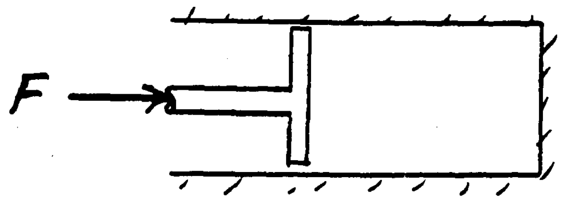
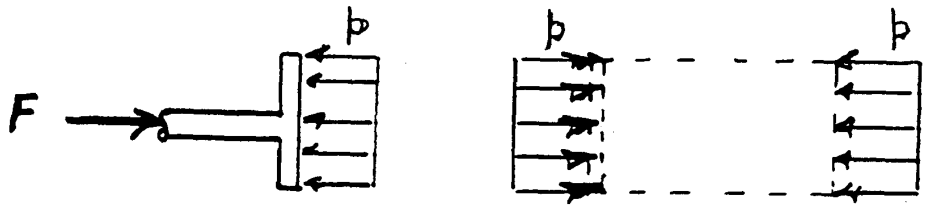
Fluid Statics
Chapter 2.1-2.9

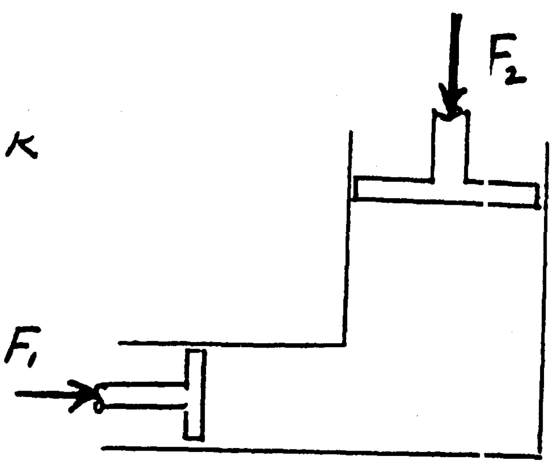
CONFINED FLUID (1.3.8 + 1.3.9)

A CONSTANT PRESSURE FROM A FORCE APPLIED AT A BOUNDARY IS COMMUNICATED THROUGHOUT THE FLUID.





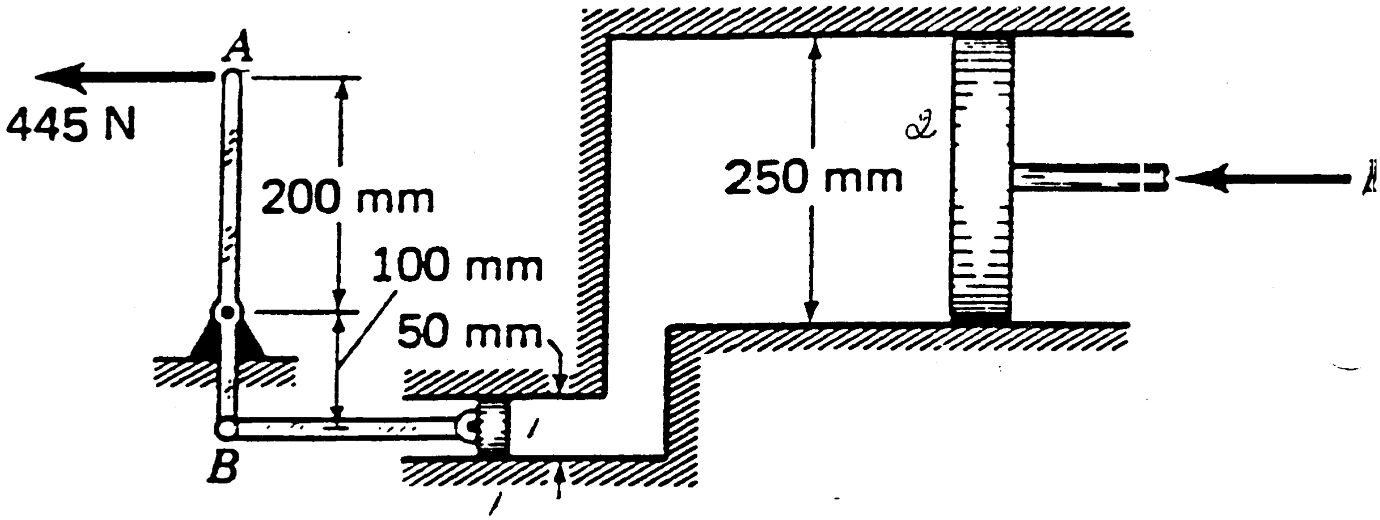
HYDRAULIC JACK



$$\frac{F_2}{F_1} = \frac{p_g A_2}{p_g A_1} = \frac{A_2}{A_1}$$

NEGLECTING GRAVITY

A force of 445 N is exerted on lever AB. End B is connected to a piston which fits into a cylinder having a diameter of 50 mm. What force P must be exerted on the larger piston to prevent it from moving in its cylinder which has a 250 mm diameter?



$$A_1 = \pi r_1^2 = (\pi)(.025)^2 \qquad A_2 = \pi r_2^2 = (\pi)(.125)^2$$

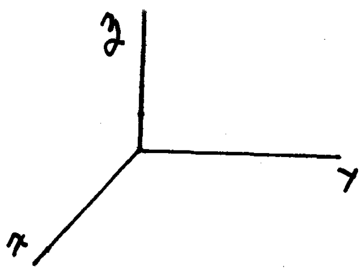
$$\frac{F_2}{F_1} = \frac{A_2}{A_1}$$

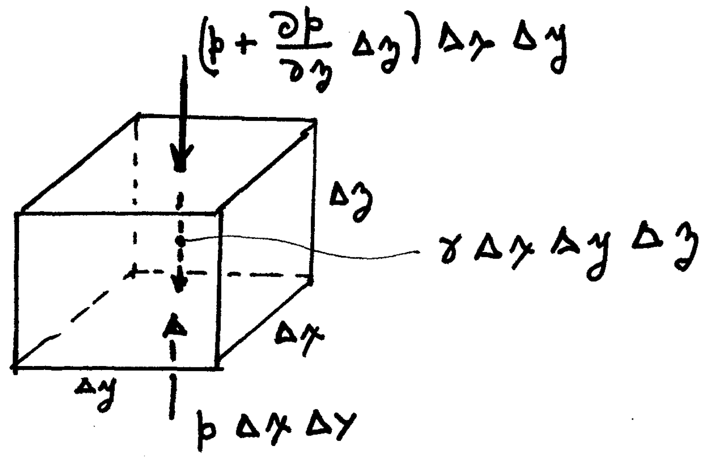
$$\therefore \; P_2 = [(2)(445)]\frac{\pi(.125)^2}{\pi(.025)^2} = 22{,}250\ N = \boxed{22.25\ kN}$$

FLUID STATICS

FORCES ON SUBMERGED SURFACES AND ON FLOATING AND SUBMERGED OBJECTS IN A FLUID AT REST

HYDROSTATIC EQUATION





$$\sum F_z = \cancel{p}\,\Delta x\,\Delta y - \left(\cancel{p} + \frac{\partial p}{\partial z}\Delta z\right)\Delta x\,\Delta y - \gamma\,\Delta x\,\Delta y\,\Delta z = 0$$

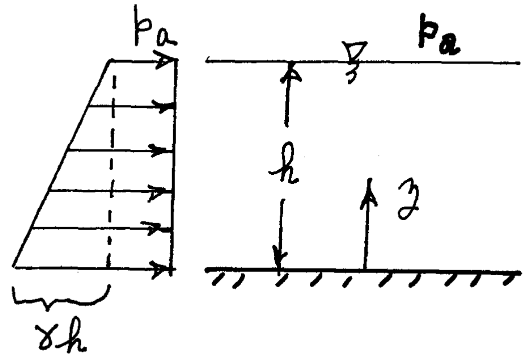
$$\frac{\partial p}{\partial z} = -\gamma = -\rho g$$

$$\sum F_x = 0 \Rightarrow \frac{\partial p}{\partial x} = 0 \qquad \sum F_y = 0 \Rightarrow \frac{\partial p}{\partial y} = 0$$

PRESSURE IS CONSTANT ON HORIZONTAL SURFACES

$$\frac{dp}{dz} = -\gamma$$ GIVES VERTICAL VARIATION

PRESSURE VARIATION FOR CONSTANT γ



$$\frac{dp}{dz} = -\gamma$$

$$p = -\gamma z + c$$

$$p_a = -\gamma h + c$$

$$p = p_a + \gamma(h - z) \qquad p_o = p_a + \gamma h$$

p = ABSOLUTE $\qquad p - p_a$ = GAUGE = p_g

ATMOSPHERIC PRESSURE VARIATION

FROM MEASUREMENTS $\quad T = T_o + m(z - z_o)$

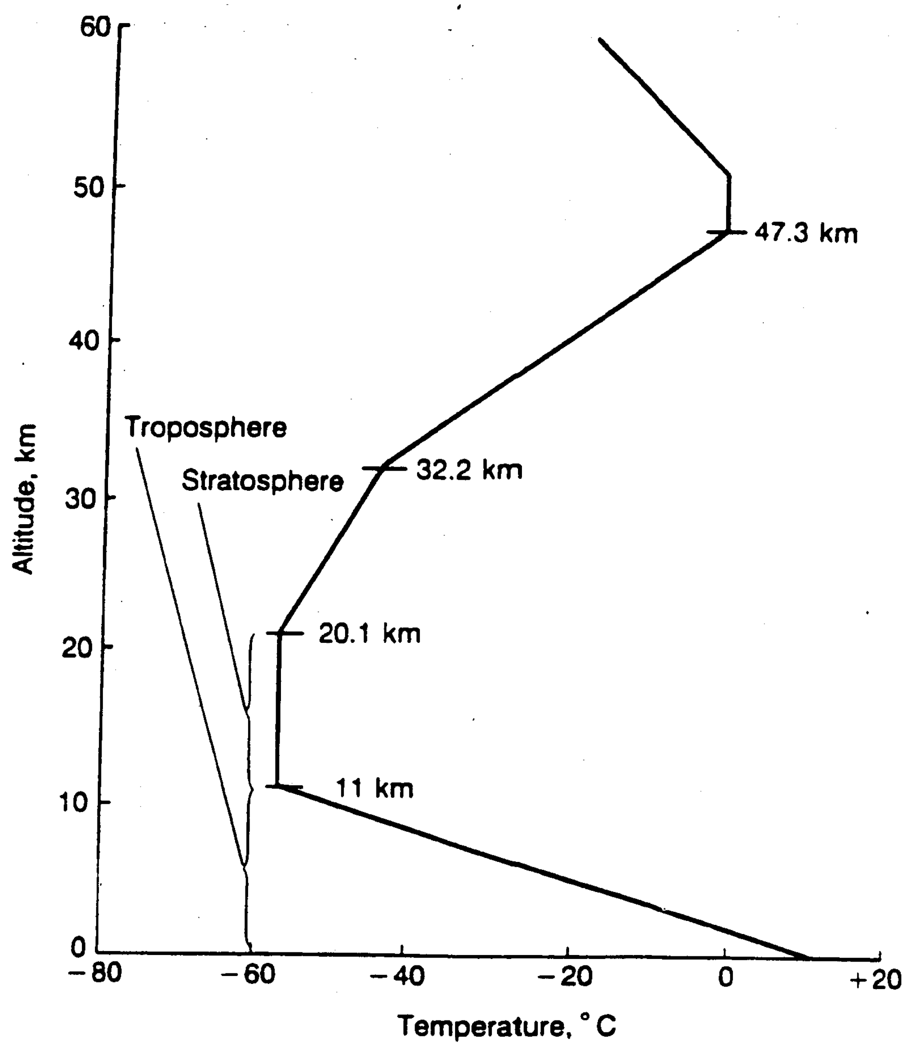
$$\frac{dp}{dz} = -\rho g = -\frac{p}{RT} g \qquad \text{PERFECT GAS: } p = \rho R T$$

