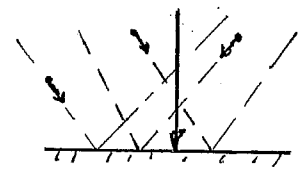
Pressure

Pressure: Force per unit area (N/m^2) acting on a surface. Caused by molecules striking the surface.

F=pA

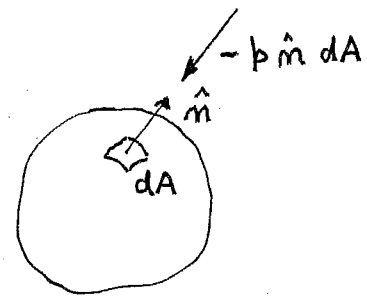


Pressure force

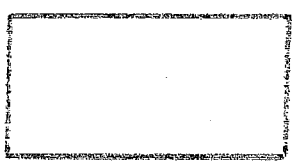
- -perpendicular to surface
- -acts on the surface

$d\vec{F} = (-\hat{n})pdA \quad \vec{n}$ = unit normal

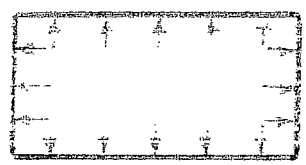
$\vec{F} = -\int_A p\hat{n}dA$



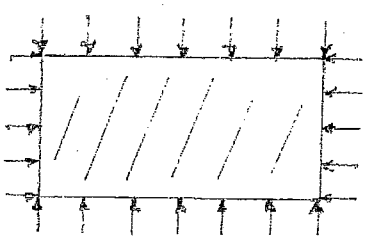
Pressure acts in the fluid:



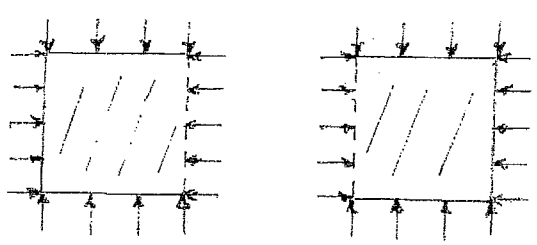
Fluid and Container



Container



Fluid



Fluid in Two Parts

Pressure in Moving Fluid:

Static Pressure = Pressure relative to moving fluid.

Equation of State

From thermodynamics

$$\rho = f(p,T)$$

$$d\rho = \frac{\partial \rho}{\partial p})_T + \frac{\partial \rho}{\partial T})_p$$

$$\frac{1}{\rho}\frac{\partial \rho}{\partial p})_T = \frac{1}{(m/\forall)}\frac{\partial (m/\forall)}{\partial p})_T = -\frac{1}{\forall}\frac{\partial \forall}{\partial p})_T = \kappa = Compressibility$$

$$-\frac{1}{\rho}\frac{\partial \rho}{\partial T})_p = -\frac{1}{(m/\forall)}\frac{\partial (m/\forall)}{\partial T})_p = \frac{1}{\forall}\frac{\partial \forall}{\partial T})_p = \beta = Thermal\ Expansion$$

$$\frac{d\rho}{\rho} = \kappa dp - \beta dT \qquad \frac{d\forall}{\forall} = -\kappa dp + \beta dT$$

Water: $\kappa = 5.8x10^{-10}/(N/m^2)$

$\beta = 1.48x10^{-4}/^0K$

$$\Delta p = 190atm, \quad \frac{d\rho}{\rho} = \kappa dp = 5.8x10^{-10}(190)(101000) = 0.01$$

$$\Delta T = 100^0K, \quad \frac{d\rho}{\rho} = -\beta dT = -1.48x10^{-4}(100) = -0.015$$

Liquid Water is Incompressible for most applications.

