Pressure

Pressure: Force per unit area \((N/m^2)\) acting on a surface. Caused by molecules striking the surface.

\[ F = pA \]

Pressure force
- perpendicular to surface
- acts on the surface
\[
d\vec{F} = (\hat{n})pdA \quad \hat{n} = \text{unit normal}
\]
\[ \vec{F} = - \int p\hat{n}dA \]

Pressure acts in the fluid:

Fluid and Container

Container

Fluid

Fluid in Two Parts

Pressure in Moving Fluid:
Static Pressure = Pressure relative to moving fluid.
Equation of State

From thermodynamics

\[ \rho = f(p, T) \]

\[ d\rho = (\frac{\partial \rho}{\partial p})_T dp + (\frac{\partial \rho}{\partial T})_p dT \]

\[ \frac{1}{\rho} \frac{\partial \rho}{\partial p} = \frac{1}{(m/\forall)} \frac{\partial (m/\forall)}{\partial p} = -\frac{1}{\forall} \frac{\partial \forall}{\partial p} = \kappa = \text{Compressibility} \]

\[ -\frac{1}{\rho} \frac{\partial \rho}{\partial T} = \frac{1}{(m/\forall)} \frac{\partial (m/\forall)}{\partial T} = \frac{1}{\forall} \frac{\partial \forall}{\partial T} = \beta = \text{Thermal Expansion} \]

\[ \frac{d\rho}{\rho} = \kappa dp - \beta dT \quad \frac{d\forall}{\forall} = -\kappa dp + \beta dT \]

Water: \( \kappa = 5.8 \times 10^{-10} / (N/m^2) \)

\( \beta = 1.48 \times 10^{-4} / ^0K \)

\( \Delta p = 190 \text{ atm}, \quad \frac{d\rho}{\rho} = \kappa dp = 5.8 \times 10^{-10} (190)(101000) = 0.01 \)

\( \Delta T = 100^0K, \quad \frac{d\rho}{\rho} = -\beta dT = -1.48 \times 10^{-4} (100) = -0.015 \)

Liquid Water is Incompressible for most applications.
Gases

Perfect Gas: \( p = \rho RT \) where \( R = \) specific gas constant

Air: \( R = 287 N \cdot m/kg \cdot ^\circ C \)

Standard sea level conditions:
\[
p = 101,000 N/m^2 \quad T = 15^\circ C
\]
\[
\rho = \frac{p}{RT} = \frac{101000}{287(15+273)} = 1.22 kg/m^3
\]

\[
\kappa = \frac{1}{\rho} \frac{\partial \rho}{\partial p} = \frac{RT}{p} \frac{\partial (p/RT)}{\partial p} = \frac{1}{p}
\]
\[
\beta = -\frac{1}{\rho} \frac{\partial \rho}{\partial T} = -\frac{RT}{p} \frac{\partial (p/RT)}{\partial T} = \frac{1}{T}
\]

\[
\frac{d\rho}{\rho} = \kappa dp - \beta dT = \frac{dp}{p} - \frac{dT}{T}
\]

At standard sea level conditions
\[
\kappa = 1/101000 = 9.9 \times 10^{-6}/(N/m^2), \quad \beta = 1/288 = 3.47 \times 10^{-3}/^\circ C
\]

For an isotopic process, \( p \propto \rho^\gamma \) or \( \frac{dp}{p} = \gamma \frac{d\rho}{\rho} \)

With the momentum equation \( dp = -\rho VdV \)
\[
\frac{d\rho}{\rho} = -\frac{1}{\gamma p} \rho VdV = -\frac{\rho}{2\gamma p} dV^2
\]

For a 2% change in density, we get \( V = 68.6 m/s \).
Air is essentially incompressible for \( V < 70 m/s \).
Flow Field Characteristics

Velocity Vector: $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$

$V = \sqrt{u^2 + v^2 + w^2}$

- $u = V \cos \alpha$
- $v = V \cos \beta$
- $w = V \cos \gamma$

Steady flow: Velocity at each point in space is constant
$\vec{V} = \vec{V}(x, y, z)$

Unsteady flow: $\vec{V} = \vec{V}(x, y, z, t)$
Examples: Oscillating flows
- Start up process
- Moving vehicle/fixed observer
- Observer moving with a vehicle
  at constant velocity = steady

Velocity measurement from displacement of small particles
Particle trajectories: \( \frac{dx}{dt} = u, \frac{dy}{dt} = v, \frac{dz}{dt} = w \)

Flow Visualization:
- Particles on a liquid surface
- Dye streams in liquids
- Smoke streams in gases

One and Two dimensional flows

![Figure 1.7 Two Dimensional Flow](image1.png)

![Figure 1.8 One Dimensional Flow](image2.png)

Shear stress and friction

\[
\tau = \lim_{\Delta t \to 0} \frac{\Delta F}{\Delta A}
\]

![Figure 1.13 Shear force](image3.png)

![Figure 1.14 Relative motion and shear stress](image4.png)
Particle Path

Patch

No patch
FUNDAMENTAL LAWS
FOR A SYSTEM

SYSTEM: A defined quantity of matter

Conservation of Matter:

\[ \frac{D}{Dt} \text{ (mass)} = 0 \]

Newton's Second Law:

\[ \frac{D}{Dt} \text{ (momentum)} = \text{Sum of Forces} \]

\[ = \text{Pressure} + \]
\[ \text{Gravity} + \]
\[ \text{Friction} \]

First Law of Thermodynamics:

\[ \frac{D}{Dt} \text{ (energy)} = \text{Heat Transfer} + \]
\[ \text{Work Done} \]

Energy = Internal + Kinetic
APPROACHES

Analytical: Fundamental Understanding
            Preliminary Design

Experimental: Fundamental Understanding
              Proof of Concept
              Model Testing
              Prototype Testing

Computational: Fundamental Understanding
              Design and Testing