$$\frac{dp}{p} = -\frac{g}{R} \frac{dz}{T_o + m(z - z_o)}$$

$$\ln p \Big|_{p_o}^{p} = -\frac{g}{mR} \ln\left[T_o + m(z - z_o)\right]_{z_o}^{z}$$

$$p/p_o = \left[\frac{T_o + m(z - z_o)}{T_o}\right]^{-g/mR}$$

STANDARD ATMOSPHERE



AT SEA LEVEL

$T = 15\ °C$

$p = 101.325\ kPa$

$\rho = 1.2232\ kg/m^3$

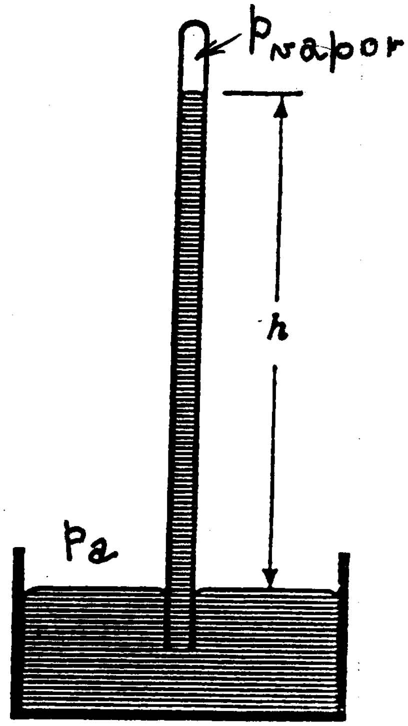
VARIATION WITH ELEVATION:

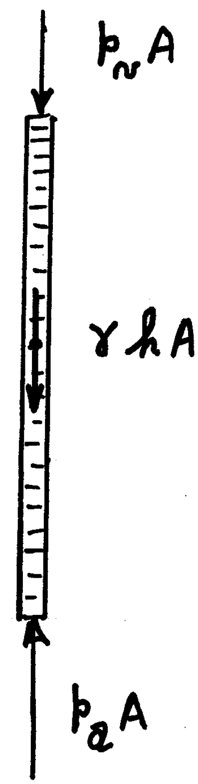
TABLE C.5 Properties of the U.S. Standard Atmosphere (SI Units)[a]

Altitude (m)	Temperature (°C)	Acceleration of gravity, g (m/s^2)	Pressure, p [N/m^2(abs)]	Density, ρ (kg/m^3)	Dynamic viscosity, μ (N·s/m^2)
−1,000	21.50	9.810	1.139 E + 5	1.347 E + 0	1.821 E − 5
0	15.00	9.807	1.103 E + 5	1.225 E + 0	1.789 E − 5
1,000	8.50	9.804	8.988 E + 4	1.112 E + 0	1.758 E − 5
2,000	2.00	9.801	7.950 E + 4	1.007 E + 0	1.726 E − 5
3,000	−4.49	9.797	7.012 E + 4	9.093 E − 1	1.694 E − 5
4,000	−10.98	9.794	6.166 E + 4	8.194 E − 1	1.661 E − 5
5,000	−17.47	9.791	5.405 E + 4	7.364 E − 1	1.628 E − 5
6,000	−23.96	9.788	4.722 E + 4	6.601 E − 1	1.595 E − 5
7,000	−30.45	9.785	4.111 E + 4	5.900 E − 1	1.561 E − 5
8,000	−36.94	9.782	3.565 E + 4	5.258 E − 1	1.527 E − 5
9,000	−43.42	9.779	3.080 E + 4	4.671 E − 1	1.493 E − 5
10,000	−49.90	9.776	2.650 E + 4	4.135 E − 1	1.458 E − 5
15,000	−56.50	9.761	1.211 E + 4	1.948 E − 1	1.422 E − 5
20,000	−56.50	9.745	5.529 E + 3	8.891 E − 2	1.422 E − 5
25,000	−51.60	9.730	2.549 E + 3	4.008 E − 2	1.448 E − 5
30,000	−46.64	9.715	1.197 E + 3	1.841 E − 2	1.475 E − 5
40,000	−22.80	9.684	2.871 E + 2	3.996 E − 3	1.601 E − 5
50,000	−2.50	9.654	7.978 E + 1	1.027 E − 3	1.704 E − 5
60,000	−26.13	9.624	2.196 E + 1	3.097 E − 4	1.584 E − 5
70,000	−53.57	9.594	5.221 E + 0	8.283 E − 5	1.438 E − 5
80,000	−74.51	9.564	1.052 E + 0	1.846 E − 5	1.321 E − 5

[a] Data from *U.S. Standard Atmosphere,* 1976, U.S. Government Printing Office, Washington, D.C.

BAROMETER - DEVICE FOR MEASURING ATMOSPHERIC PRESSURE





$$p_2 - p_1 = -\gamma(z_2 - z_1)$$

$$p_v - p_a = -\gamma h$$

$$p_a = p_v + \gamma h$$

MERCURY

S.G. = 13.6 $\quad p_v \cong 0.2$ psia

$$p_a = p_v + \gamma h$$

$$h = \frac{p_a - p_v}{\gamma} = \frac{(14.7 - 0.2)\,144}{13.6\ (62.4)} = 2.46' = 29.5''$$