Gases

Perfect Gas: $p = \rho RT$ where R = specific gas constant

Air: $R = 287 N \cdot m / kg\ ^0K$

Standard sea level conditions:

$$p = 101{,}000 N/m^2 \quad T = 15^0 C$$

$$\rho = \frac{p}{RT} = \frac{101000}{287(15+273)} = 1.22 kg/m^2$$

$$\kappa = \frac{1}{\rho}\frac{\partial \rho}{\partial p})_T = \frac{RT}{p}\frac{\partial (p/RT)}{\partial p})_T = \frac{1}{p}$$

$$\beta = -\frac{1}{\rho}\frac{\partial \rho}{\partial T})_p = -\frac{RT}{p}\frac{\partial (p/RT)}{\partial T})_p = \frac{1}{T}$$

$$\frac{d\rho}{\rho} = \kappa dp - \beta dT = \frac{dp}{p} - \frac{dT}{T}$$

At standard sea level conditions

$$\kappa = 1/101000 = 9.9x10^{-6}/(N/m^2), \ \beta = 1/288 = 3.47x10^{-3}/^0K$$

For an isotopic process, $p \propto \rho^\gamma$ or $\frac{dp}{p} = \gamma \frac{d\rho}{\rho}$

With the momentum equation $dp = -\rho V dV$

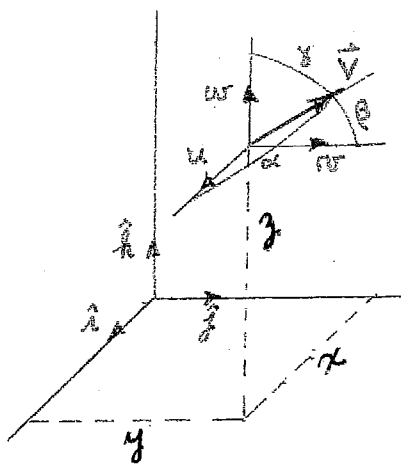
$$\frac{d\rho}{\rho} = -\frac{1}{\gamma p}\rho V dV = -\frac{\rho}{2\gamma p} dV^2$$

For a 2% change in density, we get $V = 68.6 m/s$.

Air is essentially incompressible for $V < 70 m/s$.

Flow Field Characteristics

Velocity Vector: $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$



$$V = \sqrt{u^2 + v^2 + w^2}$$

$$u = V \cos\alpha$$

$$v = V \cos\beta$$

$$w = V \cos\gamma$$

Steady flow: Velocity at each point in space is constant

$$\vec{V} = \vec{V}(x, y, z)$$

Unsteady flow: $\vec{V} = \vec{V}(x, y, z, t)$

Examples: Oscillating flows

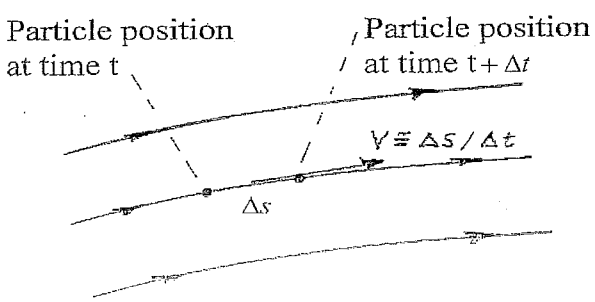
Start up process

Moving vehicle/fixed observer

Observer moving with a vehicle

at constant velocity = steady

Velocity measurement from displacement of small particles



Particle trajectories: $\frac{dx}{dt}=u,\ \frac{dy}{dt}=v,\ \frac{dz}{dt}=w$

Flow Visualization:

Particles on a liquid surface
Dye streams in liquids
Smoke streams in gases

One and Two dimensional flows

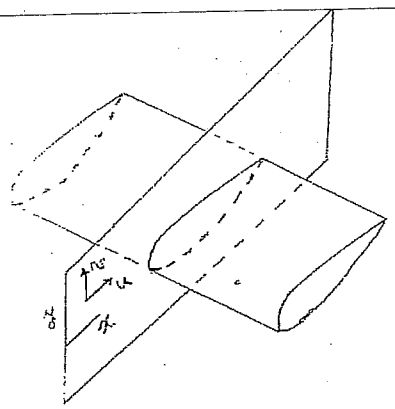


Figure 1.7 Two Dimensional Flow

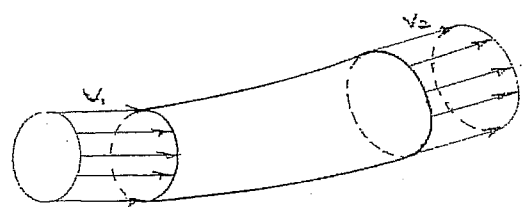


Figure 1.8 One Dimensional Flow

Shear stress and friction

$$\tau=\lim_{\Delta A\to 0}\frac{\Delta F_t}{\Delta A}$$

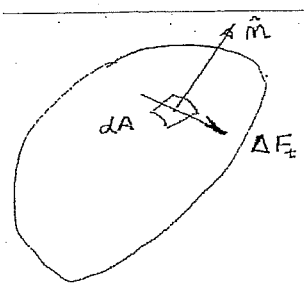


Figure 1.13 Shear force

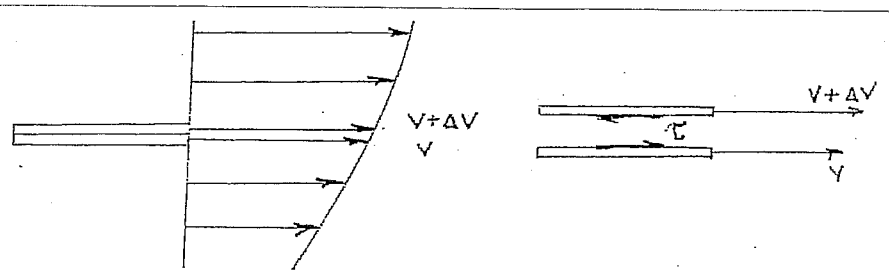
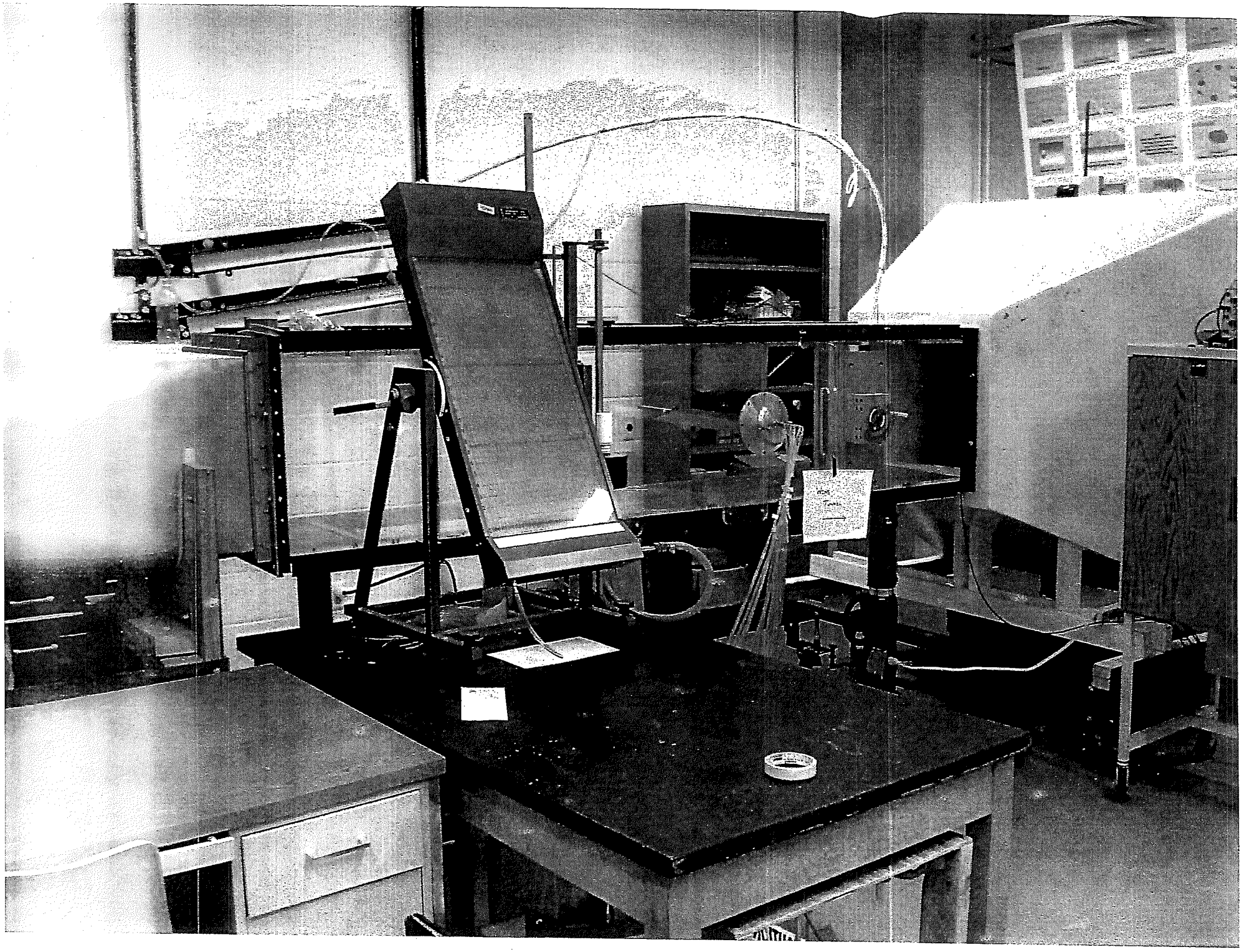
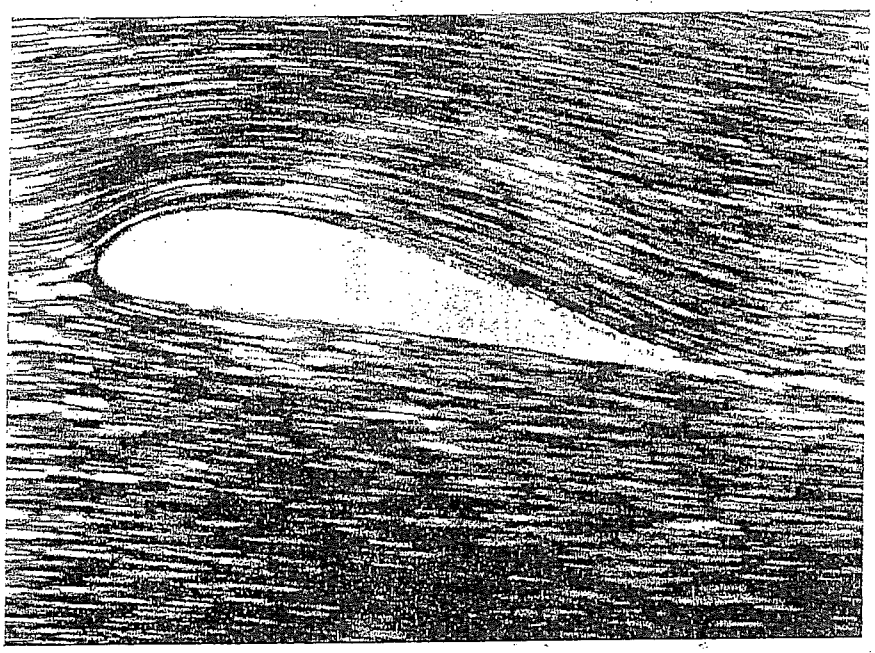
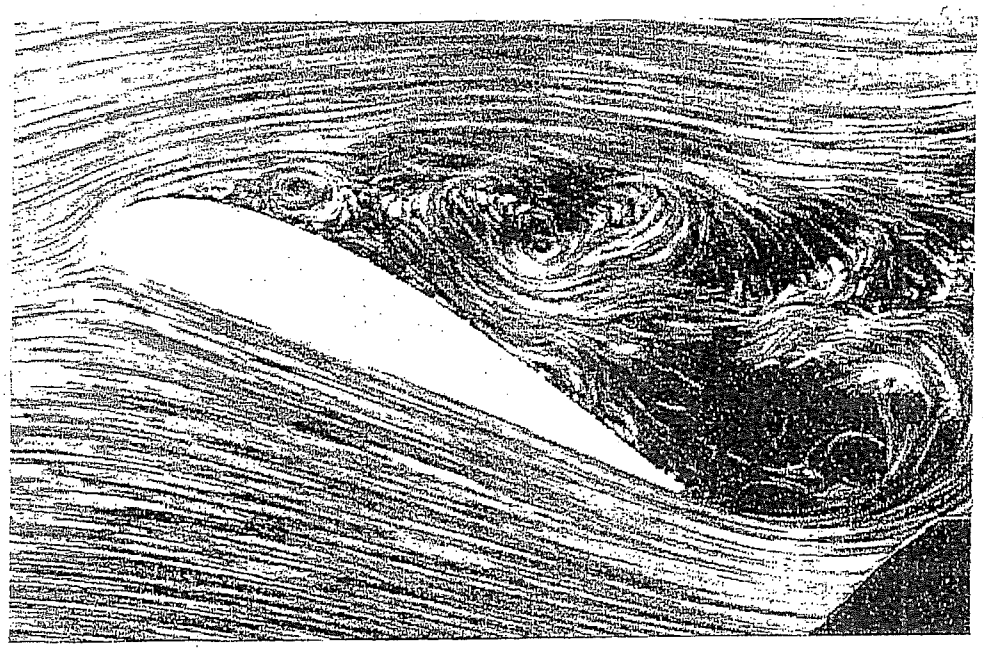


