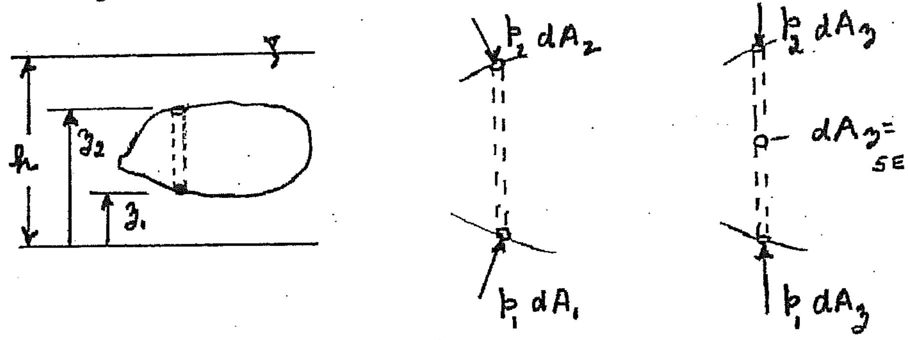
Buoyancy

Upward force on an object caused by increasing pressure with depth.

Submerged Object



Pressure force acts perpendicular to the area

$$d\vec{F}_p = -p\hat{n}dA$$

Vertical component of the pressure force

$$dF_z = d\vec{F}_p \cdot \hat{k} = -p(\hat{n}dA)\cdot\hat{k} = -pdA\cos\theta = -pdA_z$$

Net vertical force

$$\Delta F_z = (p_1 - p_2)dA_z$$

$$p_1 = p_a + \gamma(h - z_1)$$
$$p_2 = p_a + \gamma(h - z_2)$$

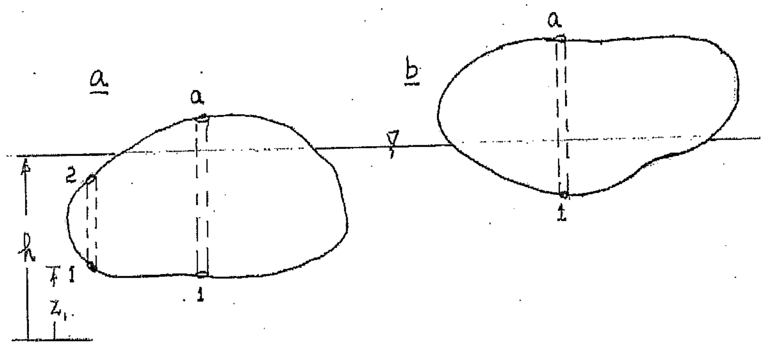
$$\Delta F_z = [\{p_a + \gamma(h - z_1)\} - \{p_a + \gamma(h - z_2)\}]dA_z = \gamma(z_2 - z_1)dA_z$$

$$F_z = \int \gamma(z_2 - z_1)dA_z = \int \gamma d\forall = \gamma\forall$$

A 1m diameter sphere completely submerged in water

$$F_b = \gamma\frac{4}{3}\pi R^3 = 9800\frac{4}{3}\pi(0.5)^3 = 5130N$$

Partially submerged or floating object in a liquid



For figure (a)

$$F_z = \int_{A_z} (p_1 - p_2) dA_z$$

$$F_z = \int_{A_{z2}} (p_1 - p_2) dA_z + \int_{A_{za}} (p_1 - p_a) dA_z$$

$$where\ A_{z2} = top\ area\ wetted\ by\ water$$

$$A_{za} = top\ area\ \text{exp}osed\ to\ air$$

$$p_1 - p_a = \gamma(h - z_1)$$

$$F_z = \gamma \int_{A_{z2}} (z_2 - z_1) dA_z + \gamma \int_{A_{za}} (h - z_1) dA_z$$

$$F_b = \gamma\forall = \gamma\forall_{submerged} = \text{weight of displaced fluid}$$

For figure (b)

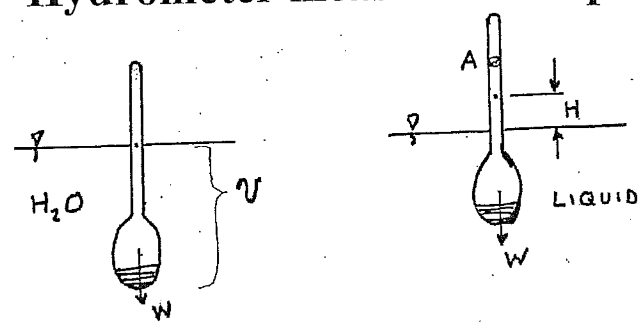
$$F_z = \int_{A_z} (p_1 - p_2) dA_z = \int_{A_{z1}} (p_1 - p_a) dA_z$$

where A_{z1} = bottom area wetted by water

$$p_1 - p_a = \gamma(h - z_1)$$

$$F_z = \int_{A_{z1}} (p_1 - p_a) dA_z = \gamma \int_{A_{z1}} (h - z_1) dA_z = \gamma\forall_{submerged}$$

Example: Hydrometer measures the specific gravity



Water: $\gamma_w \forall = W$ $\quad\forall$ = submerged volume.

Unknown Fluid: $\gamma(\forall - HA) = W$

Equating the weights: $s.g. = \dfrac{\gamma}{\gamma_w} = \dfrac{1}{1 - AH/\forall}$

H determines the specific gravity.

Example: A balloon of volume $50m^3$ is filled with helium at standard sea level temperature and pressure. If the balloon material weighs 225N, what is the tension in a line hold the balloon?

The buoyant force on the balloon in the atmosphere is

$$F_b = \gamma\forall = 1.22(9.82)(50) = 599N$$

The gas constant for helium is $R = 2077 N \cdot m / kg\,^0K$ and its density can be calculated from the perfect gas equation $p = \rho RT$

$$\rho = \frac{p}{RT} = \frac{101000}{2077(288)} = 0.169 kg/m^3$$

Then the total weight of the balloon, helium plus material, is

W=0.169(9.82)(50)+225=308N

Tension in the line = $T = F_b - W = 599 - 308 = 291N$