$$p_a = 29.5 \text{ in. } Hg$$

FOR WATER $h = \dfrac{(14.7 - 0.4)\,144}{62.4} = 33'$

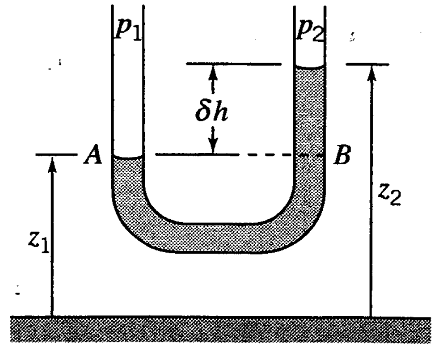
MANOMETER: A FLUID-STATIC DEVICE FOR MEASURING PRESSURE DIFFERENCES

$$\frac{dp}{dz} = -\gamma$$

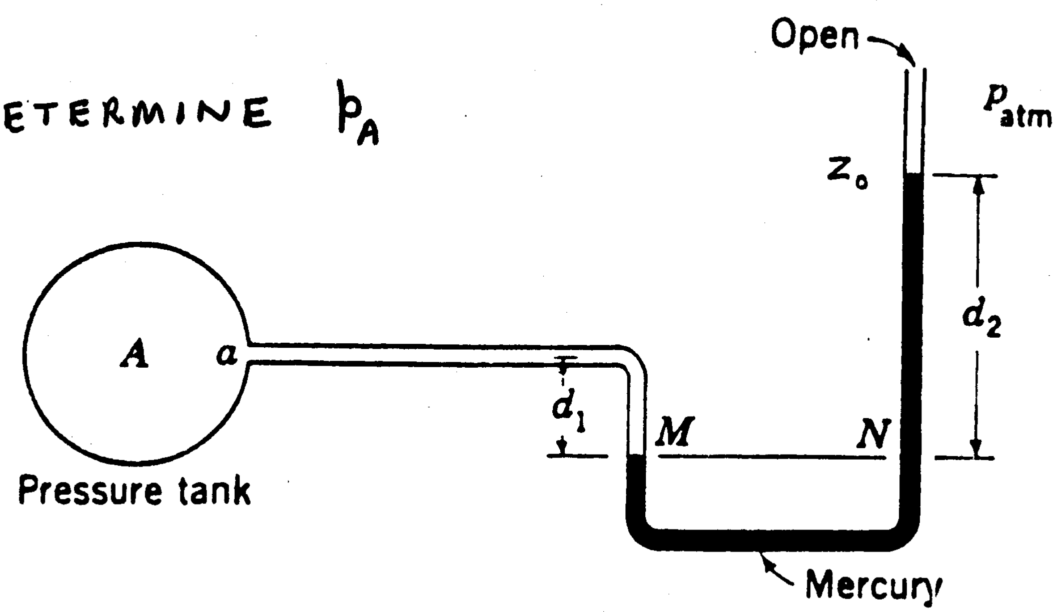
$$p_2 - p_B = -\gamma(z_2 - z_1)$$

$$p_B = p_A = p_1$$

$$p_1 - p_2 = \gamma(z_2 - z_1) = \gamma\,\delta h$$



DETERMINE p_A

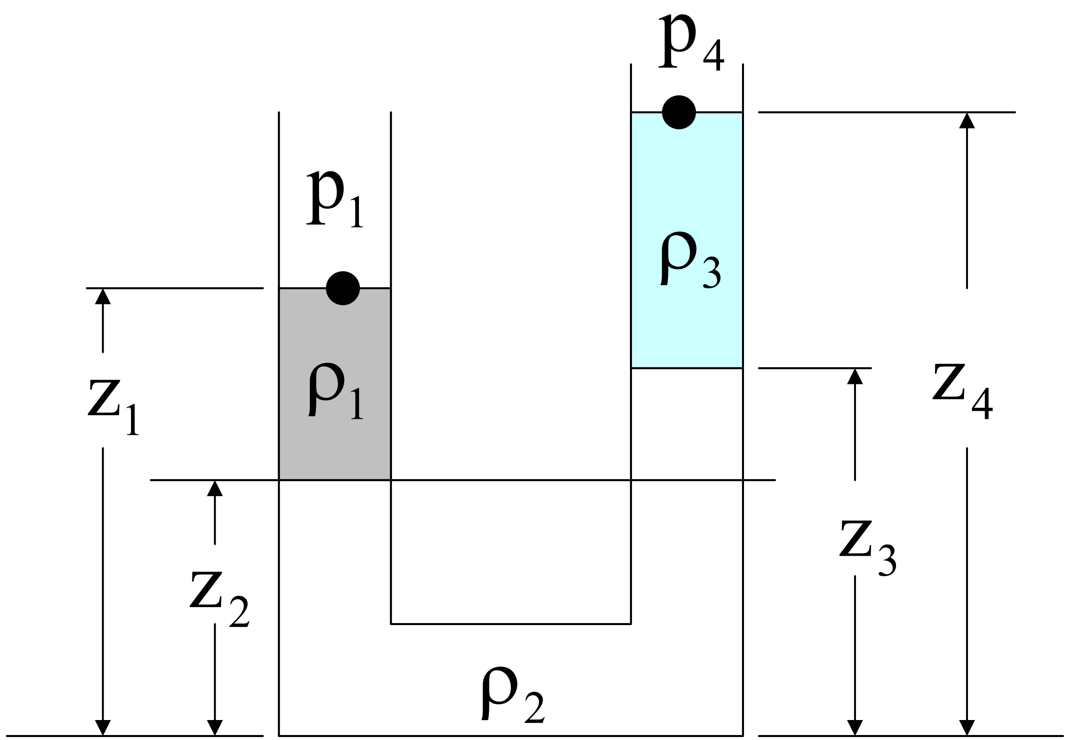


$$p_2 - p_1 = -\gamma(z_2 - z_1)$$

$$p_A - p_M = -\gamma d_1 \qquad p_a - p_N = -\gamma_{Hg} d_2$$

$$p_M = p_N \qquad p_A + \gamma d_1 = p_a + \gamma_{Hg} d_2$$

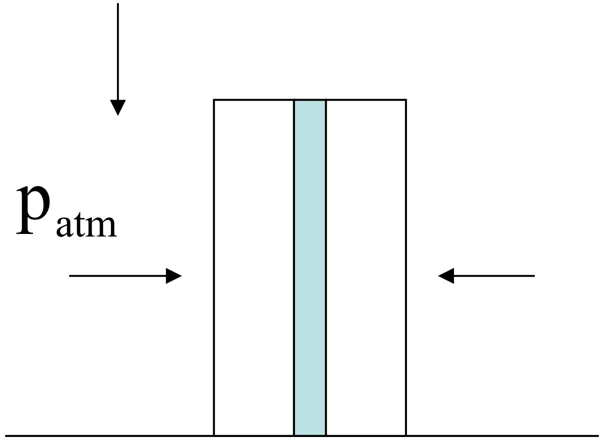
$$\therefore\ p_A - p_a = p_{A_g} = \gamma_{Hg} d_2 - \gamma d_1$$

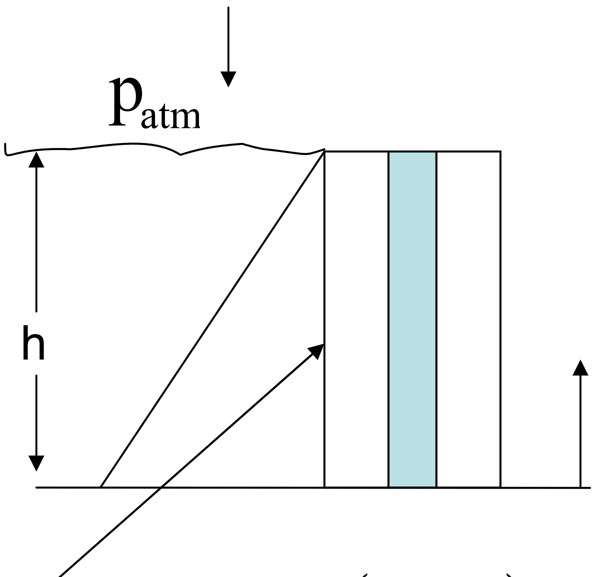


$$p_1 + \rho_1 g(z_1 - z_2) + \rho_2 g(z_2) = p_4 + \rho_3 g(z_4 - z_3) + \rho_2 g(z_3)$$

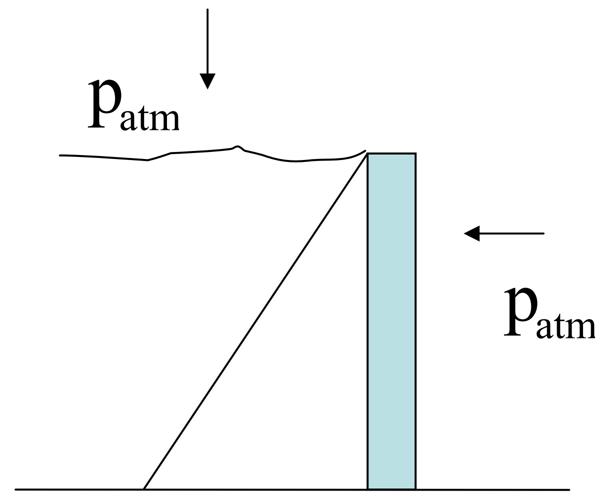
$$p_1 + \rho_1 g(z_1 - z_2) = p_4 + \rho_3 g(z_4 - z_3) + \rho_2 g(z_3 - z_2)$$

TANK OPEN TO THE ATMOSPHERE

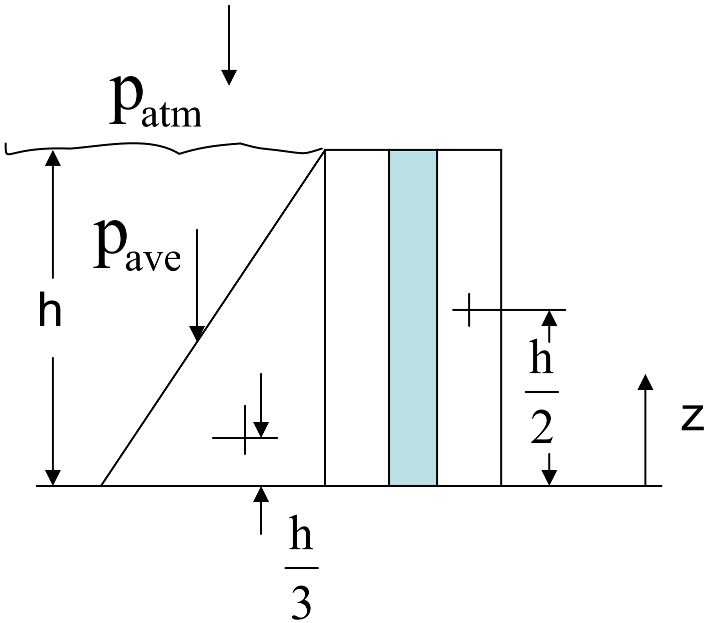




$$p = p_a + \rho g (h - z)$$



$$p = \rho g (h - z)$$



LINEAR PRESSURE VARIATION

$$p = \rho\, g\, (h - z)$$

$$F = \int p\, dA = \int_0^h \rho\, g (h - z) \times w\, dz$$

$$F = \rho\, g \frac{h^2}{2} \times h\, w$$

$$F = p_{ave} \times \text{area}$$

$$M_O = F \times \text{moment arm}$$

$$M_O = \int p \times z\, dz$$

$$M_O = \int_0^h \rho\, g\, (h - z) \times z \times w\, dz$$

$$M_O = w\, \rho\, g \int_0^h (h - z)\, z\, dz = w\, \rho\, g \left[h\, z - \frac{z^2}{2} \right]_0^h$$

$$M_O = w\, \rho\, g \frac{h^3}{6} = w\, \rho\, g \frac{h^2}{2} \times \frac{h}{3}$$

$$M_O = F \times \text{distance to centroid}$$

CONSTANT PRESSURE

$$p_{atm} = \text{constant}$$

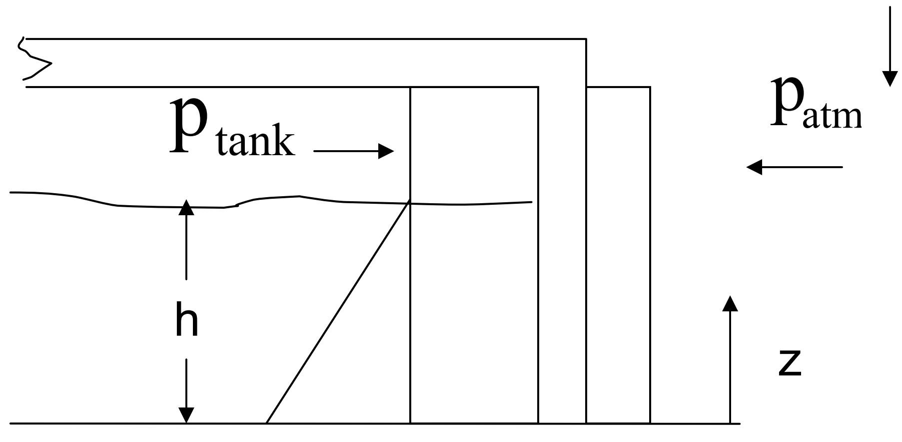
$$F = p_{atm} \int dA = p_{atm}\; w \int_0^h dz = p_{atm}\; w\, h$$

$$M_O = F \times \text{momment arm}$$

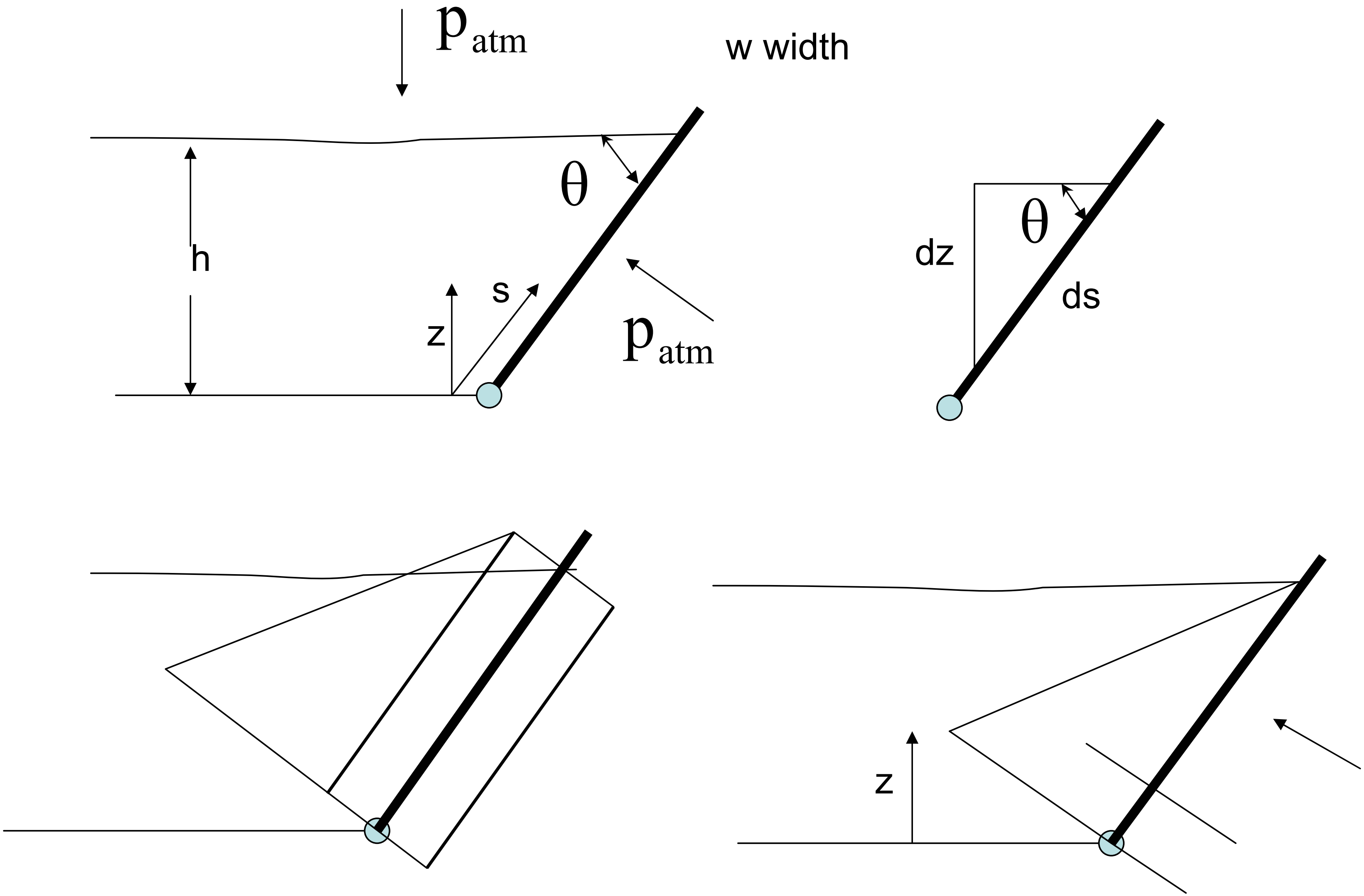
$$M_O = \int F\, z\, dz = p_{atm}\; w \int_0^h z\, dz = p_{atm}\; w\, \frac{h^2}{2}$$

