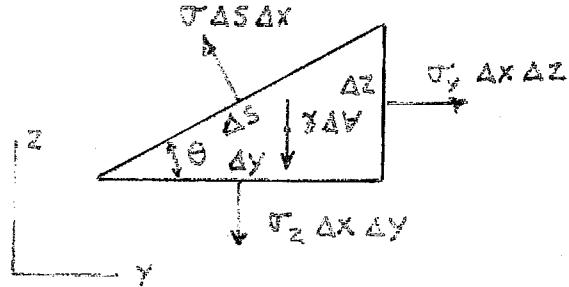


Fluid Statics

Pressure on Inclined Plane



$$\sum F_y = -\sigma_s \Delta s \Delta x \sin \theta + \sigma_y \Delta z \Delta x = 0$$

$$\sum F_z = \sigma_s \Delta s \Delta x \cos \theta - \sigma_z \Delta y \Delta x - \gamma \Delta V = 0$$

$$\sigma_s \sin \theta = \sigma_y \Delta z / \Delta s$$

$$\sigma_s \cos \theta = \sigma_z \Delta y / \Delta s - \gamma \frac{1}{2} \Delta y \Delta z / \Delta s$$

$$\Delta V = \Delta x \Delta y \Delta z / 2 \quad \cos \theta = \Delta y / \Delta s \quad \sin \theta = \Delta z / \Delta s$$

$$\sigma_s = \sigma_y$$

$$\sigma_s = \sigma_z + \frac{1}{2} \gamma \Delta s \sin \theta$$

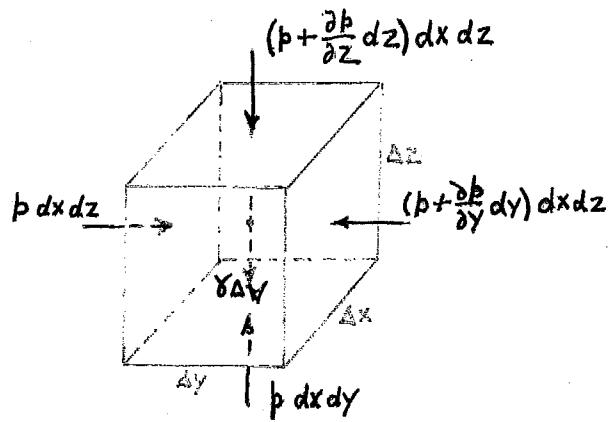
Letting $\Delta s \rightarrow 0$ while keeping the orientation the same

$$\sigma_s = \sigma_y = \sigma_z$$

and all the stresses are equal

$$\sigma_s = \sigma_y = \sigma_z = -p$$

Hydrostatic Equation



The vertical pressure change is

$$dp = \frac{dp}{dz} dz = \frac{\partial p}{\partial z} dz$$

For equilibrium

$$\sum F_x = pdydz - (p + \frac{\partial p}{\partial x} dx)dydz = 0$$

$$\sum F_y = pdxdz - (p + \frac{\partial p}{\partial y} dy)dxdz = 0$$

$$\sum F_z = pdxdy - (p + \frac{\partial p}{\partial z} dz)dxdy - \gamma dxdydz = 0$$

$$\frac{\partial p}{\partial x} = 0, \quad \frac{\partial p}{\partial y} = 0, \quad \text{and} \quad \frac{\partial p}{\partial z} = -\gamma$$

The pressure doesn't change with x and y, i.e. the pressure is constant on horizontal surfaces.

The pressure only changes with z

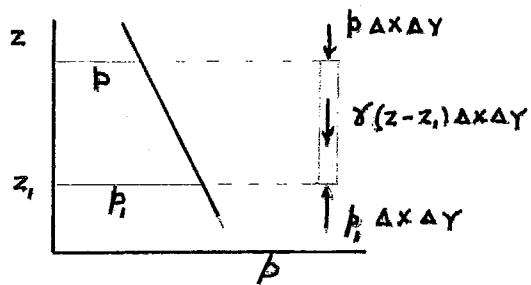
$$\frac{dp}{dz} = -\gamma$$

Pressure Distributions

Constant specific weight

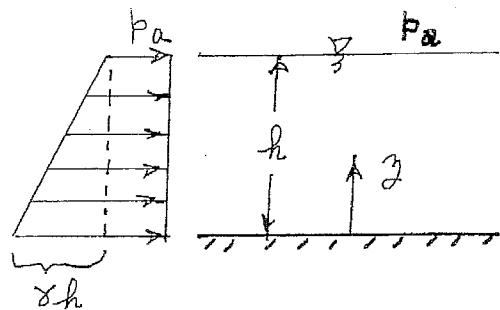
$$\int_{p_1}^p dp = -\gamma \int_{z_1}^z dz$$