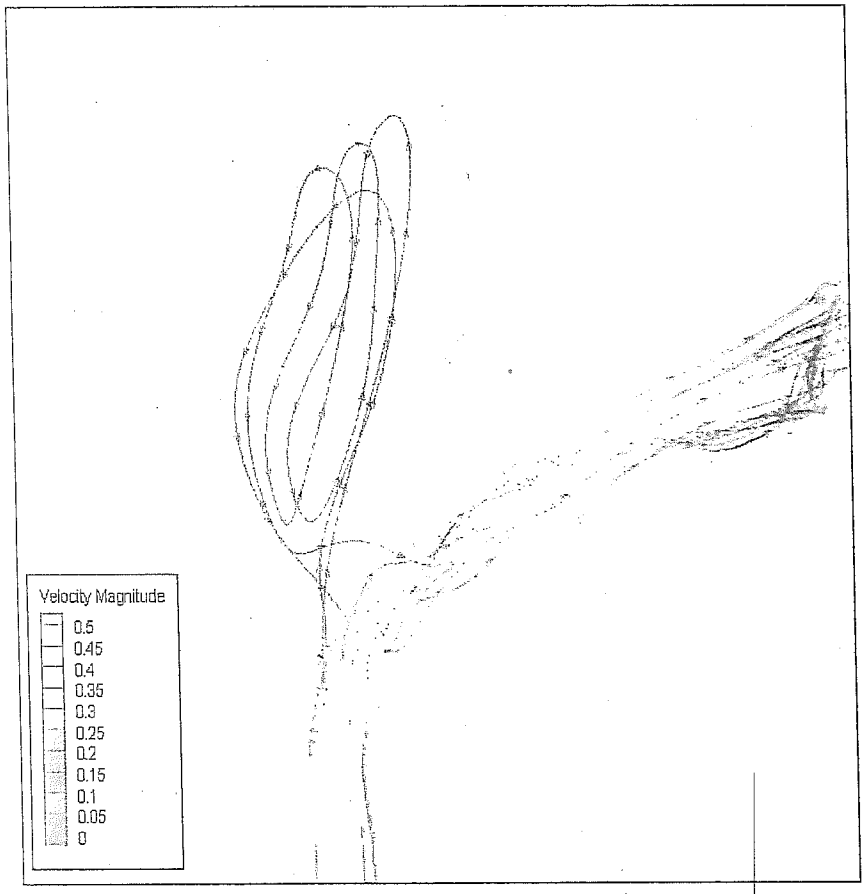
Figure 1.14 Relative motion and shear stress