$$M_O = p_{atm} w\, h \times \frac{h}{2} = F \times \text{distance to center of area}$$

CLOSED PRESSURIZED TANK



$$p = p_{tank} + \rho g (h - z)$$



p_{atm}
w width
θ
h
s
z
p_{atm}
θ
dz
ds
z

INCLINED SURFACE

FORCE

$$dA = w\, ds$$

$$ds = \frac{dz}{\sin\theta}, \quad s = \frac{h}{\sin\theta}$$

$$dA = \frac{w\, dz}{\sin\theta}$$

$$p = \rho\, g\, (h - z)$$

$$F = \int p\, dA = \int \rho\, g\, (h - z) \times w\, ds$$

$$F = \int \rho\, g\, (h - z) \times w\, \frac{dz}{\sin\theta}$$

$$F = \frac{w\, \rho\, g}{\sin\theta} \left[h\, z - \frac{z^2}{2} \right]_0^h$$

$$F = \frac{w\, \rho\, g}{\sin\theta} \frac{h}{2} = \left[\frac{w\, h}{\sin\theta} \right] \times \left[\frac{\rho\, g\, h}{2} \right]$$

$$F = A \times P_{ave}$$

MOMENT

$$\text{Moment} = F \times \text{moment arm} = F \times s$$

$$M = \frac{w}{\sin\theta} \int \rho\, g\, (h - z) \frac{z}{\sin\theta} dz$$

$$M = \frac{w\, \rho\, g}{\sin\theta^2} \int (h - z) z\, dz$$

$$M = \frac{w\, \rho\, g}{\sin\theta^2} \left[\frac{h\, z^2}{2} - \frac{z^3}{3} \right]_0^h$$

$$M = \frac{w\, \rho\, g}{\sin\theta^2} \frac{h^3}{6}$$

$$M = \left[\frac{w\, \rho\, g}{\sin\theta} \frac{h^2}{2} \right] \times \left[\frac{h}{3 \sin\theta} \right]$$

$$M = F \times \frac{s}{3} = F \times \text{centroid of pressure}$$

2.48 A certain volume of water is contained in the square vessel shown in Figure P2.48. Where the sealing edges of the inclined plate come into contact with the vessel walls, the reactive force normal and parallel to the wall is zero.

(a) Find the magnitude and direction of the single force F required to hold the plate in position. The weight of the plate may be neglected.

(b) Where does this force F act?

(c) If the inclined plate is replaced by a horizontal one, find the relative position of the new plate if the force used to maintain its position has the same magnitude as before. The volume of fluid remains the same as before.

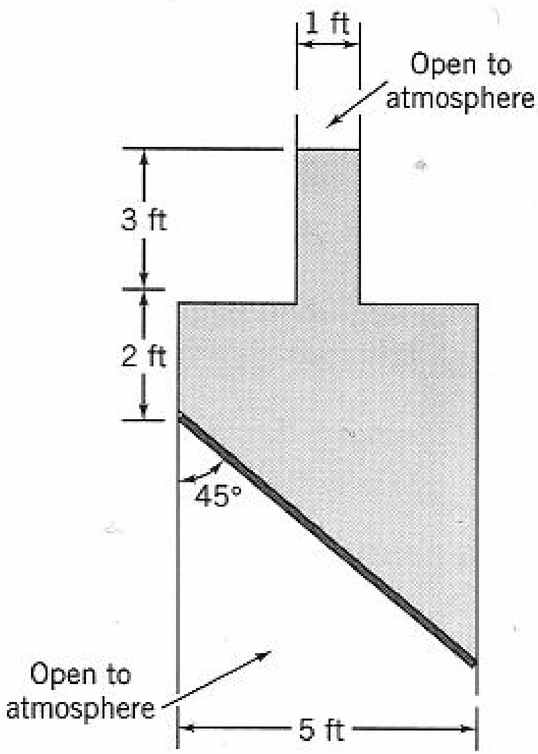
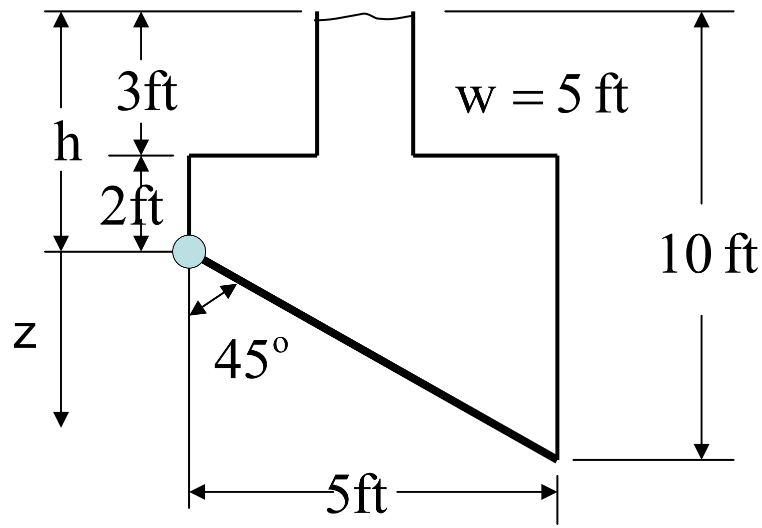


FIGURE P2.48

2.48



a) $dA = w\,ds$

$$ds = \frac{dz}{\cos\theta}$$

$$dA = w\frac{dz}{\cos\theta}$$

$$p = \rho\, g\,(h+z)$$

$$F = \int p\,dA = \int\left[\rho\, g\,(h+z)\right]\left[w\frac{dz}{\cos\theta}\right]$$

$$F = \frac{w\,\rho\, g}{\cos\theta}\int_0^5 (h+z)\,dz$$

$$F = \frac{w\,\rho\, g}{\cos\theta}\left[h\,z + \frac{z^2}{2}\right]_0^5$$

$$F = \frac{1.95\times 32.2\times 5}{.707}\left[25+5^2\right]$$

$$F = 16{,}652\ \mathrm{lb_f}$$

$$p_{ave} = p\ @\ 7.5\ \text{feet} = 1.95\times 32.2\times 7.5$$

$$p_{ave} = 470.93\ \frac{\mathrm{lb_f}}{\mathrm{ft}^2}$$

$$F = p_{ave}\times A = 470.93\times 5\times\frac{5}{\cos 45}$$

$$F = 16652.2\ \mathrm{lb_f}$$

b) $F = p\times A = \rho\, g\,(h+z)\times 5^2 = 16{,}652.2$

$$1.95\times 32.2\times(5+z)\times 25 = 16{,}652.2$$

$$(5+z) = 10.608\ \text{ft}$$

10.608 ft below the surface

2.47 If the weightless gate shown in Figure P2.47 is just on the point of opening, find an expression for B in terms of h.

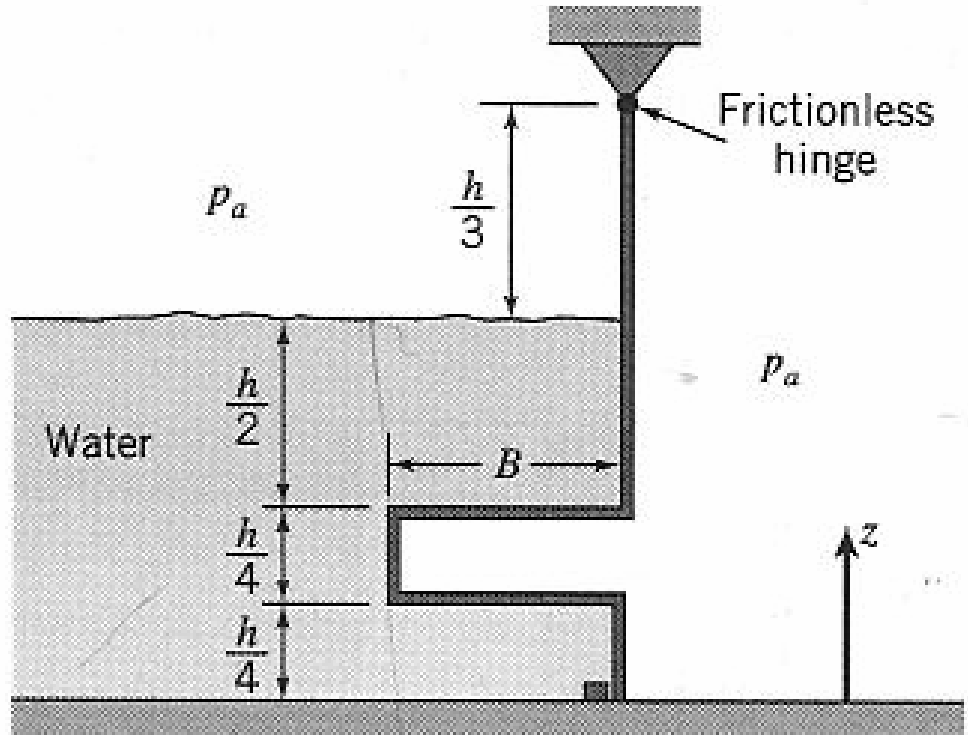
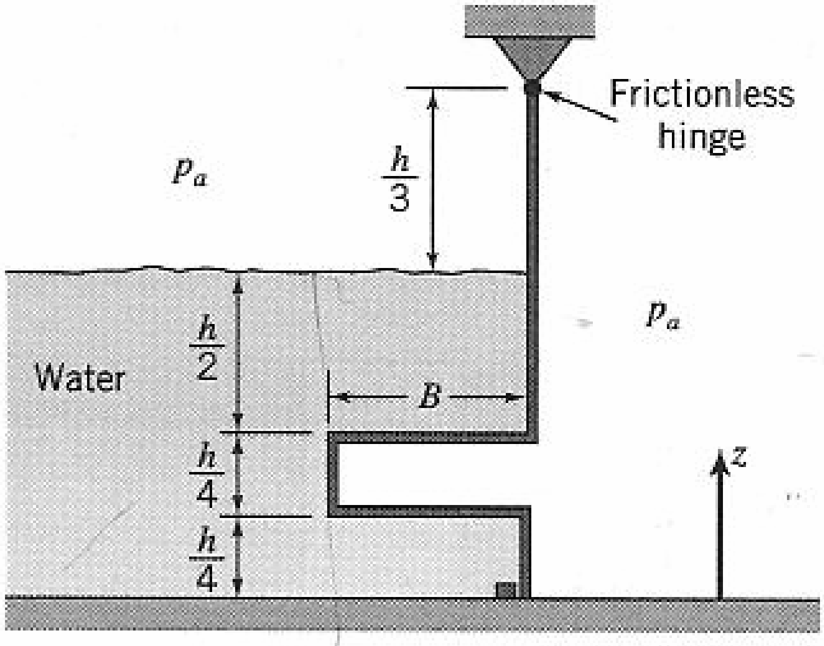


