{\rho_w}{\rho_{oil}} (H - D)$
HYDROMETER - A DEVICE FOR MEASURING SPECIFIC GRAVITY

\[
\text{In } \text{H}_2\text{O}: \quad \gamma_{\text{H}_2\text{O}} \nu = W
\]

\[
\text{In other liquid: } \quad \gamma (\nu - \text{HA}) = W
\]

\[
W = \gamma (\nu - \text{HA}) = \gamma_{\text{H}_2\text{O}} \nu
\]

SPECIFIC GRAVITY = \( \frac{\gamma}{\gamma_{\text{H}_2\text{O}}} \)

\[
\frac{\gamma}{\gamma_{\text{H}_2\text{O}}} = \frac{1}{1 - \frac{\text{HA}}{\nu}}
\]
REVIEW OF DERIVATIVES

ORDINARY DERIVATIVE \( y = f(x) \)

\[
\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}
\]

= slope = \( y'(x) \)

To approximate \( y \) at \( x + \Delta x \)

\[ y(x + \Delta x) \approx y(x) + y'(x) \Delta x \]

= a term Taylor Series

PARTIAL DERIVATIVE \( z = f(x, y) \)

\[
\frac{\partial z}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x} \quad y\text{ const.}
\]

\[
\frac{\partial z}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y} \quad x\text{ const.}
\]

\[ z(x + \Delta x, y + \Delta y) = z(x, y) + \frac{\partial z(x, y)}{\partial x} \Delta x + \frac{\partial z(x, y)}{\partial y} \Delta y \]
Exact Differential
Exact Differential

Theorem:
If the function $z = f(x, y)$ has continuous first partial derivatives in the domain, then the function $z$ has a differential

$$dz = \frac{\partial z}{\partial x} \Delta x + \frac{\partial z}{\partial y} \Delta y$$

at every point in the domain.

$$\Delta z = \left( \frac{z(x + \Delta x, y) - z(x, y + \Delta y)}{\Delta x} \right) \Delta x + \left( \frac{z(x, y + \Delta y) - z(x, y)}{\Delta y} \right) \Delta y$$

$$\lim_{\Delta \to 0} \frac{z(x + \Delta x, y) - z(x, y)}{\Delta x} = \left( \frac{\partial z}{\partial x} \right)_{y=\text{const}}$$

$$dz = \left( \frac{\partial z}{\partial x} \right)_{y=\text{const}} \, dx + \left( \frac{\partial z}{\partial y} \right)_{x=\text{const}} \, dy$$