$$p - p_1 = -\gamma(z - z_1) \quad p = p_1 \text{ at } z = z_1$$



Hydrostatic Pressure Distribution

$$p_1 - p = \gamma(z - z_1) = 998 N/m^3 (10) m = 9880 N/m^2$$



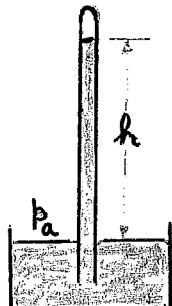
Atmospheric and Liquid Pressures

$$p = p_a + \gamma(h - z) \quad p_a = 101,000 N/m^2 = 101 kPa$$

$$p_0 = p_a + \gamma h \quad \text{at } z=0$$

$$p = p_a + \gamma \text{ depth}$$

Atmospheric Pressure



Barometer

$$p_a = p_v + \gamma h$$

$$p_v = 1380 \text{ Pa} \text{ for mercury}$$

$$h = (p_a - p_v) / \gamma = (101000 - 1380) / (13.6 \times 9810) = 0.747 \text{ m} = 747 \text{ mm}$$

$$\text{For water } p_v = 2760 \text{ N/m}^2$$

$$h = (p_a - p_v) / \gamma = (101000 - 2760) / 9810 = 10.0 \text{ m}$$

For a gas with variable specific weight

$$\frac{dp}{dz} = -\gamma = -\rho g = -\frac{p}{RT} g \quad p = \rho RT$$

$$\frac{dp}{p} = -\frac{g}{RT} dz$$

For constant temperature

$$\ln p \Big|_{p_1}^p = -\frac{g}{RT}(z - z_1) \text{ and } \frac{p}{p_1} = e^{-\frac{g}{RT}(z - z_1)}$$

For $\Delta z = 10 \text{ m}$, $\Delta p = -0.12\%$ Pressure \sim constant

Atmospheric Pressure Variation

Linear temperature variation in the troposphere, $0 < z < 11\text{km}$

$$T = T_1 + m(z - z_1)$$

$$\frac{dp}{p} = -\frac{g}{RT} dz = -\frac{gdz}{R[T_1 + m(z - z_1)]}$$

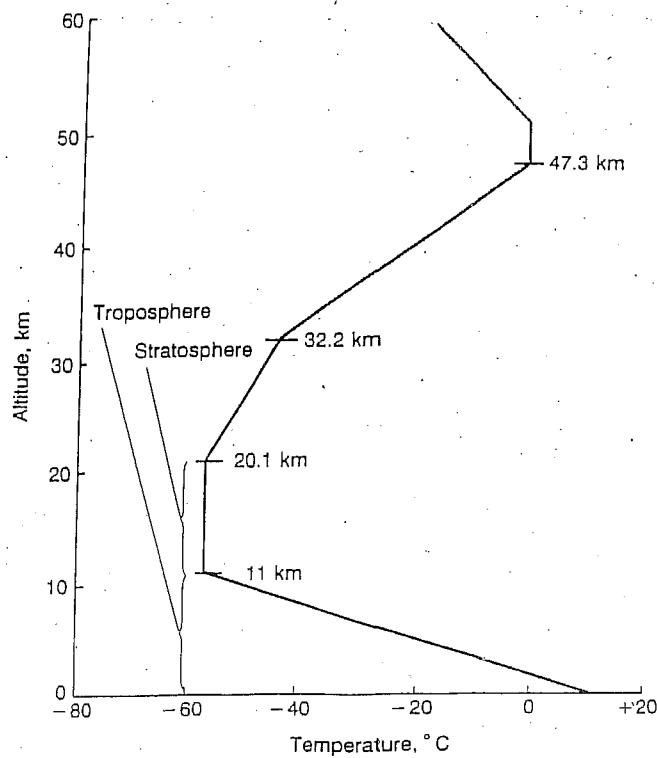
$$\frac{p}{p_1} = \left[\frac{T_1 + m(z - z_1)}{T_1} \right]^{-g/mR}$$