FIGURE P2.47



Vertical Force

$$p = \rho g (h - z)$$

$$p_{net} = \rho g \left[\left(h - \frac{3h}{4} \right) - \frac{h}{2} \right] = -\rho g \frac{h}{4}$$

$$F_{vert} = p \times A = -\rho g \frac{h}{4} \times w B$$

$$\text{momentarm} = B/2$$

$$\text{Moment}_{vert} = -w \rho g h \frac{B}{8}$$

2.47

Horizontal Force

$$p_{ave} = \rho g \frac{h}{2}$$

$$F_{horiz} = p \times A = \rho g \frac{h}{2} \times w h$$

$$\text{momentarm} = \frac{2h}{3} - \frac{h}{2} = h$$

$$\text{Moment}_{horiz} = \rho g \frac{h}{2} \times w h \times h$$

$$\text{Moment}_{horiz} = w \rho g \frac{h^3}{2}$$

$$\text{Moment}_{horiz} - \text{Moment}_{vert} = 0$$

$$w \rho \rho \frac{h^3}{2} - w \rho g h \frac{B^2}{8} = 0$$

$$B^2 = \frac{8h^3}{2h}$$

$$B = 2h$$

2.45 A rectangular gate of width w and height h is placed in the vertical side wall of a tank containing water. The top of the gate is located at the surface of the water, and a rectangular container of width w and breadth b is attached to the gate, as shown in Figure P2.45. Find d, the depth of the water required to be put into the container so that the gate is just about to open, in terms of h and b. The top and sides of the tank and container are open to the atmosphere. Neglect the weight of the container.

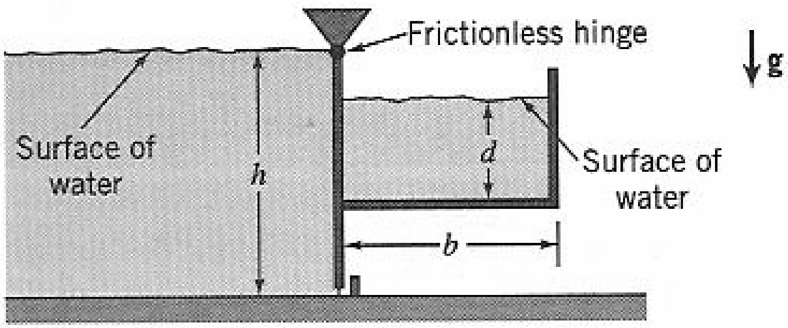
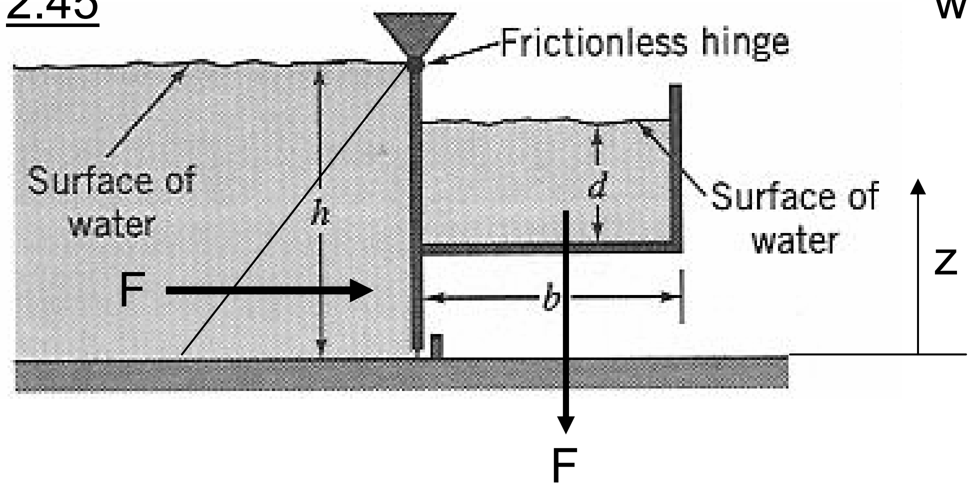


FIGURE P2.45

2.45

w depth



INSIDE

$$p_{ave} = \rho\, g\,(h - z) = \rho\, g \frac{h}{2}$$

$$F = p \times A = \rho\, g \frac{h}{2} \times w\, h$$

$$M_o = F \times \text{moment arm}$$

$$\text{moment arm} = \frac{2\,h}{3}$$

$$M_o = \rho\, g\, w \frac{h^2}{2} \times \frac{2\,h}{3}$$

$$M_o = \rho\, g\, w \frac{h^3}{3}$$

OUTSIDE

$$p = \rho\, g\, d$$

$$F = p \times A = \rho\, g\, d \times b\, w$$

$$\text{momentarm} = \frac{b}{2}$$

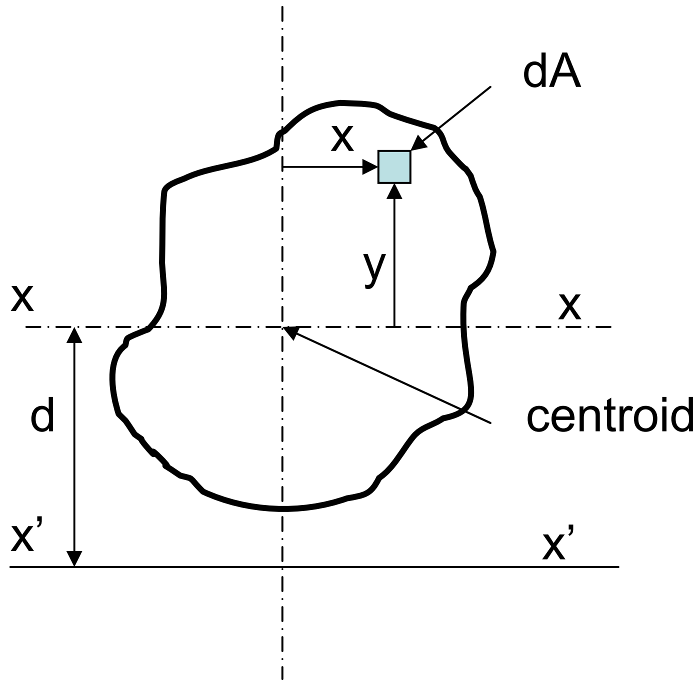
$$M_o = \rho\, g\, d\, b\, w \times \frac{b}{2}$$

$$M_{oinside} = M_{ooutside}$$

$$\rho\, g\, w \frac{h^3}{3} = \rho\, g\, d\, b\, w \times \frac{b}{2}$$

$$d = \frac{2\,h^3}{3\,b^2}$$

MOMENT OF INERTIA

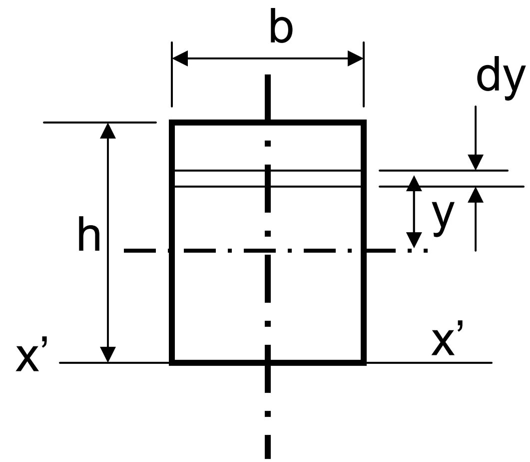


$$I_{yy} = \int x^2 d$$

$$I_{xx} = \int y^2 d$$

$$I_{xx} = \iint y^2 dxdy$$

$$I_{x'x'} = I_{xx} + Ad^2$$



$$I_{xx} = \int_{\frac{-h}{2}}^{\frac{h}{2}} y^2\, dA = \int_{\frac{-h}{2}}^{\frac{h}{2}} y^2\, b\, dy = \left[\frac{b\, y^3}{2}\right]_{\frac{-h}{2}}^{\frac{h}{2}}$$