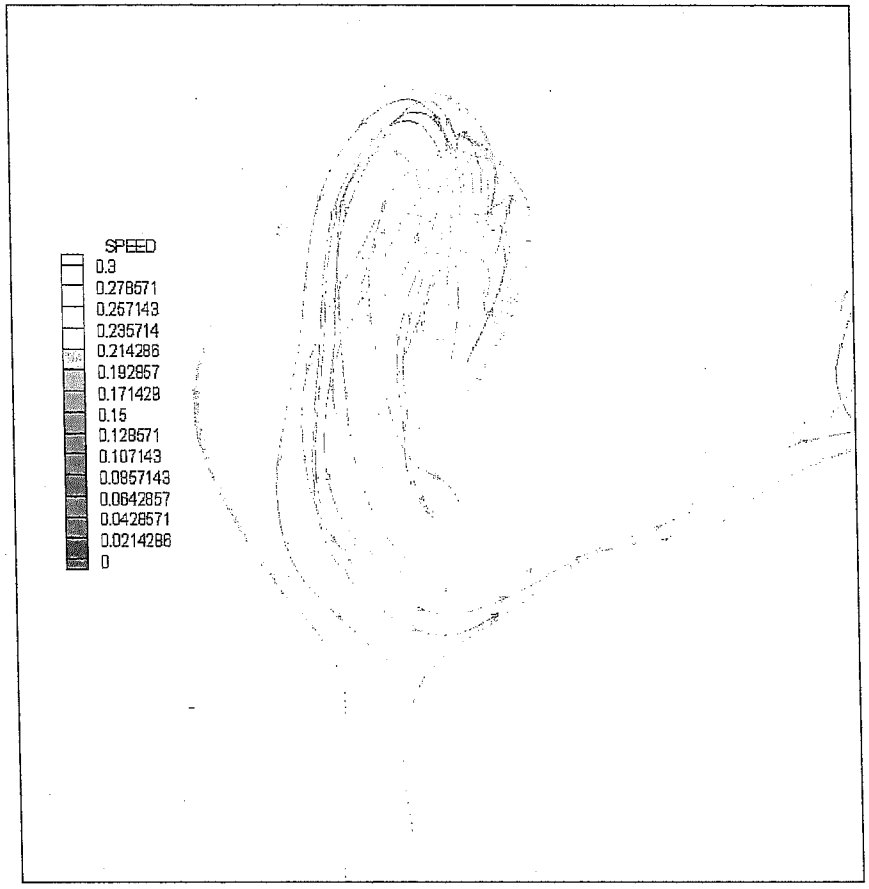




Particle Path



Patch



No patch

FUNDAMENTAL LAWS

FOR A SYSTEM

SYSTEM: A defined quantity of matter

Conservation of Matter:

D/Dt (mass) = 0

Newton's Second Law:

D/Dt (momentum) = Sum of Forces

= Pressure +
Gravity +
Friction

First Law of Thermodynamics:

D/Dt (energy) = Heat Transfer +
Work Done

Energy = Internal + Kinetic

APPROACHES

Analytical:	Fundamental Understanding
	Preliminary Design
Experimental:	Fundamental Understanding
	Proof of Concept
	Model Testing
	Prototype Testing
Computational:	Fundamental Understanding
	Design and Testing