For the troposphere $z \leq 11.0\text{km}$

$$T = 288 - 6.5z \quad p = 101300(1 - 0.02257z)^{5.25}$$

For the stratosphere $11.0 \leq z \leq 20.0\text{m}$ $T_1 = -56.5^{\circ}\text{C} = 216.5^{\circ}\text{K}$

$$p = 22630e^{0.1577(z-11.0)} \quad 11.0 \leq z \leq 20.0$$

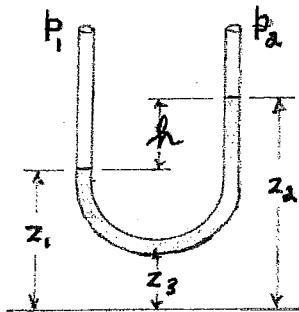


Properties of the U.S. Standard Atmosphere (SI Units)^a

Altitude (m)	Temperature (°C)	Acceleration of gravity, <i>g</i> (m/s ²)	Pressure, <i>p</i> [N/m ² (abs)]	Density, <i>ρ</i> (kg/m ³)	Dynamic viscosity, <i>μ</i> (N·s/m ²)
-1,000	21.50	9.810	1.139 E + 5	1.347 E + 0	1.821 E - 5
0	15.00	9.807	1.103 E + 5	1.225 E + 0	1.789 E - 5
1,000	8.50	9.804	8.988 E + 4	1.112 E + 0	1.758 E - 5
2,000	2.00	9.801	7.950 E + 4	1.007 E + 0	1.726 E - 5
3,000	-4.49	9.797	7.012 E + 4	9.093 E - 1	1.694 E - 5
4,000	-10.98	9.794	6.166 E + 4	8.194 E - 1	1.661 E - 5
5,000	-17.47	9.791	5.405 E + 4	7.364 E - 1	1.628 E - 5
6,000	-23.96	9.788	4.722 E + 4	6.601 E - 1	1.595 E - 5
7,000	-30.45	9.785	4.111 E + 4	5.900 E - 1	1.561 E - 5
8,000	-36.94	9.782	3.565 E + 4	5.258 E - 1	1.527 E - 5
9,000	-43.42	9.779	3.080 E + 4	4.671 E - 1	1.493 E - 5
10,000	-49.90	9.776	2.650 E + 4	4.135 E - 1	1.458 E - 5
15,000	-56.50	9.761	1.211 E + 4	1.948 E - 1	1.422 E - 5
20,000	-56.50	9.745	5.529 E + 3	8.891 E - 2	1.422 E - 5
25,000	-51.60	9.730	2.549 E + 3	4.008 E - 2	1.448 E - 5
30,000	-46.64	9.715	1.197 E + 3	1.841 E - 2	1.475 E - 5
40,000	-22.80	9.684	2.871 E + 2	3.996 E - 3	1.601 E - 5
50,000	-2.50	9.654	7.978 E + 1	1.027 E - 3	1.704 E - 5
60,000	-26.13	9.624	2.196 E + 1	3.097 E - 4	1.584 E - 5
70,000	-53.57	9.594	5.221 E + 0	8.283 E - 5	1.438 E - 5
80,000	-74.51	9.564	1.052 E + 0	1.846 E - 5	1.321 E - 5

^a Data from *U.S. Standard Atmosphere, 1976*, U.S. Government Printing Office, Washington, D.C.

Manometry and Pressure Measurement



Manometer

$$\text{For the left leg } p_1 - p_3 = -\gamma g(z_1 - z_3)$$

$$\text{For the right leg } p_2 - p_3 = -\gamma g(z_2 - z_3)$$

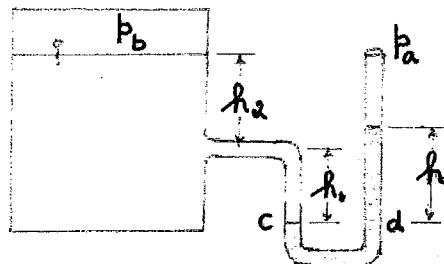
$$p_1 - p_2 = \gamma g(z_2 - z_1) = \gamma g h$$

For oil (specific gravity of 0.7) with $h=10\text{cm}$

$$p_1 - p_2 = 0.7(9810)(0.10) = 687\text{Pa}.$$

$$\text{For } p_2 = p_a \quad p_1 - p_2 = p_1 - p_a = p_{1g} = \gamma g h.$$

Measurement of Pressure in a Tank with Liquid



$$p_c = p_d \text{ where } p_c = p_b + \gamma(h_1 + h_2) \text{ and } p_d = p_a + \gamma g h$$

$$p_b - p_a = p_{bg} = \gamma g h - \gamma(h_1 + h_2)$$