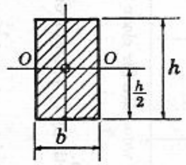
$$I_{xx} = \frac{b\left(\frac{h}{2}\right)^3}{2} - \frac{b\left(-\frac{h}{2}\right)^3}{2}$$

$$I_{xx} = \frac{b\, h^3}{12}$$

$$I_{x'x'} = \frac{bh^3}{12} + bh\frac{h^2}{4} = \frac{bh^3}{3}$$

AREAS AND MOMENTS OF INERTIA OF AREAS AROUND CENTROIDAL AXES

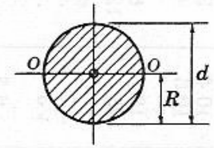
RECTANGLE



$A = bh$

$I_o = \frac{bh^3}{12}$

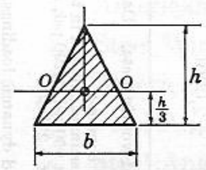
CIRCLE



$A = \pi R^2$

$I_o = \frac{J}{2} = \frac{\pi R^4}{4}$

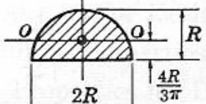
TRIANGLE



$A = \frac{bh}{2}$

$I_o = \frac{bh^3}{36}$

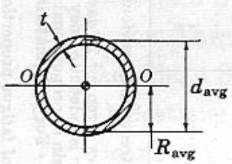
SEMICIRCLE



$A = \frac{\pi R^2}{2}$

$I_o = 0.110 R^4$

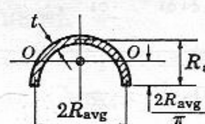
THIN TUBE



$A = 2\pi R_{avg} t$

$I_o = \frac{J}{2} \doteq \pi R_{avg}^3 t$

HALF of THIN TUBE

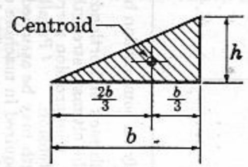


$A = \pi R_{avg} t$

$I_o \doteq 0.095 \pi R_{avg}^3 t$

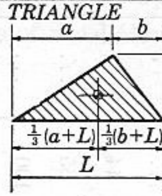
AREAS AND CENTROIDS OF AREAS

TRIANGLE



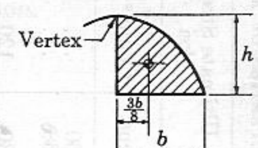
$A = \frac{bh}{2}$

TRIANGLE



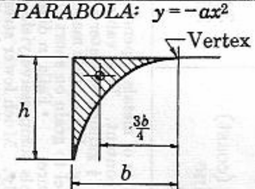
$A = \frac{hL}{2}$

PARABOLA



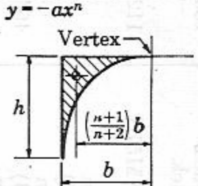
$A = \frac{2bh}{3}$

PARABOLA: $y = -ax^2$



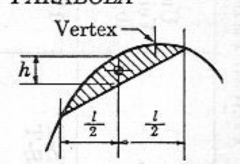
$A = \frac{bh}{3}$

$y = -ax^n$



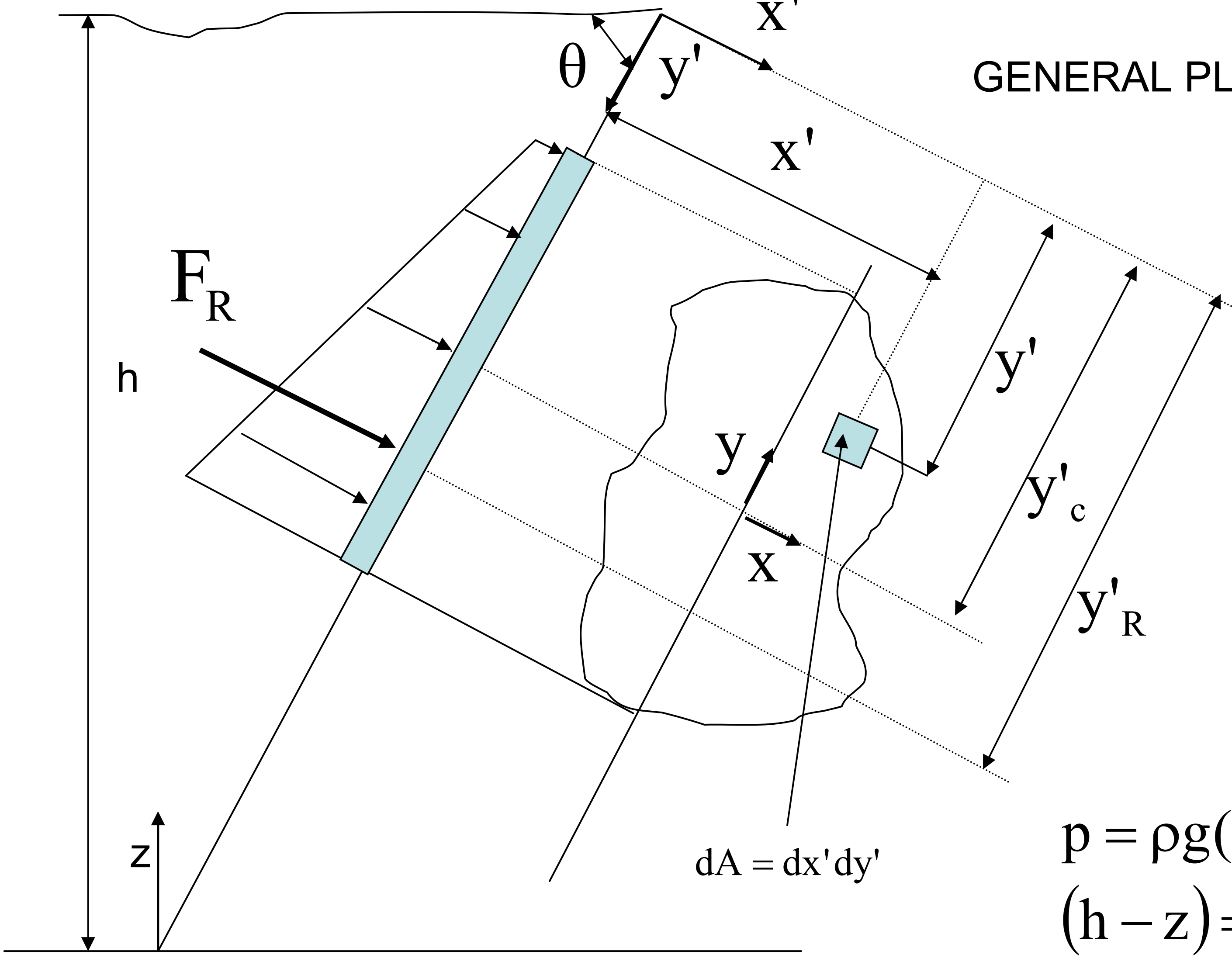
$A = \left(\frac{1}{n+1}\right) bh$

PARABOLA



The area for any segment of a parabola is $A = \frac{2hl}{3}$

GENERAL PLANE SURFACE



$$p = \rho g(h - z)$$

$$\left(h - z\right) = y'\sin\theta$$

GENERAL PLANE SURFACE

$$(h-z) = y'\sin\theta$$

$$p = \rho\, g (h-z) = \rho\, g\, y'\sin\theta$$

$$F_{inside} = \int p\, dA$$

$$F_{inside} = \rho\, g \iint (h - z) dx'\, dy'$$

$$F_{inside} = \rho\, g\, \sin\theta \iint y'\, dx'\, dy'$$

$$\iint y'\, dx'\, dy' = \text{centroid} \times A = y'_{centroid}\, A$$

$$F_{inside} = \rho\, g\, \sin\theta\, y'_{centroid}\, A = p_{centroid} \times A$$

$$M_O = \int \text{moment arm} \times F$$

$$M_O = \iint y' p\, dx'\, dy' = \iint y' \rho\, g\, y' \sin\theta\, dy'\, dx'$$

$$M_O = \rho\, g\, \sin\theta \iint y'^2 dy'\, dx'$$

$\iint y'^2 dy'\, dx'$ is the second moment of area around the x' axis

$$M_O = \rho\, g\, \sin\theta\, I_{x'x'}$$

$$M_{O\,R} = M_{O\,inside}$$

$$p_c\, y'\, A = \rho\, g\, I_{x'x'}\, \sin\theta$$

$$I_{x'x'} = I_{xx} + A\, y_c^2$$

$$p_c y'\, A = \rho\, g\, \sin\theta \left(I_{xx} + A y_c^2\right)$$

$$p_c y'\, A = \rho\, g\, \sin\theta\, I_{xx} + \rho\, g\, \sin\theta\;\; A y_c^2$$

since $p_c = \rho\, g\, y_c \sin\theta$i $\quad \rho\, g\, \sin\theta = \dfrac{p_c}{y_c}$

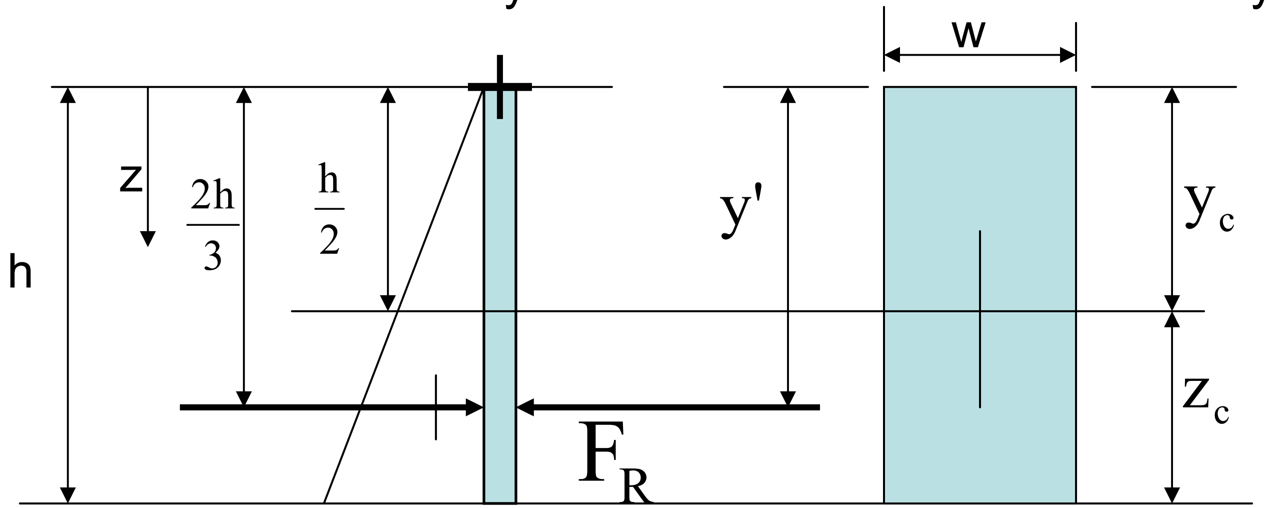
$$p_c y'\, A = \rho\, g\, \sin\theta\, I_{xx} + \frac{p_c}{y_c} A\, y_c^2$$

$$p_c\, y'\, A - p_c\, y_c\, A = \rho\, g\, \sin\theta\, I_{xx}$$

$$y' - y_c = \frac{\rho\, g\, \sin\theta\, I_{xx}}{p_c\, A}$$

Vertical Surface Analysis

General Surface Analysis



$$\sin\theta = 1,$$

$$I_{xx} = \frac{w\,h^3}{12}$$

$$A = w\,h$$

$$p = \rho g z$$

$$F = p_{centroid} A$$

$$F = \rho g \frac{h}{2} wh = w\rho g \frac{h^2}{2}$$

$$M_O = F \times y_{centriod} = w\rho g \frac{h^2}{2}\frac{2h}{3}$$

$$M_O = w\rho g \frac{h^3}{3}$$

$$z_R - z_c = \frac{2h}{3} - \frac{h}{2} = \frac{h}{6}$$

$$p = \rho\, g z, \qquad p_c = \frac{\rho\, g\, h}{2}$$

$$F = p_{centroid} \times A = \frac{\rho\, g\, h}{2} wh = w\rho\, g \frac{h^2}{2}$$

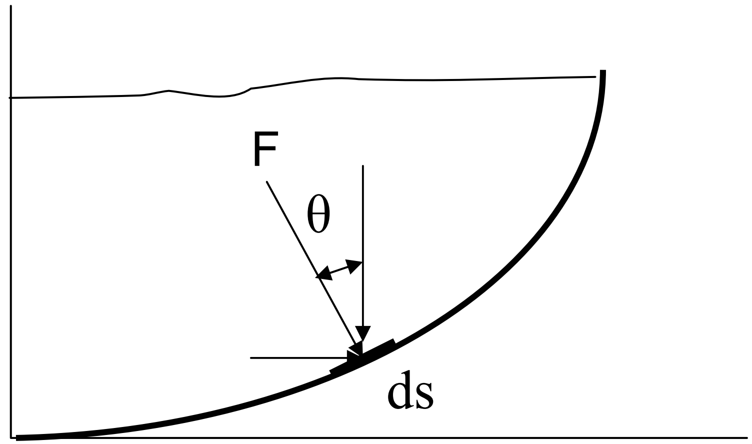
$$M_O = \rho\, g \sin\theta\, I_{x'x'} = \rho\, g (I_{xx} + Ad^2)$$

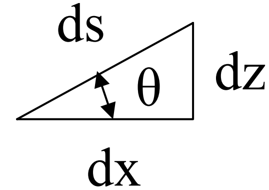
$$M_O = \rho\, g \frac{wh^3}{12} + wh\frac{h^2}{4} = w\rho\, g \frac{h^3}{3}$$

$$y' - y_c = \frac{\rho\, g \sin\theta\, I_{xx}}{p_c\, A}$$

$$y' - y_c = \rho\, g \frac{w\,h^3}{12} \frac{1}{\frac{\rho\, g\, h}{2}} \frac{1}{w\,h} = \frac{h}{6}$$

FORCES ON A CURVED SURFACE





$$dx = ds\cos\theta$$

$$dz = ds\sin\theta$$

$$dF = pwds = w\rho g(h - z)ds$$

HORIZONTAL FORCE

$$F_x = F\sin\theta = w\,\rho\,g\int(h - z)\sin\theta\,ds$$

$$F_x = w\,\rho\,g\int(h - z)\,dz$$

$$F_x = \text{force on a vertical surface}$$

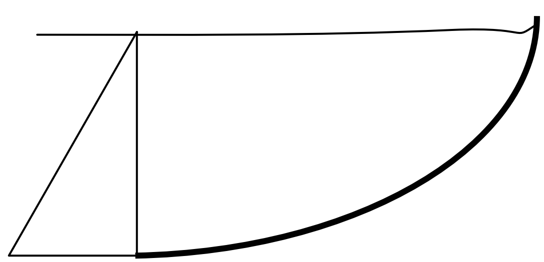
VERTICAL FORCE

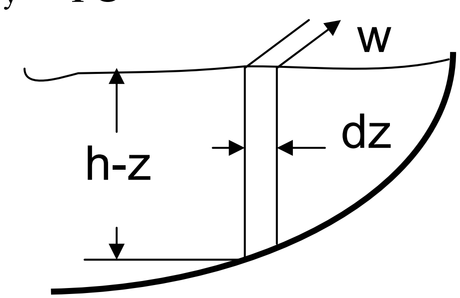
$$F_y = F\cos\theta = w\,\rho\,g\int(h - z)\cos\theta\,ds$$

$$F_y = w\,\rho\,g\int(h - z)\;dx$$

$$F_y = \int(p\,g)\times(w\,(h - z)\,dx)$$

$$F_y = pg \times \text{volume above surfce}$$





VERTICAL FORCE LOCATION

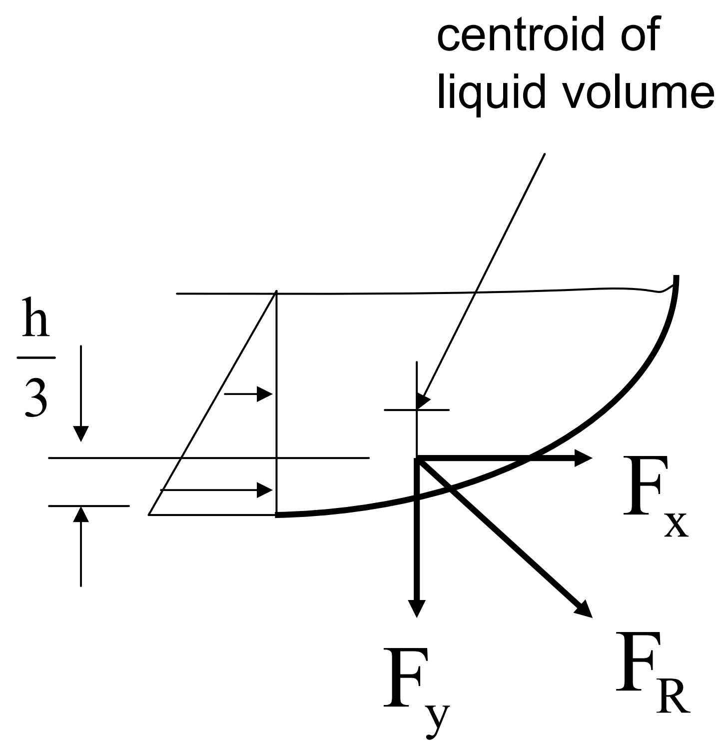
$$M_O = -\int x\, p\, w\, dx = -\int_0^L x\, \rho\, g\, (h - z)\, w\, dx$$

$$M_O = -\rho\, g \int_0^L x \times (h - z)\, w\, dx = \rho\, g \int_0^L x\, d\text{Volume}$$

$$M_O = -\rho\, g\, \text{Volume} x_c$$

$$M_O = -F_z x_c$$

the vertical force component acts along the x coordinate direction of the centroid of the volume of fluid abovethe surface



RESULTANT

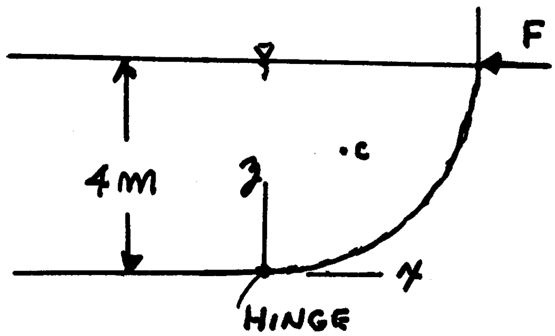
the horizontal force acts through the centroid of the pressure – depth diagrm

$$F_x = \rho\, g \frac{h}{2} wh$$

$$M_O = -\frac{h}{3} F_x - x_c |F_z|$$

$$|F_z| = \rho\, g\, \text{Volume}$$

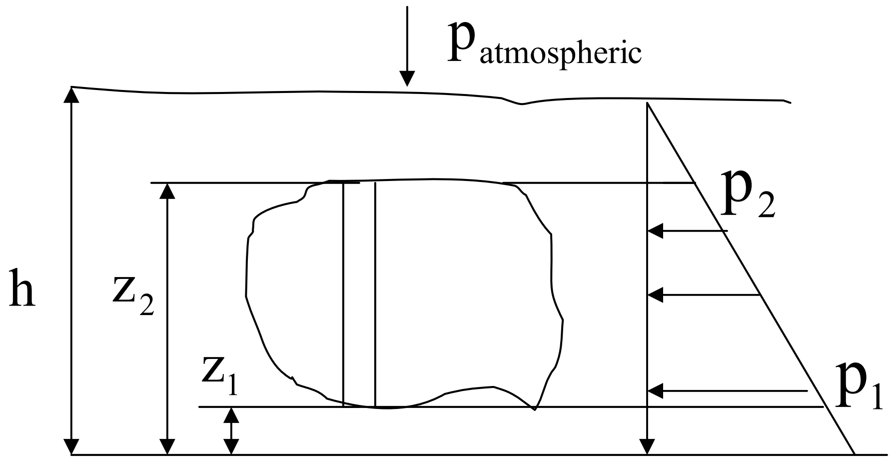
EXAMPLE: FIND THE FORCE TO HOLD THE QUARTER-CIRCULAR WALL



WIDTH = 3 m

$x_c = 0.424\ R$

$\gamma_{H_2O} = 9800\ N/m^2$



BOUYANCY

From the Hydrostatic Equation a pressure difference exists between the top and bottom of a submerged body

$$dF_z = (p_1 - p_2)dA_z = (p_a + \rho g(h - z_1)) - (p_a + \rho g(h - z_2))$$

$$dF_z = \rho g(z_2 - z_1)dA_z = \rho g \text{Volume}_{\text{object}}$$

$$F_{\text{bouyancy}} = \text{weight of fluid displacd}$$

Momment about y axis

$$dM = xFdy = x \times \rho\, g\, dV$$

$$M = \rho g \int x dV = \rho\, g\, x_c V = x_{\text{centroid}} F_y$$

The bouyant force acts through the centroid of the body volume

2.42 A float and lever system is used to open a drain valve, as shown in Figure P2.42. The float has a volume $\forall$ and a density ρ_f. The density of the water is ρ_w. Find the maximum force available to open the drain valve, given that the hinge is frictionless. Hint: first consider the forces acting on the float, then consider the free-body diagram of the gate.

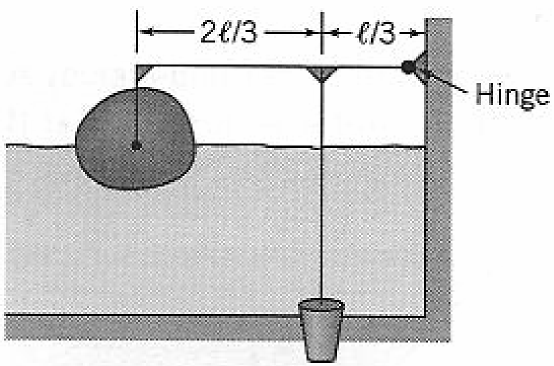


FIGURE P2.42

maximum force with ball submerged

$$F_{float} = V\,\rho_w\,g - V\,\rho_{float}\,g$$

$$M_{Ofloat} = F_{float} \times L = L\,V\,g\left(\rho_w - \rho_{float}\right)$$

$$M_{valve} = F_{float} \times \frac{L}{3}$$

$$M_{Ofloat} = M_{valve}$$

$$F_{float} \times \frac{L}{3} = L\,Vg\left(\rho_w - \rho_{float}\right)$$

$$F_{\substack{float\\ maximum}} = 3\,V\,g\left(\rho_w - \rho_{float}\right)$$

2.35 A rectangular barge of length L floats in water (density ρ_w) and when it is empty, it is immersed to a depth D, as shown in Figure P2.35. Oil of density ρ_o is slowly poured into the barge until it is about to sink. Find a relationship for the depth of oil at this point in terms of H, D, ρ_w, and ρ_o.

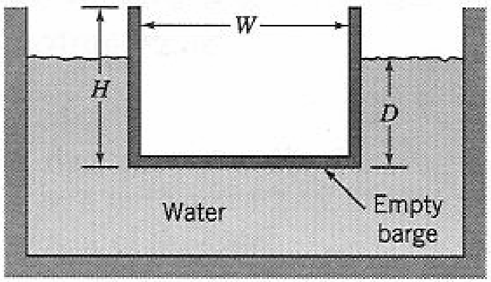


FIGURE P2.35

$$\text{barge weight} = L\,W\,D\,\rho_w g$$

$$\text{"sinking" barge weight} = L\,W\,H\,\rho_w g$$

$$\text{oilweight} = L\,W\,d_{oil}\,\rho_{oil}\,g$$

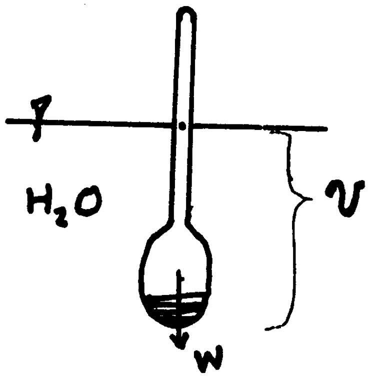
$$\text{"sinkging" weight} = \text{barge wieght} + \text{oil weight}$$

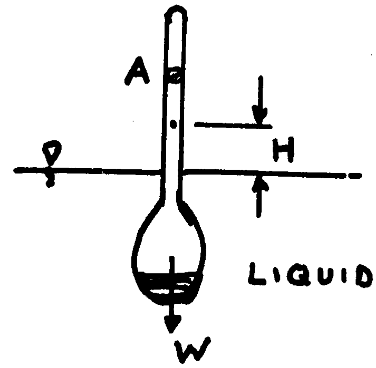
$$L\,W\,H\,\rho_w\,g = L\,W\,D\,\rho_w\,g + L\,W\,d_{oil}\,\rho_{oil} g$$

$$H\rho_w = D\rho_w + d_{oil}\rho_{oil}$$

$$d_{oil} = \frac{\rho_w}{\rho_{oil}}(H - D)$$

HYDROMETER - A DEVICE FOR MEASURING SPECIFIC GRAVITY





IN H_2O: $\gamma_{H_2O}\mathcal{V} = W$

IN OTHER LIQUID: $\gamma(\mathcal{V} - HA) = W$.

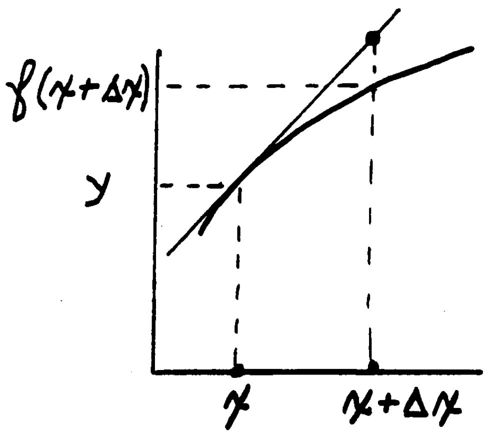
$$W = \gamma(\mathcal{V} - HA) = \gamma_{H_2O}\mathcal{V}$$

$$\text{SPECIFIC GRAVITY} = \frac{\gamma}{\gamma_{H_2O}}$$

$$\frac{\gamma}{\gamma_{H_2O}} = \frac{1}{1 - \frac{AH}{\mathcal{V}}}$$

REVIEW OF DERIVATIVES

ORDINARY DERIVATIVE $y = f(x)$



$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \text{slope} = y'(x)$$

TO APPROXIMATE y at $x+\Delta x$

$$y(x+\Delta x) \triangleq y(x) + y'(x)\,\Delta x$$

= 2 term Taylor Series

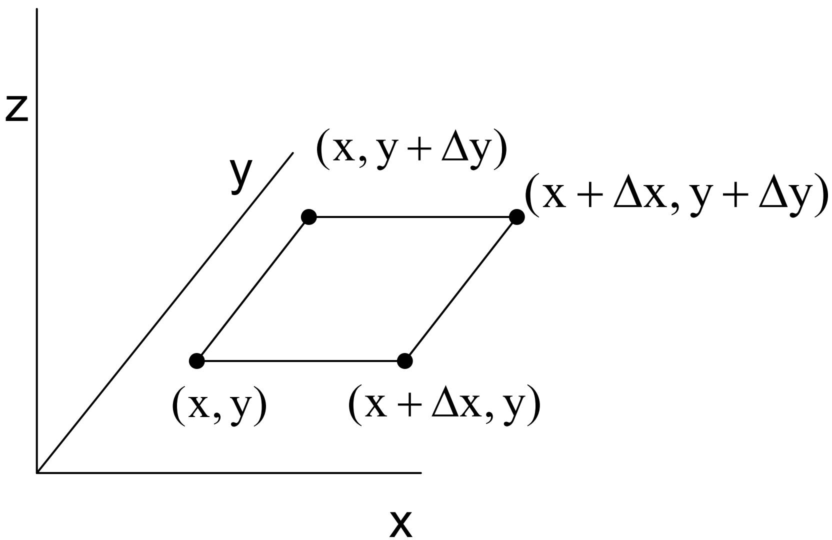
PARTIAL DERIVATIVE $z = f(x, y)$

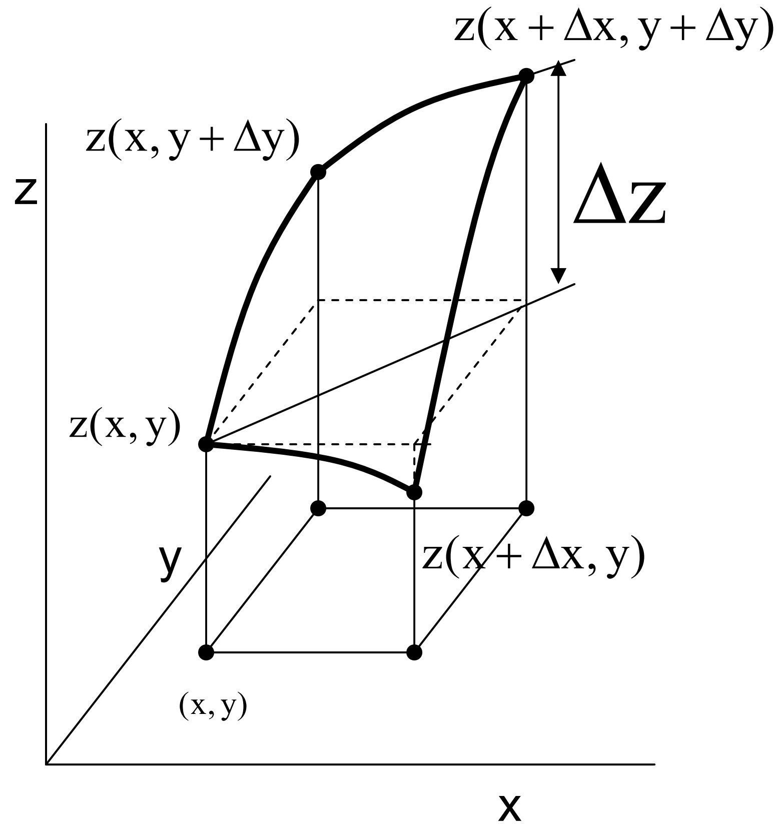
$$\frac{\partial z}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x, y)}{\Delta x} \quad y \text{ const.}$$

$$\frac{\partial z}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x, y)}{\Delta y} \quad x \text{ const.}$$

$$z(x+\Delta x, y+\Delta y) = z(x, y) + \frac{\partial z(x,y)}{\partial x}\Delta x + \frac{\partial z(x,y)}{\partial y}\Delta y$$

Exact Differential





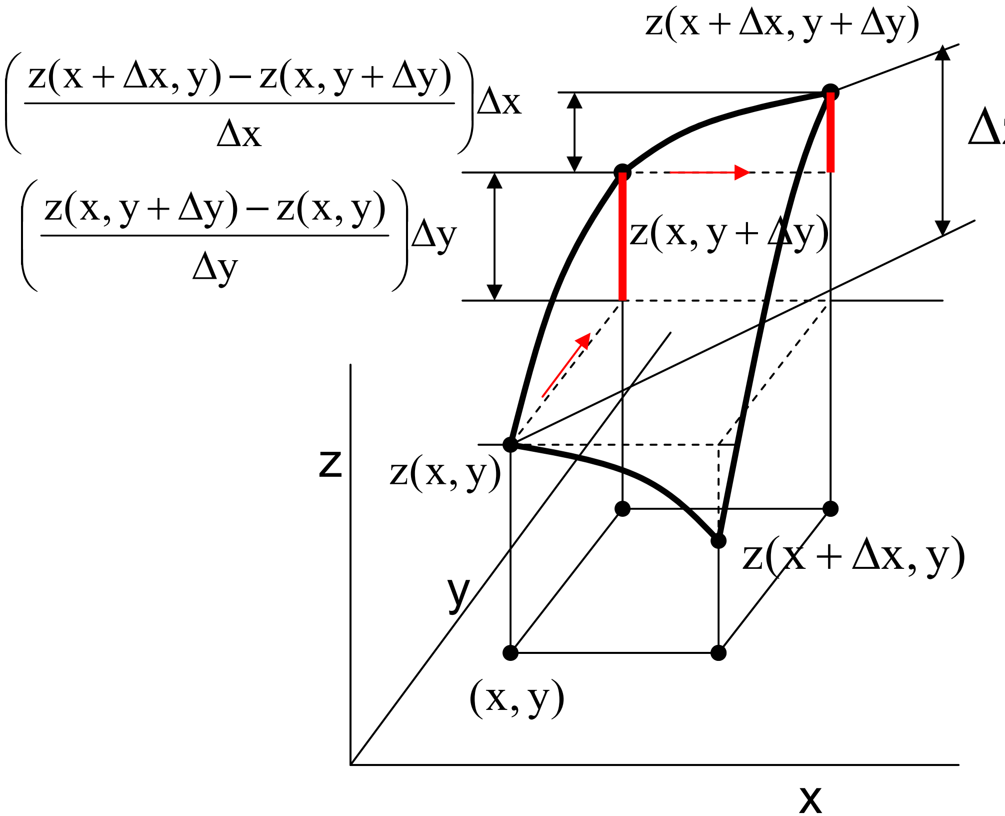
Exact Differential

Theorm :

If the function $z = f(x, y)$ has continuous first partial deriviatives in the domain, then the function z has a differential

$$dz = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y$$

at every point in the domain.



$$\Delta z = \left(\frac{z(x+\Delta x, y) - z(x, y+\Delta y)}{\Delta x}\right)\Delta x + \left(\frac{z(x, y+\Delta y) - z(x, y)}{\Delta y}\right)\Delta y$$

$$\operatorname*{limit}_{\Delta=0} \frac{z(x+\Delta x, y) - z(x, y)}{\Delta x} = \left(\frac{\partial z}{\partial x}\right)_{y=\text{const}}$$

$$dz = \left(\frac{\partial z}{\partial x}\right)_{y=\text{const}} dx + \left(\frac{\partial z}{\partial y}\right)_{x=\text{const}} dy$$